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Metallicity gradients and the matter distribution in elliptical galaxies

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Abstract. In this work I discuss on the basis of the instantaneous recycling approximation the possibility of determining a relation between matter distribution (luminous and dark) in elliptical galaxies and the metallicity distribution.

Two different contexts are considered depending on the behaviour of the gas velocity.

Key words: galaxies: abundances – galaxies: elliptical and lenticular, cD – galaxies: evolution – galaxies: structure

1. Introduction

Metallicity gradients in elliptical galaxies are inferred from the variation of metallic indices as a function of radius (see Danziger et al. 1993; Carollo et al. 1993 and references therein for previous works on the subject). The mechanism often invoked to explain the formation of these gradients is dissipative processes in the gas during galaxy formation (Larson 1974a,b, Carlberg 1984, Lynden-Bell 1975, Pagel 1997). In this framework, an abundance gradient should arise because the stars form everywhere in collapsing clouds and then remain in orbit with a little inward motion whereas the gas sinks further in because there is dissipation. This gas contains the metals ejected by evolving stars so that an abundance gradient develops in the gas. As stars continue to form their composition reflect the gaseous abundance gradient.

Franx and Illingworth (1990) found that, for their sample of 17 galaxies, the local (B-R) (U-R) colours are functions of the local escape velocity v_{esc} for all of galaxies. This result was confirmed later by Carollo et al. (1993) and Davies et al. (1993) using a more direct indicator of metallicity (Mg_2) than colours and a more appropriate calculation of v_{esc} for each galaxy. This relation suggests that metallicity gradients could have arisen because of the different time that gas spends in different galactic regions. In fact this time depends both on the energy injected in the interstellar medium (ISM) by supernovae (of both type I and II) and on the binding energy of gas. Hence, in the regions where v_{esc} is low (i.e. where the local potential is shallow), this time is low and the gas is less processed than in the regions where v_{esc} is higher.

This mechanism, completely different from dissipative collapse, can also produce metallicity gradients depending on the local binding energy of gas and therefore on the matter distribution. On the basis of this idea Martinelli et al. (1998) reproduced metallicity gradients in good agreement with observational data, by using the Jaffe distribution for luminous matter (Jaffe 1983) and a dark matter distributed on a diffuse halo. In their work they do not take into account the processes occurred during galaxy formation such as collapse or merging. They proposed a model starting with the total mass already present at the beginning. The main result found in that work is that the differential occurrence of galactic winds alone can explain the observed abundance gradients.

The aim of this work is to exploit the previous idea in the frame-work of analytical models (based on the *IRA* approximation) for deriving a relation between metallicity distribution and matter distribution in elliptical galaxies within the effective radius R_0.

The models considered are defined and discussed in Sect. 2 together with the related equations for the chemical evolution in the *IRA* approximation. Then in Sect. 3 the relation between matter and metallicity distribution is given for the models considered. Finally some conclusions and remarks are done in Sects. 4 and 5.

2. Basic equations

In the following, we consider elliptical galaxies to have a spherical symmetry. We assume that at the initial time (t=0) these systems consist only of primordial gas and the star formation starts at the same time in all galactic regions.

Let indicate the gas density and its velocity at position **r** and time t respectively with g(**r**,t) and **v**(**r**,t). In the IRA approximation the evolution of gas is given by the following equation (Edmunds and Greenhow 1995, hereafter EG95)

$$\frac{\partial g}{\partial t} = -\alpha\Psi - \nabla(g\mathbf{v}) \quad (1)$$

where $\Psi = \Psi$(g,**r**,t) is the star formation rate (SFR) and α is the fraction of material formed into stars which will remain as long lived stars (all the stars with mass less than the solar mass, in the IRA approximation) or stellar remnants.

The mass fraction z of heavy elements in the interstellar medium is governed by the following equation, by assuming $z << 1$ (EG95)

$$\frac{\partial(zg)}{\partial t} = (p-z)\alpha\Psi - \nabla(zg\mathbf{v}) \quad (2)$$

where p is the *yield* i.e. the *ratio between the mass of new ejected metals and the mass which remains locked up in low mass unevolving stars and remnants.*

We consider an initial mass function (IMF) which does not depend on **r**. Therefore both the parameter α and p are constant in **r**.

