

Is the Universe a fractal?

Results from the Southern Sky Redshift Survey 2

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Abstract. We perform a fractal analysis of the Southern Sky Redshift Survey 2, following the methods prescribed by Pietronero and collaborators, to check their claim that the galaxy distribution is fractal at all scales, and we discuss and explain the reasons of some controversial points, through tests on both galaxy samples and simulations. We confirm that the amplitude of the two-point correlation function does not depend on the sample depth, but increases with luminosity. We find that there is no contradiction between the results of standard and non-standard statistical analysis; moreover, such results are consistent with theoretical predictions derived from standard CDM models of galaxy formation, and with the hypothesis of homogeneity at large scale ($\sim 100 h^{-1}$ Mpc). However, for our SSRS2 volume-limited subsamples we show that the first zero-point of the autocorrelation function $\xi(s)$ increases linearly with the sample depth, and that its value is comparable to the radius of the maximum sphere which can be completely included in the sampled volume; this implies that the true zero-crossing point of $\xi(r)$ has not been reached. We conclude that the apparent fractal behavior is due to a combination of a luminosity-dependent correlation amplitude and the recovering of power at larger scales in deeper samples.

Key words: cosmology: observations – cosmology: large-scale structure of the Universe

1. Introduction

One of the pillars of the standard cosmological models is the large-scale homogeneity of the Universe (e.g. Peebles 1993). The standard statistical methods to analyze the large-scale structure, as the two-point correlation function $\xi(r)$, are described by Peebles (1980); they rely on the definition of a mean galaxy density \bar{n} , which is meaningful only if the assumption of large-scale homogeneity is true. However, at small scales the autocorrelation function of the galaxy distribution is positive and can be fitted by a power-law, which is a property of a fractal set (see Peebles 1980). On the basis of the observational evidence that

galaxies are clustered in ever increasing systems, from groups and clusters to superclusters, de Vaucouleurs (1970, 1971) presented “the case for a hierarchical cosmology”¹. Mandelbrot (1982) developed this concept, suggesting that the galaxy distribution in the Universe is fractal, with dimension $D = 1$.

In addition, in the last 20 years redshift surveys at increasing depths have revealed ever larger structures and voids (see e.g. Davis et al. 1982; de Lapparent et al. 1986, da Costa et al. 1994, Vettolani et al. 1997, and references therein). Einasto et al. (1986) found evidence that r_0 increases with sample volume, but they estimated that it should reach a value $\sim 10h^{-1}$ Mpc for a fair sample of the Universe. Pietronero (1987) stressed that, if homogeneity is not reached, the correlation length r_0 cannot be taken as a measure of the clustering amplitude, and suggested a slightly but significantly different definition of the autocorrelation function. Adopting this approach, Coleman et al. (1988) reanalyzed the CfA1 redshift survey (Huchra et al. 1983), concluding that the distribution of galaxies was fractal to at least $20 h^{-1}$ Mpc. On the other hand, Davis et al. (1988) found that r_0 increased as $r_0^{0.5}$, and not linearly with the depth as predicted for a simple fractal (see also Maurogordato et al. 1992; Martínez et al. 1993). Others advocated the need for a multifractal approach (e.g. Balian & Schaeffer 1989; Martínez & Jones 1990; Martínez et al. 1990); this is however another issue and we will not discuss it: see for example the review by Borgani (1995) and references therein.

The first redshift surveys sampled relatively small volumes, and the reality and nature of correlation amplitude variations with depth remained an open -and much debated- question. Could these variations simply reflect fluctuations due to local structures? Were they a consequence of the fractal distribution of galaxies? Or were they an indirect consequence of the dependence of clustering on galaxy luminosity?

¹ The idea of a hierarchical Universe has a long history, which dates back to the XVIIIth century (see E. Harrison, 1987, *Darkness at night, A Riddle of the Universe*, Harvard University Press, Cambridge); in this century, Fournier d’Albe and Charlier were the first to propose a hierarchical (now we would say fractal) model of the Universe (see Mandelbrot 1982).

It should be pointed out that even at a scale $\sim 1000 h^{-1}$ Mpc we do not expect a *perfect* homogeneity, as COBE has found evidence of anisotropy in the CMB radiation at a level $\delta T/T \sim 10^{-5}$ (Smoot et al. 1992), a value necessary and sufficient, at least in some standard cosmological models, to explain the formation of the observed structures. The existence of very large structures in the Universe implies that the galaxy or cluster autocorrelation function must be positive to a scale corresponding to the size of these structures; but this is not necessarily inconsistent with the measured value of the galaxy autocorrelation length $r_0 \sim 5-6 h^{-1}$ Mpc for $L \sim L_*$ galaxies (as claimed by Pietronero and collaborators), as larger structures have also a lower contrast, and the correlation of luminous matter may be significantly amplified relatively to the underlying dark matter correlation function (Kaiser 1984; Bardeen et al. 1986). Therefore, we expect that the galaxy and cluster distribution will not be perfectly homogeneous even at large scales. The claim for a fractal Universe is obviously much more stronger than that: it implies that there is no convergence to homogeneity and that it is not possible to define a mean density \bar{n} of the Universe.

In the last years Pietronero and his collaborators applied statistical indicators which do not imply a universal mean density on an ever increasing number of catalogs (see e.g. Di Nella et al. 1996, Sylos Labini, Montuori & Pietronero 1996, Montuori et al. 1997), claiming evidence for a galaxy fractal distribution at all scales. Their results are impressive, as recently stressed by Coles (1998), but they appear to be at variance with the results of other groups who analyzed the same catalogs with standard indicators. It is clear that the situation is unsatisfactory, despite many articles and even public debates on the subject (see Pietronero et al. 1997; Davis 1997; Guzzo 1997); while the strongest support to large-scale homogeneity comes indirectly from the high level of isotropy of the CMB radiation and from two-dimensional catalogs (Peebles 1996), there is still confusion on the interpretation of the available three-dimensional data, mainly due to the difference in the statistical indicators, which do not allow a direct and quantitative comparison of the results.

Therefore we analyzed the Southern Sky Redshift Survey 2 (SSRS2; da Costa et al. 1994) following the approach of Pietronero and collaborators, in order to independently check their claims and to answer their criticisms (Sylos Labini et al. 1997, hereafter SMP) about the work of Benoist et al. (1996, hereafter Paper I). In Sect. 2 we discuss the apparent dependence of r_0 on sample depth; in Sect. 3 we describe the different statistics, and their relations; in Sect. 4 we present and discuss our fractal analysis of the SSRS2 in comparison with the standard analysis; in Sect. 5 we show that theoretical predictions derived from the standard CDM model can reproduce our results, and are therefore consistent with a homogeneous Universe; our conclusions are in Sect. 6.