If we consider radial flows (i.e. **v**=v**r**/r), in spherical polar coordinates and by assuming the following expression for the SFR

$$\Psi(g,\mathbf{r},t) = \nu(r)g^k \quad (3)$$

Eqs. (1) and (2) become

$$\frac{\partial g}{\partial t} = -\alpha\nu g^k - v\frac{\partial g}{\partial r} - g\frac{\partial v}{\partial r} - 2\frac{gv}{r} \quad (4)$$

$$\frac{\partial z}{\partial t} + v\frac{\partial z}{\partial r} = p\alpha\nu g^{k-1} \quad (5)$$

We now want to consider the fact that, since the IRA approximation sets to zero the life of massive stars, it introduces inevitably a time-spatial resolution. In other words previous equations (and in particular Eqs. (4) and (5)) are correct only if the gas velocity is not larger than a precise value depending on the spatial dimension within which we are interested to study metallicity and gas density variations. In fact, if the velocity is so high that in a time equal to the life of a massive star the gas is able to cover a distance l, is clear that we cannot use above equations to compute metallicity variations within the distance l. Eqs. (4) and (5) become totally inadequate to compute metallicity gradients when in a time equal to the life of a massive star the gas is able to leave the entire galaxy. In order to take into account the previous remark we consider two different contexts. In the first one we suppose the gas velocity very low (zero) until a given time t_{GW} (which depends on r). After this time the velocity becomes suddenly high and the gas is able to leave the galaxy in a very short time. We call the models belonging to this context GW-models. Previous equations are inadequate to describe GW-models: in the next subsection we give a method to compute metallicity gradients in GW-models. The second context is the opposite situation. The velocity is sufficiently low and its behaviour is smooth in time. Therefore in this case we can compute metallicity in the gas by Eqs. (4) and (5). We will refer to models belonging to this context as F-models. F-models do not produce any metallicity gradients in the gas if the parameters in Eq. (3) are $k=1$ (i.e. linear star formation) and $\frac{d\nu}{dr}=0$. This result is pointed out by Edmunds (th 3 in EG95).

In the following we consider three different models:

1. F-model with $k=1$ and $\nu(r) = \nu_0 e^{-r/R_0}$ (M1), where R_0 is the effective radius.
2. F-model with $k=2$ (quadratic star formation) and $\frac{d\nu}{dr}=0$ (M2);
3. GW-models for every Ψ (M3) (in this case the relation between matter and metallicity distribution does not depend on the star formation rate);

In next sections we compute the metallicity gradients in the two contexts considered.

2.1. F-models

The metallicity in gas z and the gas density g can be computed as function of r and t by solving directly Eqs. (4) and (5), once the function $v(r)$ is given. Let assume here (as in EG95) the following expression:

$$v(r) = v_0\left(\frac{R_0}{r}\right)^n \quad (6)$$

By introducing the quantities $x=\frac{r}{R_0}$ and $u_0=\frac{v_0}{R_0}$ Eqs. (4) and (5) become respectively in M1 and M2 models:

$$\frac{\partial g}{\partial t} + u_0 x^{-n}\frac{\partial g}{\partial x} = -\alpha\nu_0 e^{-x} g + \frac{(n-2)gu_0}{x^{n+1}} \quad (7)$$

$$\frac{\partial z}{\partial t} + u_0 x^{-n}\frac{\partial z}{\partial x} = p\alpha\nu_0 e^{-x} \quad (8)$$

and

$$\frac{\partial g}{\partial t} + u_0 x^{-n}\frac{\partial g}{\partial x} = -\alpha\nu_0 g^2 + \frac{(n-2)gu_0}{x^{n+1}} \quad (9)$$

$$\frac{\partial z}{\partial t} + u_0 x^{-n}\frac{\partial z}{\partial x} = p\alpha\nu_0 g \quad (10)$$

These are linear equations in the partial derivatives. We assume zero metallicity at $t=0$ everywhere in the galaxy (i.e. $z(x,0)=0$) and we set the gas density at the initial time $g(x,0)=g_0 f(x)$ where $f(x)$ is a dimensionless function. The solutions of the above equations are:

-M1

$$g = g_0 f(ax) a^{2-n} exp\left[-\frac{\alpha\nu_0}{u_0}\int_{ax}^{x} dy e^{-y} y^n\right] \quad (11)$$

$$\frac{z}{p} = \frac{\alpha\nu_0}{u_0}\int_{ax}^{x} dy e^{-y} y^n \quad (12)$$

-M2

$$g = g_0 f(ax)\left[a^{n-2} - \frac{\alpha\nu_0 g_0 f(ax)(a^{2n-1}-1)x^{n+1}}{u_0(2n-1)}\right]^{-1} \quad (13)$$

$$\frac{z}{p} = \alpha\nu_0\int_0^t d\tau g(xa(x,t-\tau),\tau) \quad (14)$$

where a depends on x and t by the following expression:

$$a(x,t) = \left[1 - \frac{u_0 t(n+1)}{x^{n+1}}\right]^{\frac{1}{n+1}} \quad (15)$$

A convenient parameterization is to measure t in units of a star formation time-scale: $t = f(\alpha\nu_0)^{-1}$ and $t = f(\alpha\nu_0 g_0)^{-1}$ respectively in M1 and M2 models.

Eqs. (12) and (14) yield the metallicity in the gas. However, we are interested to know the mean stellar metallicity. The mass-averaged metallicity of a composite stellar population in x is defined following Pagel and Patchett (1975) as:

$$< z >_* = \frac{\int_0^t g(x,\tau)^k z(x,\tau) d\tau}{\int_0^t g(x,\tau)^k d\tau} \quad (16)$$

By substituting Eqs. (11), (12) and (13), (14) in (16) we can compute the metallicity in stars respectively in M1 and in M2 models.

2.2. GW-models

In this context we can compute chemical evolution in each galactic region as in the Simple Model (Tinsley 1980) until the time t_{GW}, when the gas has the necessary energy to leave the galaxy. In order to compute the metallicity in the gas and in the stars the galaxy is partitioned in zones having the shape of spherical shell and thickness δr with $\frac{\delta r}{r} << 1$.

The equation for the gas evolution in each region is (Simple Model)

$$dg = -\alpha ds \quad (17)$$

where $ds = \Psi dt$.

The equation for the evolution of metals is (under the assumption that $z << 1$):

$$d(gz) = (p - z)\alpha ds \quad (18)$$

Eqs. (17) and (18) have the following solutions

$$g = g(x, 0) - \alpha s = g_0 f(x) - \alpha s \quad (19)$$

$$z = -p ln(\frac{g}{g(x, 0)}) \quad (20)$$

Substituting previous equations in (16) we can compute the metallicity in stars in each galactic region for each $t < t_{GW}$ (or $s < s_{GW}$ where αs_{GW} is the mass of the stars and stellar remnants when galactic wind occurs). We obtain

$$\frac{< z >_*}{p} = 1 + \left(\frac{g_0 f(x)}{\alpha s} - 1\right) ln \left(1 - \frac{\alpha s}{g_0 f(x)}\right) \quad (21)$$

3. Relation between matter and metallicity distribution

To obtain informations on the matter distribution from the metallicity distribution we must compare the thermal energy in the gas at time t injected by supernovae (of type I and II) with the binding energy due to gravitational attraction: the latter depends, of course, on the matter distribution.

We deal F-Models and GW-models separately again.

3.1. F-models

In F-models the behaviour of the gas velocity is smooth in time, namely the injection of energy into the ISM is an adiabatic process. Therefore we can impose, at each time, that the amount of energy injected by supernovae in the time dt is exactly equal to the work done by the gas in the gravitational field in that time. Namely we impose the condition:

$$dE_{th_{SN}} = dL_{Grav} \quad (22)$$

Let consider the gas in the shell at galactocentric distance r and thickness δr. The total thermal energy in the gas at the time t is (Matteucci and Tornambè 1987):

$$E_{th_{SN}}(r, t) = \int_0^t \epsilon_{th_{SN}}(t - x) R_{SN}(x) dx \quad (23)$$

where $R_{SN}(x)$ is the SN rate (either type I or II) and $\epsilon_{th_{SN}}(t_{SN})$ is the fraction of the initial blast wave energy which is transferred by the SN into the ISM as thermal energy. A detailed description of the SN rate is available in Matteucci and Greggio (1986).