2. Does r_0 depend on sample depth?

If the Universe is homogeneous at large scales, then for any fair sample the statistical properties of the galaxy distribution

should be the same; in particular, the autocorrelation function of any given class of galaxies should not depend on the sample depth. On the contrary, if the Universe is a simple fractal at all scales, the mean density in a sphere of radius r centered on a randomly chosen galaxy will vary as

$$n(r) \propto r^{D-3} \quad (1)$$

where D is the so-called correlation dimension.

The autocorrelation function will vary as

$$\xi(r) \propto r^{(3-D)} - 1 \quad (2)$$

and the correlation length r_0 will increase with the sample depth:

$$r_0 = [(3 - \gamma)/6]^{1/\gamma} R_s \quad (3)$$

where $\gamma = 3 - D$ and R_s is a measure of the sample size.

In practice, there is the problem to define a sample not dominated by a few prominent structures, and to discriminate between a dependence of r_0 on the sample depth or on the absolute luminosities of the galaxies, as deeper volume-limited samples extracted from a magnitude-limited catalog will progressively include more luminous galaxies. The most recent results appear to confirm the original claims by Hamilton (1988) and Börner et al. (1989) about a real dependence of r_0 on the luminosity. In particular, in Paper I we analyzed the SSRS2 and showed that r_0 has a significant increase for $L \geq L_*$ galaxies. SMP criticize these results and point out that a value of r_0 as large as $16 h^{-1}$ Mpc, as found in our Paper I for the $M \leq -21$ galaxies, has never been found in previous surveys as the CfA1. We stress however that a) the previous, shallower surveys do not contain enough galaxies with $M \leq -21$ galaxies which, being rare, require a larger volume; b) as shown in Fig. 5 of Paper I, the results obtained by Hamilton on the CfA1 are consistent with the new SSRS2 results -up to $M = -20.5$, representing the maximum luminosity reachable in a volume-limited sample large enough for a statistical analysis in the CfA1-. Notice that the CfA1 and the SSRS2 are limited respectively to $m = 14.5$ and $m = 15.5$, so volume-limited samples containing galaxies with the same absolute luminosity have different depths; their consistency at a fixed absolute magnitude is another argument in favor of luminosity segregation and against a fractal distribution. However, SMP criticize our Paper I also because we tested the independence of the correlation length from the sample depth only for a few subsamples; moreover, they notice that the results obtained by Sylos Labini & Pietronero (1996) on the Perseus-Pisces catalog show a systematic dependence of r_0 on the sample depth, and not on luminosity. Indeed the same test performed by SMP on the Perseus-Pisces is possible also for all the subsamples of the SSRS2. In Fig. (1) we present r_0 with its bootstrap error as a function of the absolute magnitude for volume-limited samples of the SSRS2 selected with apparent limiting magnitudes of 14.5 and 15.5 (see Tables 1 and 2). Notice that the distance limits for the volume-limited samples are calculated assuming $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $q_0 = 0.5$, and taking into account

Table 1. Volume-limited subsamples from the SSRS2

M_{lim}	D_L	D_{com}	z_{lim}	N_{gal}
-16.0	19.70	19.58	0.0066	284
-16.5	24.73	24.53	0.0082	279
-17.0	31.01	30.69	0.0103	302
-17.5	38.84	38.35	0.0129	329
-18.0	48.60	47.83	0.0161	488
-18.5	60.72	59.52	0.0202	773
-19.0	75.72	73.87	0.0251	766
-19.5	94.23	91.38	0.0312	755
-20.0	116.95	112.60	0.0386	593
-20.5	144.71	138.12	0.0477	268
-21.0	178.41	168.52	0.0587	133

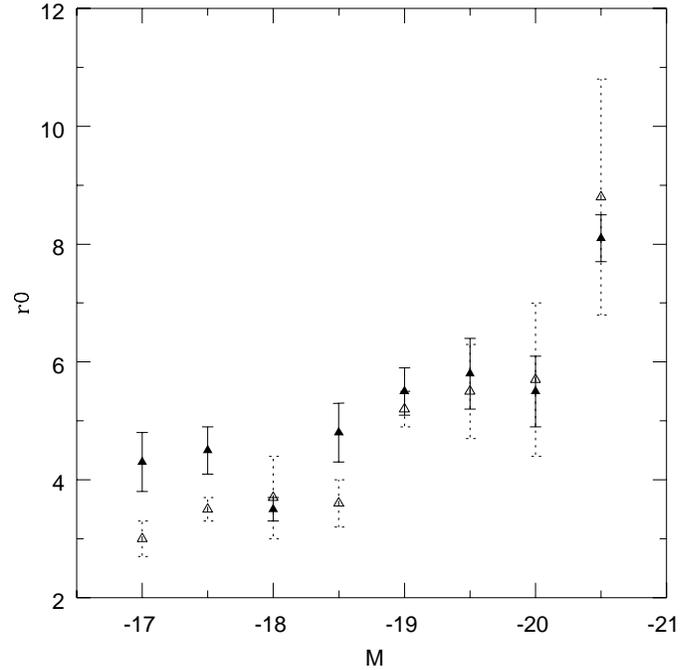
Table 2. Volume-limited subsamples from the SSRS2 with a limiting apparent magnitude 14.5

M_{lim}	D_L	D_{com}	z_{lim}	N_{gal}
-16.5	15.69	15.61	0.0053	97
-17.0	19.70	19.58	0.0066	164
-17.5	24.73	24.53	0.0082	156
-18.0	31.00	30.69	0.0103	155
-18.5	38.84	38.34	0.0129	141
-19.0	48.60	47.83	0.0161	185
-19.5	60.72	59.51	0.0215	212
-20.0	75.72	73.87	0.0251	134
-20.5	94.23	91.38	0.0312	68

the K-correction, and they are conservatively fixed to include the galaxies of all the morphological types (for more details see Paper I).

As discussed in Paper I, the less deep subsamples (including galaxies with a faint absolute magnitude) may be affected by the dominance of local structures. From Fig. 1, there is indeed marginal evidence for sampling variations in subsamples including galaxies with absolute magnitude fainter than $M = -19$, while for deeper subsamples galaxies with the same absolute magnitude have the same value of r_0 within the errors, independently of the sample depth. This consistency internal to the SSRS2, in addition to the external consistency between subsamples extracted from the CfA1 and the SSRS2, is a further and robust confirmation that the correlation amplitude depends on the luminosity and not on the sample depth. This result is obviously in disagreement with the claims by SMP, based on subsamples of the Perseus-Pisces supercluster, which show indeed very large differences between the correlation length r_0 of galaxies with the same absolute magnitude but in volumes at different depths (see their Fig. 40). SMP assume a constant error $\Delta r_0 = 0.5 h^{-1}$ Mpc; while a more realistic error would have been preferable, it cannot be a large underestimate given the size of their subsamples.

While there is no *a priori* reason to prefer one catalog to another, the type of information which can be extracted depends on the particular selection criteria of each catalog. In this

**Fig. 1.** Correlation length r_0 for different volume-limited subsamples extracted from the SSRS2, assuming an apparent limiting magnitude $m_{lim} = 14.5$ (open triangles) and $m_{lim} = 15.5$ (solid triangles)

case, contrarily to the “anonymous” CfA or SSRS redshift surveys, the name Perseus-Pisces redshift survey reminds us that it is a region of the sky selected because of the presence, at about 5500 km/s, of a large filamentary structure (Giovanelli & Haynes 1991). While many interesting statistical results can -and could- be obtained, their interpretation must take into account this peculiarity of the survey. Guzzo (1997) discusses the effect of the Perseus-Pisces supercluster on the galaxy counts, questioning the interpretation of Pietronero et al. (1997). As concerning the variation of r_0 with sample depth, we notice that a volume-limited sample including all galaxies brighter than -19 has a depth of ~ 5000 km/s at an apparent limiting magnitude of $m = 14.5$, and a depth of ~ 8000 km/s at an apparent limiting magnitude of $m = 15.5$. In the case of the Perseus-Pisces redshift survey, this means that in the first case we include the void in front of the supercluster but not the supercluster itself, while in the second case we include the whole supercluster. This might at least partly explain the discrepancy with our results; therefore a fractal galaxy distribution does not appear as the most plausible explanation for the systematic variations of r_0 in the Perseus-Pisces redshift survey.

3. Statistical tools

As we mentioned in the introduction, the statistical indicators used to assess the fractality of the Universe are different from -but not independent of- the standard ones. In this section we will clarify the relations between the two sets of indicators.

A simple dimensionless measure of galaxy clustering is the two-point correlation function $\xi(r)$, which is defined by the

joint probability δP of finding a galaxy in a volume element dV_1 and another galaxy in a volume element dV_2 at separation r : $\delta P = \bar{n}^2[1 + \xi(r)]dV_1dV_2$, where we take into account only the magnitude of the separation but not its direction, assuming as usual that the galaxy distribution is a stationary point-process (see Peebles 1980). The two-point correlation function $\xi(r)$ of a sample is measured generating a catalog of randomly distributed points in the same volume, with the same selection function, then counting the number $N_{gg}(r)$ of distinct galaxy-galaxy pairs at separation r , and the number $N_{gr}(r)$ of galaxy-random pairs at separation r (Davis & Peebles 1983):

$$\xi(r) = 2 \frac{n_R N_{gg}(r)}{n_G N_{gr}(r)} - 1 \quad (4)$$

where n_G is the mean galaxy density and n_R the density of points in the random catalog. Hamilton (1994) suggested a slightly different estimator of $\xi(r)$ less dependent on the density, and best suited to measure $\xi(r)$ at large separations, which we adopted in Paper I, and which we use also in this paper. This is what we will call in the following the “direct” estimate of $\xi(r)$.

The galaxy two-point correlation function is well approximated by a power-law, $\xi(r) = (r/r_0)^{-\gamma}$, where $\gamma \sim 1.8$, at least at scales less than $\sim 10h^{-1}$ Mpc: this implies that the correlation dimension $D = 3 - \gamma$ is ~ 1.2 .

As discussed in the previous section, if the Universe is a fractal, it is impossible to find a “universal” value of r_0 , because larger samples will have larger voids and a larger r_0 . For this reason Pietronero (1987) has defined the conditional average density $\Gamma(r)$:

$$\Gamma(r) = \frac{1}{N} \sum_{i=1}^N \frac{1}{4\pi r^2 dr} \int_r^{r+dr} n(r_i + r) dr \quad (5)$$

which measures the average density at distance r from an occupied point. $\Gamma(r)$ is in principle related to $\xi(r)$:

$$\xi(r) = \Gamma(r)/n - 1 \quad (6)$$

One implication (trivial but important) of relation (6) is that, if $\xi(r)$ is a power-law, $\Gamma(r)$ is not, and vice-versa. Moreover, there is a practical difference in the way of estimating the two functions (see e.g. Coleman & Pietronero 1992). $\Gamma(r)$ is calculated only within the sample limits, while $\xi(r)$, as apparent from Eq. (4), is computed using random catalogs to correct for volume boundaries, thus implicitly assuming homogeneity. Even if the Universe is not a fractal, this can be a problem when the sample size is still small relatively to the scale where homogeneity is achieved; as a consequence, an indirect estimate of $\Gamma(r)$ obtained from $\xi(r)$ through Eq. (6) may be wrong.

$\xi(r)$ is however a simple statistical measure of clustering; other useful functions representing variously defined integrals of $\xi(r)$ can be directly measured on a sample.

For example, the mean number of objects $\bar{N}_p(r)$ in a volume V centered on a randomly chosen object, is

$$\bar{N}_p(r) =$$

$$\int_0^r [1 + \xi(x)] dV = 4\pi \int_0^r x^2 [1 + \xi(x)] dx = nV + 4\pi J_3 \quad (7)$$

where $J_3(r)$ is the so-called J_3 integral:

$$J_3(r) = \int_0^r x^2 \xi(x) dx \quad (8)$$

$4\pi J_3$ represents the mean number of excess galaxies around a galaxy within a radius r .

Another important integral quantity is the volume-averaged two-point correlation function $\bar{\xi}(r)$:

$$\bar{\xi}(r) = \frac{1}{V^2} \int_V \xi(r_1 r_2) d^3 r_1 d^3 r_2 \quad (9)$$

Contrarily to $\xi(r)$, $\bar{\xi}(r)$ is usually computed through counts in randomly placed spherical cells, taking into account only the spheres totally included within the sample boundaries. No random catalog is used, analogously to the estimate of $\Gamma(r)$.