In the *IRA* approximation we can write

$$R_{SN}(x) = 4\pi r^2 \delta r F \nu g^k \quad (24)$$

where

$$F = A \int_{M_{Bm}}^{8} \phi(M) dM + \int_8^{100} \phi(M) dM \quad (25)$$

and A and M_{Bm} are defined in Matteucci and Greggio (1986). The *IRA* approximation related to the SNI is questionable because the SNI progenitors have mass not very large and consequently their lifetime is not negligible. However, for small values of t the contribution of SNI is negligible with respect to SNII (see Fig. 1 of Matteucci and Greggio 1986) and this condition is acceptable in the hypothesis that metallicity gradients form in a time not larger than a few Gyr.

Substituting (24) in (23) we obtain

$$E_{th_{SN}}(r, t) = 4\pi r^2 \delta r F \nu \int_0^t \epsilon_{th_{SN}}(t - y) g(r, y)^k dy \quad (26)$$

If we adopt the Larson (1974b) prescription for $\epsilon_{th_{SN}}(x)$ ($\epsilon_{th_{SN}}(x) = 0.1\epsilon_0 = 0.1\ 10^{51} erg$), we have

$$E_{th_{SN}}(r, t) = 4\pi r^2 \delta r F \nu 0.1\epsilon_0 \int_0^t g(r, y)^k dy \quad (27)$$

By differentiating this last equation with respect to the time we can write the condition given in Eq. (22) as:

$$4\pi r^2 \delta r 0.1\epsilon_0 F \nu g^k = 4\pi r^2 \delta r g G \frac{M(r)}{r^2} v \quad (28)$$

where $M(r)$ is the total mass (gas, stars and dark matter) within the radius r. By differentiating Eq. (28) with respect to r and adopting the expression given in (6) for v we obtain the following expressions for the total mass density respectively in the M1 and M2 models:

$$\rho_T(x) = \frac{0.1\epsilon_0 F\nu_0}{4\pi G u_0 R_0^2}(2+n-x)x^{n-1}e^{-x} \tag{29}$$

and

$$\rho_T(x) = \frac{0.1\epsilon_0 F\nu_0}{4\pi G u_0 R_0^2}\left[(2+n)g + x\frac{dg}{dx}\right]x^{n-1} \tag{30}$$

3.2. GW-models

Chemical evolution in a given galactic region proceeds until the gas has the necessary energy to leave the galaxy, namely until the galactic wind starts in that region. In fact Ψ is different from zero until $t < t_{GW}$, or equivalently $s < s_{GW}$. Therefore, we can establish the following relation between metallicity in stars and s_{GW} from Eq. (21)

$$\frac{< z >_*}{p} = 1 + \frac{1-\tau}{\tau} ln(1-\tau) \tag{31}$$

where $\tau = \frac{\alpha s_{GW}}{g_0 f(x)}$. s_{GW} is related to t_{GW} by the solution of the equation $ds = \Psi dt$. The values of t_{GW} depend on r. In fact in the region at galactocentric distance r, t_{GW} is given by the following condition:

$$E_{th_{SN}}(r, t_{GW}) = E_{Bgas}(r, t_{GW}) \tag{32}$$

where $E_{th_{SN}}(r, t_{GW})$ is given by Eq. (27) with $t = t_{GW}$ and $E_{Bgas}(r, t_{GW})$ is the binding energy due to the gravitational attraction.