The equivalent of $J_3(r)$ and $\bar{\xi}(r)$ in the fractal approach is the integrated conditional density $\Gamma^*(r)$ defined as:

$$\Gamma^*(r) = \frac{3}{4\pi r^3} \int_0^r 4\pi x^2 \Gamma(x) dx \quad (10)$$

which “gives the behavior of the average density of a sphere of radius r centered around an occupied point averaged over all occupied points” (Coleman & Pietronero 1992).

We notice that $\Gamma^*(r)$ is related to $J_3(r)$: using relations (6), (8) and (10) we have:

$$\Gamma^*(r) = n[1 + \frac{3}{r^3} J_3(r)] \quad (11)$$

which represents the integral counterpart of Eq. (6).

As in the case of $\xi(r)$, $J_3(r)$ is also computed using random samples, i.e. assuming homogeneity beyond the sample limits, so Eq. (11) cannot be used to obtain $\Gamma^*(r)$ from J_3 .

Assuming now that $\xi(r)$ is a perfect power law, we have the following relations:

$$J_3(r) = \frac{r^3}{3(1 - \gamma/3)} \xi(r) \quad (12)$$

and

$$\bar{\xi}(r) = \frac{2^{-\gamma}}{(1 - \gamma/3)(1 - \gamma/4)(1 - \gamma/6)} \xi(r) \quad (13)$$

We have then:

$$\Gamma^*(r) = n[1 + 2^\gamma (1 - \gamma/4)(1 - \gamma/6) \bar{\xi}(r)] = n[1 + \frac{\xi(r)}{(1 - \gamma/3)}] \quad (14)$$

Eqs. (12-14) are useful because they allow us to estimate $\xi(r)$ from the integral indicators; in the next section we will use them, with Eq. (6), to compare the different functions we have defined above.

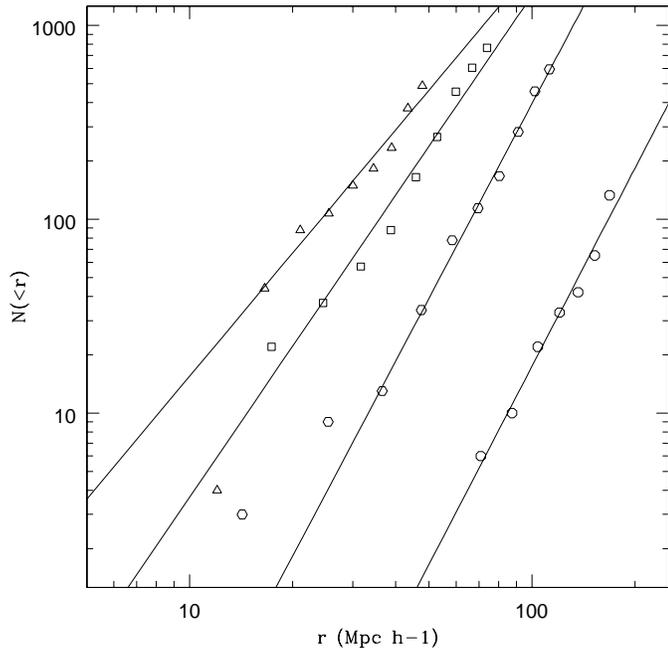


Fig. 2. Integrated number of galaxies as a function of comoving distance r for 4 volume-limited subsamples of the SSRS2 ($M \leq -18$, $M \leq -19$, $M \leq -20$, $M \leq -21$). The solid lines are least-squares fits to the data with the law $N(<r) \propto r^D$, with respectively $D = 2.11 \pm 0.27$, $D = 2.59 \pm 0.21$, $D = 3.34 \pm 0.18$, $D = 3.40 \pm 0.35$.

4. Fractal analysis of the SSRS2

A preliminary test -and the easiest one- in order to compute the fractal dimension of the galaxy distribution can be performed counting the number $N(r)$ of objects within a distance r from us: from Eq. (1), we expect $N(r) \propto r^D$. The results for the 4 volume-limited SSRS2 samples with absolute magnitude limits $M \leq -18$, $M \leq -19$, $M \leq -20$, $M \leq -21$, are shown in Fig. 2. A formal least-squares fit to the data gives $D = 2.11 \pm 0.27$, $D = 2.59 \pm 0.21$, $D = 3.34 \pm 0.18$, $D = 3.40 \pm 0.35$; the errors were computed through bootstrap resampling of the original samples. These results apparently tell us that from the shallowest to the deepest sample we have a change in the fractal dimension, with convergence to homogeneity. Even considering the uncertainty associated to these values, we can conservatively state that such results are more consistent with homogeneity at scales $\sim 100 h^{-1}$ Mpc, than with a fractal Universe.

We have also computed $\Gamma(s)$ and $\Gamma^*(s)$ for the same 4 volume-limited samples. In what follows we will use redshift space estimates. First of all, we are interested here in a relative comparison between different indicators, and not in determining a true value; second, we want to compare our results with those of Pietronero and collaborators, always performed in redshift space; finally, the SSRS2 is not affected by large peculiar motions (see Willmer et al. 1998).

The results are shown in Fig. 3. A least-square fit in the small-scale range (less than $10 h^{-1}$ Mpc) gives a slope $\gamma = 3 - D \sim 1$, i.e. $D \sim 2$. At larger scales, the slope fluctuates

from sample to sample, and within the same sample, but does not show a clear *plateau* (as should be expected if homogeneity was reached); moreover, positive values of $\Gamma(s)$ and $\Gamma^*(s)$ are measured to larger scales for deeper samples. These results are consistent with those found by Pietronero and collaborators on other catalogs.

Does all this mean that the Universe is fractal, and that the hypothesis of large-scale homogeneity is falsified?

First of all, we have to take into account the behavior of $N(r)$ we have discussed above, which is consistent with homogeneity at large scales. $\Gamma(s)$ and $\Gamma^*(s)$ are more general tests, as they average on all the galaxies, while $N(r)$ is centered on our position; nevertheless, with $N(r)$ we can test larger scales ($> 100 h^{-1}$ Mpc), while $\Gamma(s)$ and $\Gamma^*(s)$ are limited to $\sim 40 h^{-1}$ Mpc. At this scale we still have a positive correlation function (see below) and we should not expect to measure $D = 3$; therefore what we can infer from all these tests is that there is still power at a scale of $\sim 40 h^{-1}$ Mpc, but no evidence for significant power at $\sim 100 h^{-1}$ Mpc.

A further problem for the fractal model is the discrepancy between the slope of $\xi(s)$, which implies $D \sim 1$ (for the SSRS2 see our results in Table 1 of Paper I), and that of $\Gamma(s)$ and $\Gamma^*(s)$, implying $D \sim 2$. SMP claim that people fit $\xi(s)$ in the “wrong range” around r_0 , where $\xi(s)$ would deviate from a power-law. We notice that people usually fit $\xi(s)$ where it can be reasonably approximated by a power-law, excluding both small and large scales; therefore the explanation of SMP (see their Fig. 9) does not seem plausible.

In interpreting these results, it should perhaps be taken into account an effect which is trivial but which we point out for sake of clarity. In Fig. 4, we show $\xi(r)$ and $\Gamma(r)$ assuming that $\xi(s)$ is a perfect power-law with $\gamma = -2$, and points are obtained assuming an error of 20%. It is obvious that the quantity $\Gamma(r) \propto [1 + \xi(r)]$ can be approximated by 2 power-laws; in this case, in the range $2-10 h^{-1}$ Mpc it has a slope $\gamma = -1.3$ or $D = 1.7$. At larger scales, where $\xi(r)$ becomes very small, Γ becomes constant. However, in real samples $\xi(s)$ deviates from a power-law at small and large scales (see below), and the regime where $\Gamma(r)$ is constant may not be reached. This simple exercise tells us that if $\xi(s)$ is a genuine power-law, there is a range around r_0 where $\Gamma(s)$ can be approximated by a power-law with a flatter slope, which is what is observed.

We will check now the internal consistency between standard and non standard statistics, examining the behavior of the functions described in the previous section. In Fig. 5, for the volume-limited subsample including all galaxies with $M \leq -20$, we show the values of $\xi(s)$ estimated directly using the Hamilton (1994) formula, from $\Gamma(s)$ through Eq. (6), and from the integral functions $J_3(s)$, $\bar{\xi}(s)$ and $\Gamma^*(s)$ using Eqs. (12), (13), (14) (i.e., assuming that $\xi(s)$ is a perfect power law). In the standard scenario, all these estimates applied on the same sample should in principle measure the same quantity, and the only differences should be due to the biases introduced by each method (this is why no error bars are shown in the figure).

Fig. 5 shows that there are two main effects: at small scales, there is a systematic difference between the estimates of $\xi(s)$

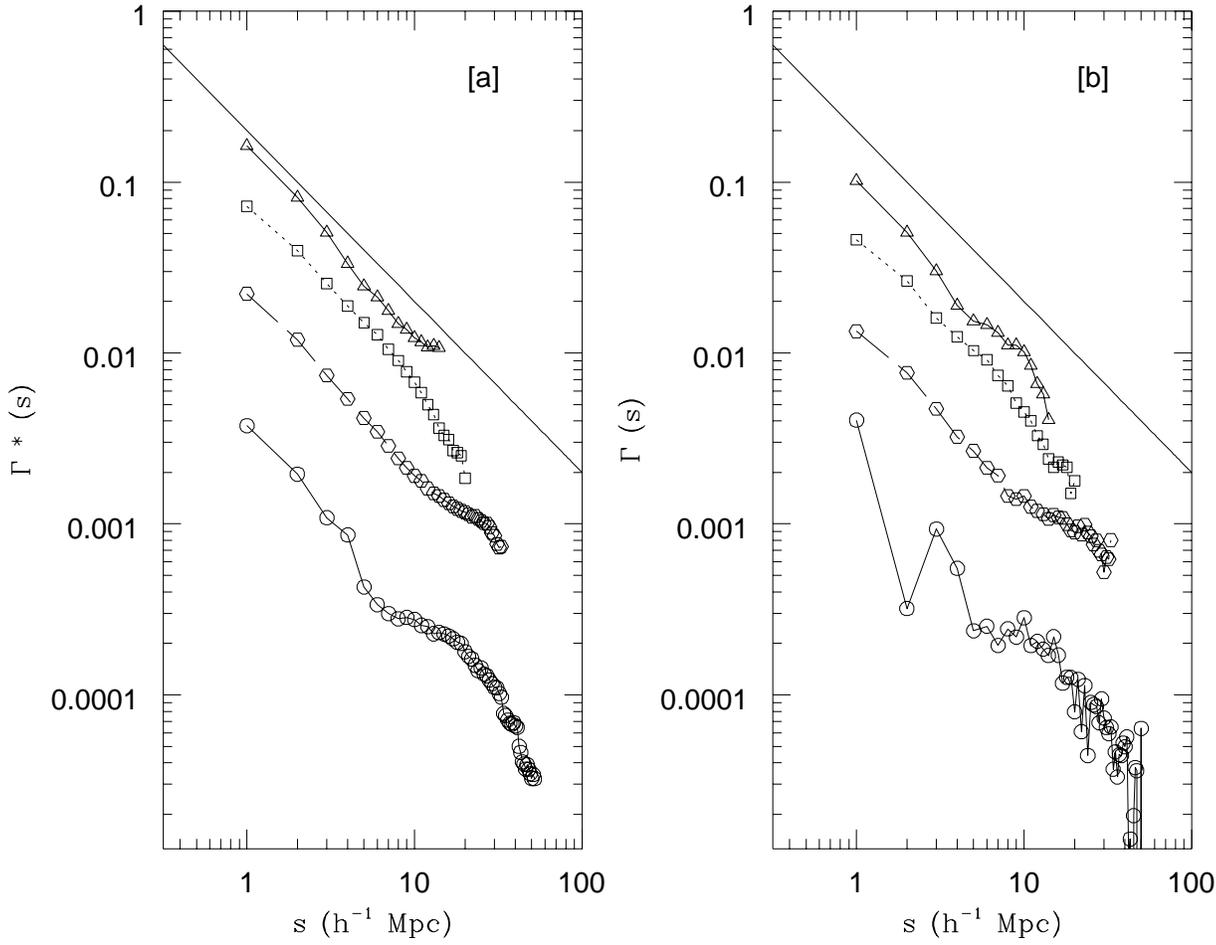


Fig. 3. $\Gamma^*(s)$ (panel a) and $\Gamma(s)$ (panel b) for the SSRS2 volume-limited samples including galaxies with $M \leq -18$ (triangles), $M \leq -19$ (squares), $M \leq -20$ (hexagons), $M \leq -21$ (circles). The solid line has a slope $\gamma = -1$ (i.e. $D = 2$).

obtained from the integrated functions and the two estimates obtained directly or from $\Gamma(s)$; at large scales, there is a systematic difference between the estimates based on random catalogs and the other ones which only take into account the volumes fully included within the sample limits.