We define $E_{Bgas}(r, t_{GW})$ as the energy necessary to carry the gas in a given galactic region at galactocentric distance r outside the galaxy. Therefore we can write

$$E_{Bgas}(r, t_{GW}) = 4\pi r^2 \delta r G g(r, t_{GW}) \int_r^\infty \frac{M(r')}{r'^2} dr' \tag{33}$$

Eq. (27) expressed in terms of s and at $t = t_{GW}$ becomes

$$E_{th_{SN}}(r, s_{GW}) = 4\pi r^2 \delta r F 0.1\epsilon_0 s_{GW} \tag{34}$$

Substituting Eqs. (33) and (34) in Eq. (32), by using Eq. (19) and by differentiating with respect to r we obtain finally

$$\frac{M(r)}{r^2} = -\frac{0.1F\epsilon_0}{\alpha G R_0}\frac{\tau'}{(1-\tau)^2} \tag{35}$$

where the prime denotes the derivation with respect to $x = (\frac{r}{R_0})$

Eqs. (31) and (35) can be used to obtain informations on the matter distribution in the following way. We consider a given value of r. From the Eq. (31) we can calculate the corresponding value of τ and τ', once the functions $\frac{<z>_*}{p}$ and $\frac{<z>'_*}{p}$ are known (this is possible because the function in Eq. (31) is bijective, i.e. invertible (Fig. 6)). By substituting these values of τ and τ' in the Eq. (35) we obtain the quantity $M(r)$.

4. Discussion

The luminous mass density profile in elliptical galaxies is given by the following analytical expression (Jaffe 1983)

$$\rho_{Lum}(r) = \frac{1}{r^2(1+r)^2} \tag{36}$$

where r is normalized to 1 at the radius containing half the total emitted light in the space. This radius equals $R_0/0.763$.

We can compute for our models the density of luminous matter (star and gas), as a function of the radius. We obtain

-M1

$$\rho_l(x,t) = \alpha_* \nu_0 e^{-x} \int_0^t d\tau g(x,\tau) + g(x,t) \tag{37}$$

-M2

$$\rho_l(x,t) = \alpha_* \nu_0 \int_0^t d\tau g(x,\tau)^2 + g(x,t) \tag{38}$$

-M3

$$\rho_l(x,t) = \alpha_* s_{GW} = \frac{\alpha_*}{\alpha} g_0 f(x)\tau \tag{39}$$

where α_* is the fraction of material formed into stars which will remain as long lived stars (all the stars with mass less than the solar mass, in the IRA approximation).

Let suppose an initial gas density profile given by the following expression:

$$f(x) = x^{-\theta} \tag{40}$$

For each model we choice the best value for θ by imposing the condition $\rho_l(x) = \rho_{lum}(x)$. We obtain for all the models values of θ in the range [2,2.5]. These values are obtained in F-models by integrating eq (37) and (38) up to $t_0 = 20(\alpha\nu_0)^{-1}$ and $t_0 = 20(\alpha\nu_0 g_0)^{-1}$ respectively in M1 and M2 models. In M3 model the calculation of $\rho_l(x,t)$ requires the knowledge of the function $\tau(x)$. We obtained this function from equation (31), where $\frac{<z>_*}{p}$ is obtained from observational data (Davies et al. 1993), as explained in the following.

In order to compare our results with observational data we have to specify the value of the yield. At this regard we observe that our models produce metallicity always less than the yield, on the contrary to what happens in models of galaxy formation involving dissipative collapse (Larson 1974a, Carlberg 1984, Lynden-Bell 1975, Pagel 1997) without galactic wind, where the overall mean approximates the yield and the central abundance is several times the yield. In GW-models in fact the chemical evolution occurs as in the Simple Model until $t = t_{GW}$. Now in the Simple Model it is easy to show that the mass-weighted mean stellar abundance is less than the yield and is equal to it in the limit of gas exhaustion (Edmunds 1990). Also in F-models we have the same result. In order to prove this it is sufficient to demonstrate that this is true in the most internal region because we consider only models with negative gradient (hence with the highest metallicity in the central region). In the most internal

Table 1. Values of β and γ related to the data of Davies et al. (1993). 2^{st} and 3^{d} column regard the projected metallicities along the line of sight, whereas 4^{th} and 5^{th} column regard local metallicities.