The direct estimate of $\xi(s)$ is consistent with that computed from $\Gamma(s)$ to about $10h^{-1}$ Mpc. At small scales peculiar motions affect $\xi(s)$, and the number of pairs at small separations becomes very small; therefore we expect significant deviations of $\xi(s)$ from a power-law, and this explains why the indirect estimates of $\xi(s)$ from $\Gamma^*(s)$, $\bar{\xi}(s)$, and $J_3(s)$ based on the assumption of a perfect power-law, differ systematically from the other two estimates.

At scales larger than $\sim 10h^{-1}$ Mpc the estimates of $\xi(s)$ obtained from $\Gamma(s)$, $\Gamma^*(s)$ and $\bar{\xi}(s)$ decline more rapidly than the direct estimate of $\xi(s)$ and that obtained from $J_3(s)$. As the former indicators take into account only the volumes fully included within the sample limits, they are limited to smaller scales -their maximum scale corresponds to the radius of the largest sphere which can be included in the sample-; moreover, at large scales the randomly placed spheres have a significant overlap, and are no more independent, thus giving a biased (lower) estimate of

the variance. It can be noticed that the estimate of $\xi(s)$ from $\Gamma(s)$ drops somewhat more rapidly than the estimates from $\Gamma^*(s)$ and $\bar{\xi}(s)$, because the latter are derived from integral measures, thus are smoother and have a better signal to noise ratio. It is also clear that the estimate of $\xi(s)$ from $J_3(s)$, which is an integrated measure and is based on random catalogs, can reach the largest scales, and appears to be consistent with a power-law extending to $\sim 50h^{-1}$ Mpc. The direct standard estimate of $\xi(s)$ is noisier and oscillates; it drops beyond $10h^{-1}$ Mpc, in a way similar to the integral estimates, then has a second oscillation, following the $J_3(s)$ estimate.

We can conclude from the above discussion that there is no disagreement between the results obtained with standard and non-standard indicators, when taking into account the range of validity of each one of them.

In Fig. 6, for the volume-limited subsamples from $M = -16$ to $M = -21$ (step 0.5) we show the correlation length r_0 , the first zero-point R_c of $\xi(s)$ (i.e. the scale where $\xi(s)$ becomes negative), and the radius of the largest sphere totally included in the sample, R_{max} , as a function of the sample depth D_s . The values of r_0 with their bootstrap error bars are the same as in Fig. 1, while the values of R_c simply correspond to the last

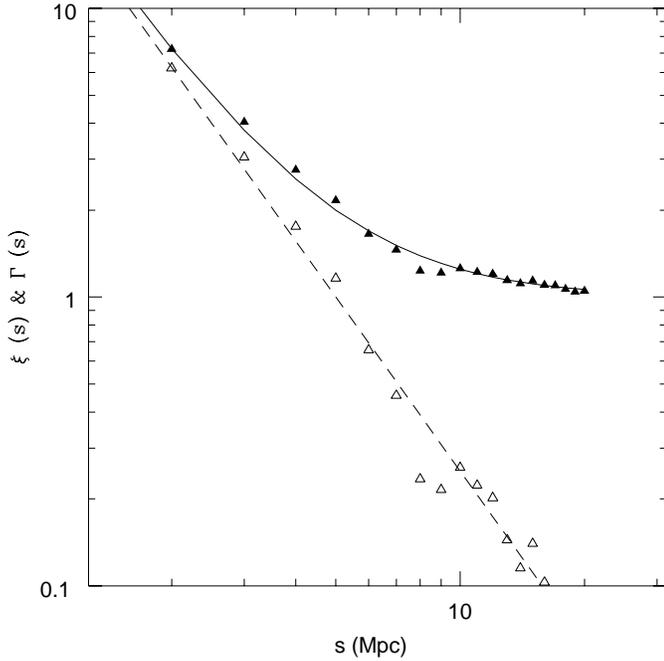


Fig. 4. $\xi(r)$ (dashed line) and $\Gamma(r) = n[1 + \xi(r)]$ (solid line) assuming $\xi(r) = (r_0/r)^2$, with $r_0 = 5h^{-1}$ Mpc and $n = 1$. Open and filled triangles simulate a measure of the above two functions assuming a 20% error.

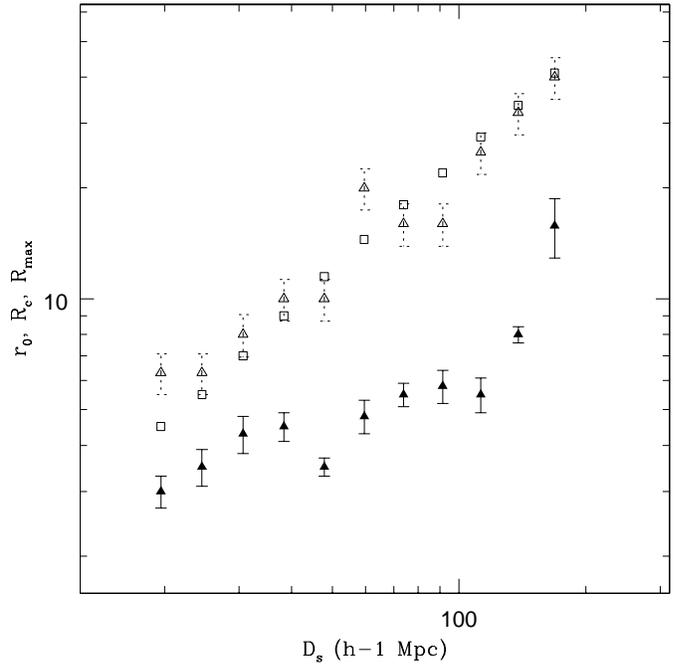


Fig. 6. The correlation length r_0 (filled triangles), the zero-point R_c (open triangles), and the maximum radius R_{max} (open squares) as a function of the sample depth D_s , for volume-limited samples of the SSRS2 (from $M \leq -16$ to $M \leq -21$, with a step of 0.5 magnitudes).

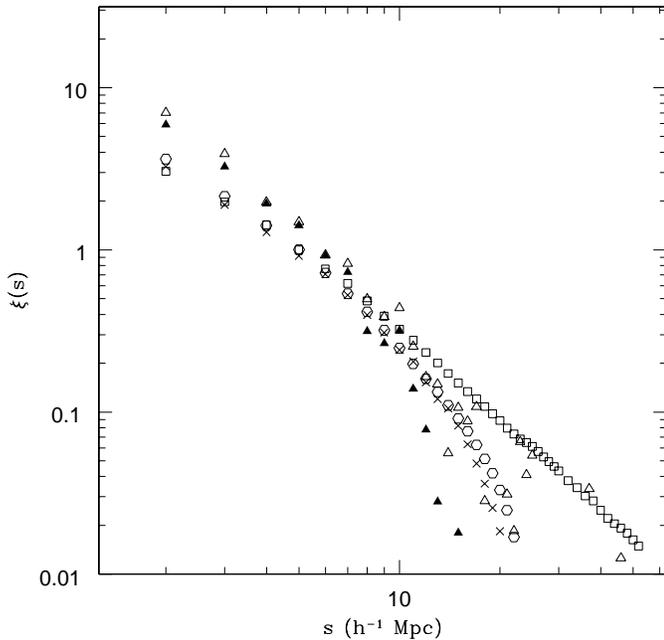


Fig. 5. $\xi(s)$ derived with different statistical tools for the $M = -20$ subsample of the SSRS2. Open triangles: direct standard estimate; filled triangles: computed from Γ ; open squares: computed from J_3 ; open hexagons: computed from $\bar{\xi}$; crosses: computed from Γ^* .

bin where $\xi(s)$ is positive, and their error bars indicate the bin width.

It is trivial that $R_{max} \propto D_s$. The correlation length r_0 is not proportional to the depth, contrarily to what expected for a frac-

tal; the less deep subsamples are probably not “fair”, and r_0 may be underestimated (even if a luminosity effect for low luminosity galaxies cannot be excluded); then r_0 is more or less constant in an intermediate range of luminosities, and increases rapidly (more rapidly than the depth) for samples including galaxies brighter than M_* . This is the luminosity effect we have shown in Paper I (see also Fig. 1). The most striking result of Fig. 6 is that the value of the zero-point R_c clearly increases linearly with the depth, and that its value corresponds in a striking way to R_{max} .

This quantitative correspondence between R_c and R_{max} means that the standard estimate of the correlation function $\xi(s)$ is in fact reliable only at scales smaller than the largest sphere contained in the sample: it is at this scale which $\xi(s)$ is forced to become negative, then starts to oscillate. We stress that this effect should be taken into account; often an oscillating correlation function is interpreted as evidence of a real characteristic scale, but if this scale corresponds to the R_{max} of the sample taken into account, the unavoidable conclusion (as in the case of our subsamples) is that the real zero-point of $\xi(s)$ has not been reached.

5. Comparison with theoretical predictions

Are the results of our fractal analysis consistent with standard models of galaxy formation?

The behavior of Γ and Γ^* can be checked by using N-body simulations, which are based on standard Friedmann-Lemaître cosmological models, and assume homogeneity at large scales.

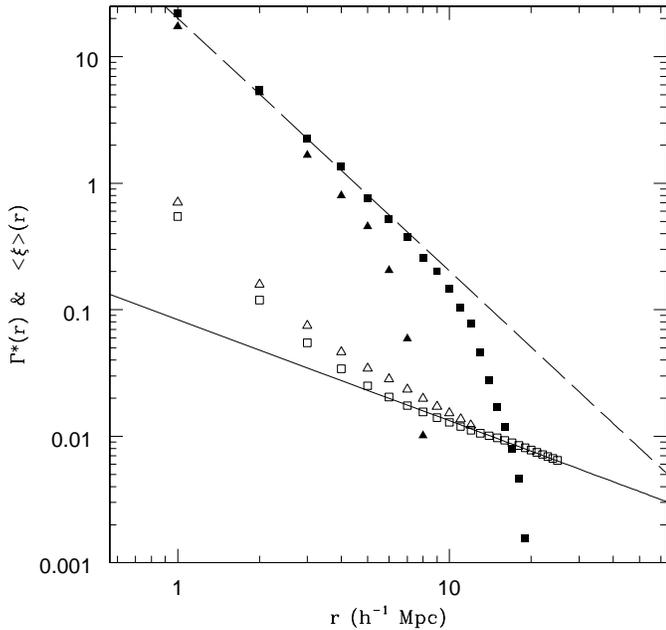


Fig. 7. $\Gamma^*(r)$ and $\bar{\xi}(r)$ measures for one realization of a CDM universe. Filled and open triangles represent respectively $\bar{\xi}$ and Γ^* measured in a box with $L = 25 h^{-1}$ Mpc; filled and open squares represent respectively $\bar{\xi}$ and Γ^* measured in a box $L = 50 h^{-1}$ Mpc. The solid line has a slope $\gamma = -0.8$; the dashed line has a slope $\gamma = -2.0$.

We used the Adaptive P^3M code of Couchman (1991) to evolve a standard Cold Dark Matter universe. We started from the initial linear power spectrum of Bond & Efstathiou (1984) with $\Omega h = 0.5$. The box size was $100 h^{-1}$ Mpc, with 64^3 particles. We analyzed the simulation at a time when its correlation function was roughly comparable to the observed one, diluting it to approach the observed galaxy densities. Fig. 7 shows $\bar{\xi}(r)$ and $\Gamma^*(r)$ for 2 subsamples extracted from the simulation: the first with a box size $L = 25 h^{-1}$ Mpc, the second with $L = 50 h^{-1}$ Mpc. The best-fit slope for Γ^* is $\gamma = -1.0$, i.e. $D = 2$ in the first case; $\gamma = -0.8$, i.e. $D \sim 2.2$, in the second case. $\bar{\xi}(r)$ shows the typical cutoff when approaching the radius of the largest sphere which can be contained in the sample: in fact, the deeper sample allows to recover power at larger scales. These results are comparable to our results on the SSRS2, and also to those presented by SMP. We have already mentioned that a problem for a simple fractal Universe is the discrepancy between the fractal correlation dimension implied by $\xi(s)$, $D = 3 - \gamma \sim 1$, and that derived from $\Gamma(s)$ or $\Gamma^*(s)$, $D \sim 2$. We have also noted that such a discrepancy cannot be due simply to an improper fitting of $\xi(s)$ (as claimed by SMP) in a range where it deviates from a power-law.