$Galaxy$	β_p	γ_p	β	γ
NGC 315	0.416	0.125	0.427	0.145
NGC 741	0.232	0.216	0.243	0.247
NGC 3379	0.254	0.224	0.266	0.256
NGC 3665	0.276	0.120	0.283	0.139
NGC 4374	0.320	0.209	0.335	0.239

region there is not inflow because we consider only positive gas velocities. Therefore this region satisfies the hypothesis of the following theorem (T(8) of Edmunds 1990) *The mass-weighted mean stellar abundance in a model with outflow is always less than that of the simple model*.

In order to have good agreement with observational data, we are forced to fix the value of the yield to 0.05. This value corresponds to have a highest value of Mg_2 equal to 0.35 (obtained by substituting $z = p = 0.05$ in the calibration formula (43) below).

To explain this high value we have three possibilities:

1. our models are not correct;
2. elliptical galaxies have high yields with respect to galaxies with different symmetry (i.e. the IMF is flatter in ellipticals than in the other galaxies);
3. the yield is high in all of galaxies but in the disk galaxies there is some mechanism able to lower the metallicity.

The dependence of $< z >_*$ on r can be reasonably chosen as

$$\frac{< z >_* (r)}{p} = \beta \left(\frac{r}{R_0}\right)^{-\gamma} \tag{41}$$

in accordance with many observational data (Carollo et al. 1993, Davies et al. 1993) in the range $0.1 < \frac{r}{R_0} < 1$. The quantities β and γ can be determined experimentally. We take into account that observational data refer to projected and not local mean abundances.

Let indicate with $< z >_{*proj} (R)$ the projected metallicity:

$$\frac{< z >_{*proj} (R)}{p} = \beta_p \left(\frac{R}{R_0}\right)^{-\gamma_p} \tag{42}$$

In Table 1 we report the values of β_p and γ_p (2^{st} and 3^{d} column) related to the data of Davies et al. (1993).

We computed these parameters by doing a least squares fit (between (42) and observational data) and by using the following calibration formula (Buzzoni et al. 1992)

$$log\left(\frac{z}{z_\odot}\right) = 5.85 Mg_2 - 1.66 \tag{43}$$

where $z_\odot$ is the solar metallicity.

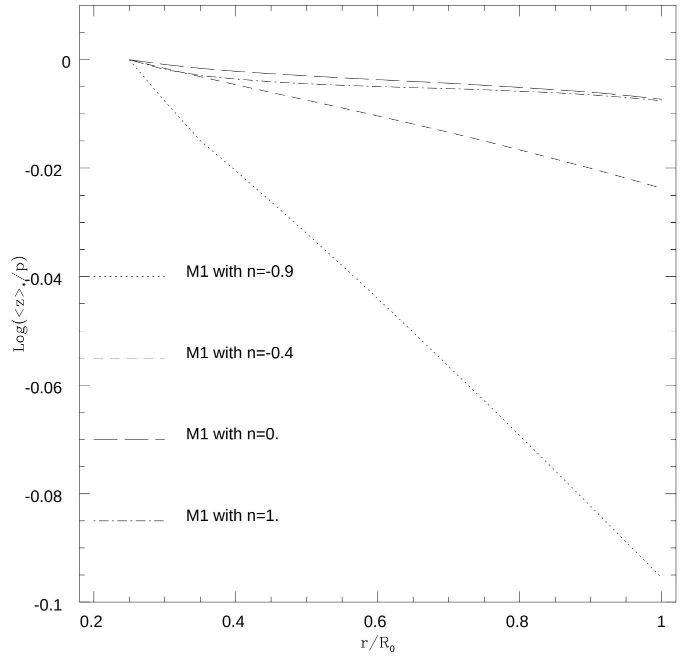

Fig. 1. Abundance gradients in the stars in M1 model for various values of n. The models are calculated at time $t = 20t_s = 20(\alpha\nu_0)^{-1}$.

The local metallicity is related to the projected one by the following expression (Ciotti, Stiavelli and Braccesi 1995)

$$\frac{< z >_* (r)}{p} = \frac{1}{p} \frac{\int_r^\infty \frac{d< z >_{*proj}(R) I(R)}{dR} \frac{dR}{\sqrt{(R^2-r^2)}}}{\int_r^\infty \frac{dI(R)}{dR} \frac{dR}{\sqrt{(R^2-r^2)}}} \tag{44}$$

where $I(R)$ is the apparent light intensity per unit area emitted by the galaxy (we use the de Vaucouleurs (1948) law, i.e. $I(R) = exp\left[-7.67\left(\frac{R}{R_0}^{\frac{1}{4}} - 1\right)\right]$). In 4^{th} and 5^{th} column of Table 1 we report β and γ obtained as the values for which there is the best agreement between (44) and (41).

Once the metallicity distribution is known (namely once β and γ are specified), it is possible to find the distribution of matter.

4.1. F-models

Eqs. (29) and (30) yield the density profiles in M1 and M2 models. To use these expressions we must know the value of n: we can find the best values for n by computing metallicity in stars as explained in Sect. 2.1 and by comparing our theoretical results with the expression (41).

The results related to M1 model are shown in Fig. 1,2.

We plot in Fig. 1 the metallicity in the stars as function of the radial distance from galactic centre for various values of the parameter n. We can observe that decreasing n tends to steepen the logarithmic gradient. Moreover we remark that for $n > 0$ the logarithmic gradient is nearly zero.

In Fig. 2 the mass density profiles related to the same models of Fig. 1. are shown together with the Jaffe luminous mass density profile.

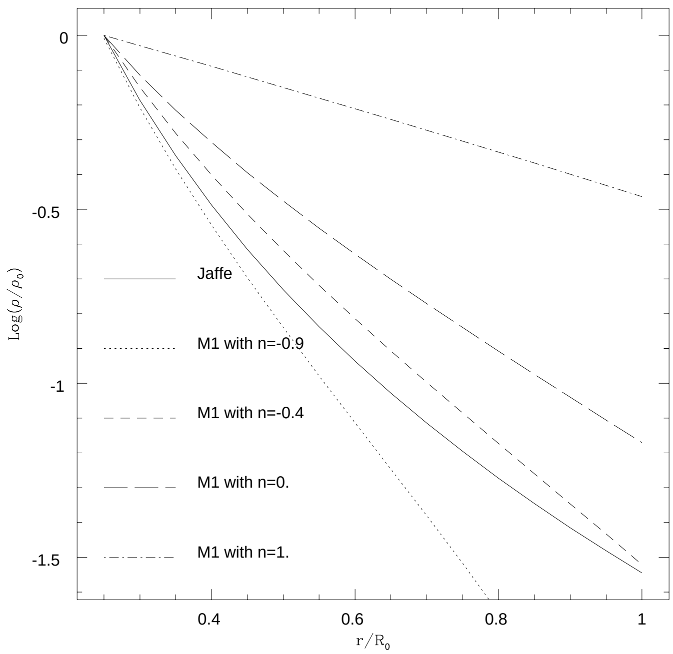

Fig. 2. Mass density profiles related to M1 model for various n together with the Jaffe mass density profile. The models are calculated at time $t = 20t_s$.