Now we have shown that the observed slopes of $\xi(s)$ and $\Gamma(s)$ can be reproduced by a standard CDM N-body simulation: this confirms the implications of our example in Fig. 4, and the consistency of both $\xi(s)$ and $\Gamma(s)$ with gravitational clustering in a Universe homogeneous at large-scale.

We have not addressed, of course, the question of the variations of r_0 . In our simulation, r_0 at any fixed time has a unique,

constant value; when analyzing deeper subsamples of our simulation, r_0 rapidly converges to its true value (no significant difference in fact is found in the value r_0 measured in a box of $50 h^{-1}$ Mpc and in another at $75 h^{-1}$ Mpc), while the scale of the zero-point increases significantly.

As we have previously shown, in the real Universe the correlation length r_0 of galaxies does depend on their luminosity (or, in other terms, there is evidence of luminosity segregation). Our simple answer to the criticism by SMP –“the authors [...] have never presented *any quantitative argument that explains the shift of r_0 with sample size*”–, is that shallow samples are affected by the presence of local structures, as the discrepancy between the Perseus-Pisces and the other surveys reminds us; moreover, when a sample is not deep enough, $\xi(s)$ can be artificially truncated, thus affecting the estimate of r_0 ; finally, and it is our main quantitative argument, from Fig. 1 it is clear that r_0 does not depend on sample depth, but on galaxy luminosity (as shown in paper I), for $L > L_*$ galaxies.

On the other hand, there are various reasons why we should expect luminosity segregation. It has been shown that the correlation function can be amplified as a consequence of a “statistical bias” (Kaiser 1984), which can explain in a natural way the cluster autocorrelation function. As concerning galaxies, White et al. (1987) used N-body simulations to show that more massive galaxies are more clustered; they called this phenomenon “natural bias”. Recent semi-analytical models (Mo & White 1996, Kauffmann et al. 1997) predict this bias if galaxies form in sufficiently massive halos, with $M > M_*$; in this case, the trend is similar to that observed in the SSRS2 (see Fig. 5 of Paper I), even if they do not succeed in fitting both the low and high luminosity samples. This is not surprising, given all the theoretical uncertainties, particularly on the dark halo-galaxy relation.

In addition to the luminosity-dependent correlation amplitude, there are other 2 important points which should be considered when analyzing the galaxy distribution: the K-correction and the cosmological model (see Guzzo 1997; Scaramella et al. 1998). While in our analysis of the SSRS2 samples we have fixed an $H_0 = 100$, $q_0 = 0.5$ model and systematically applied K-corrections, Pietronero and collaborators usually perform their analysis in Euclidean space and without K-correction. As concerning the cosmological model, for a fractal Universe one should find and apply the appropriate formulae for an isotropic but inhomogeneous Universe (unless one wants to question the validity of General Relativity). On the other hand, the K-correction is an empirical correction to the magnitude of a galaxy, as in a fixed wavelength range we observe different parts of the spectrum at different redshifts. The effect is still small in the deepest sample of the SSRS2; if we use Euclidean distances and do not apply the K-correction, from the relation $N(< r) \propto r^D$ we find $D = 3.28$ instead of $D = 3.40$. It becomes critical in deeper surveys as the ESO Slice Project (Vetolani et al. 1997; Zucca et al. 1997), as shown by Scaramella et al. (1998), who do not confirm the results obtained by SMP on that sample as well as on Abell clusters.

Moreover, the K-correction cannot be ignored when interpreting deep counts; the claim by Sylos Labini et al. (1996) that

deep counts are consistent with a fractal distribution can be reversed, concluding that the agreement with the observed count slope would be lost if at least the K-correction had been applied (not to speak of cosmological and evolutionary corrections: they are model-dependent, but we know that galaxies evolve).

Why do the choice of an Euclidean space and the absence of a K-correction give results more consistent with a fractal distribution? There are essentially two main reasons: first of all, Euclidean distances become systematically larger than comoving ones at increasing redshift: this means also that one will have larger voids and structures. Second, when applying the K-correction we can recover galaxies that otherwise would not have been included in a volume-limited subsample; if the K-correction is not applied, a systematically larger fraction of galaxies will be lost at larger distances. The combination of the two effects lowers the density at increasing redshift, and flattens the slope of the $\log(N) - \log(r)$ relation, thus following the same trend of a fractal distribution.

6. Conclusions

In this paper, we have carefully examined the claim that the Universe is a fractal, analyzing the SSRS2 and using the same statistical approach of Pietronero and collaborators.

Here are our main results:

- the SSRS2 has a luminosity dependent correlation amplitude;
- results obtained with the standard and fractal approach give consistent results;
- concerning large-scale homogeneity, observational evidence is -for the moment- consistent with theoretical predictions derived from standard CDM models of galaxy formation, and does not require a fractal Universe;
- on the other hand, the first zero-point of the correlation function we measure in SSRS2 volume-limited subsamples increases linearly with the sample depth, and has approximately the same value of the radius of the largest sphere which can be included in the sample. This implies that the zero-point of $\xi(s)$ is beyond $40 h^{-1}$ Mpc, a lower limit set by very luminous galaxies (see Benoist et al. 1996; Cappi et al. 1998) and consistent with results from clusters (e.g. Cappi & Maurogordato 1992); on this point, we agree with SMP that measurements of $\xi(s)$ beyond a scale corresponding to the radius of the largest sphere included in the sample are not reliable.

We conclude that the dependence of the galaxy correlation function on the galaxy luminosity and the power still present at large scales ($\geq 40 h^{-1}$ Mpc) can satisfactorily explain those effects interpreted by SMP as evidence of an inhomogeneous, fractal Universe.

After this paper was accepted, two new preprints concerning the same subject have been submitted to the astro-ph database: Martínez et al. (1998) apply the K function (related to J_3 and Γ^*) to 4 galaxy catalogs, while Wu et al. (1998) derive various

constraints on the isotropy and homogeneity of the Universe. Both these papers confirm the consistency of the available observational evidence with homogeneity on large scales.

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