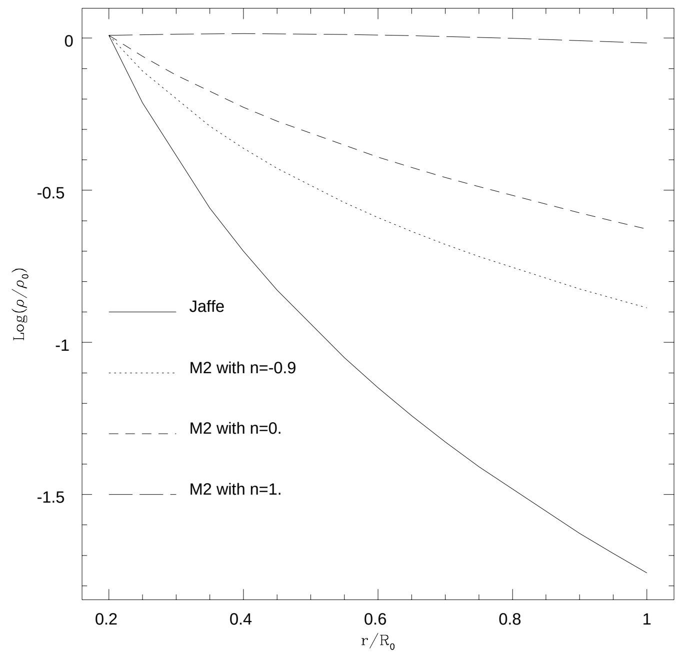

Fig. 4. Mass density profiles related to M2 model for various n together with the Jaffe mass density profile. The models are calculated at time $t = 20t_s$.

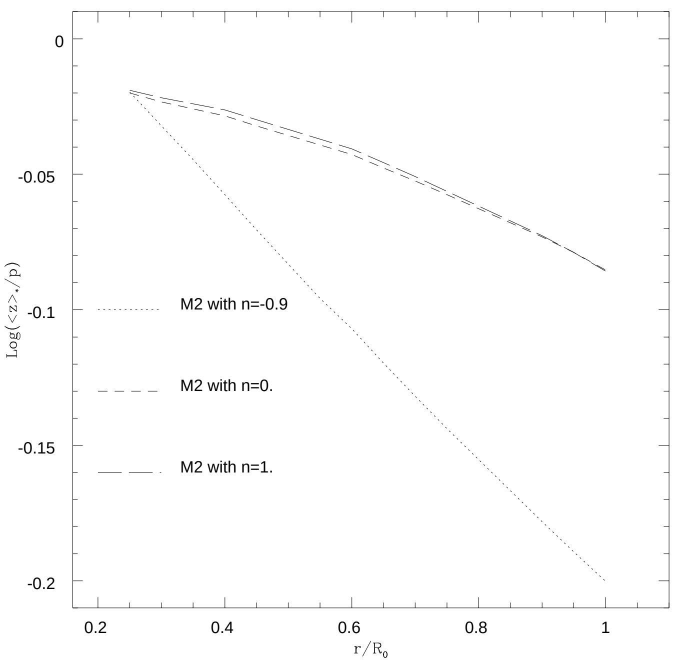

Fig. 3. Abundance gradients in the stars in M2 model for various values of n. The models are calculated at time $t = 20t_s = 20(\alpha\nu_0 g_0)^{-1}$.

We normalized the density profiles by imposing the condition $\rho_T(0.2) = \rho_{Lum}(0.2)$. For values of n between -0.9 and -0.4 there is a very good agreement with the jaffe distribution.

The results related to M2 model are shown in Fig 3,4.

Also in this case we observe that decreasing n tends to steepen the logarithmic gradient. However in M2 model $\rho_T(x)$ is always higher than the Jaffe profile for each value of n.

We observe from Fig 1,2,3,4 that models producing a flat metallicity gradient show also a flat density profile.

From the γ values in Table 1 we can see that the largest range in metallicity over a factor of 5 in radius is 1.51 (NGC3379) and the smallest is 1.21 (NGC3665). These values correspond to a logarithm difference of the metallicity of respectively 0.18 and 0.10. These values are of the same order of those found in F-models when $n = -0.9$ (Fig1,3). This is a good result because when $n \simeq -0.9$ the matter distribution approaches the Jaffe profile, as shown before.

4.2. GW-models

By differentiating Eq. (31) with respect to x we obtain

$$\tau' = -\frac{<z>'_*}{p}\frac{\tau^2}{ln(1-\tau)+\tau} \qquad (45)$$

Substituting this equation in (35), by using Eq. (41) we obtain finally

$$\frac{M(r)}{r^2} = -\frac{0.1F\epsilon_0}{\alpha G R_0}\frac{\tau^2}{(1-\tau)^2}\frac{\gamma\beta x^{-(\gamma+1)}}{ln(1-\tau)+\tau} \qquad (46)$$

where τ is related to x by the following equation

$$\beta x^{-\gamma} = 1 + \frac{1-\tau}{\tau}ln(1-\tau) \qquad (47)$$

In Fig. 5 we plot the functions $\rho_T(r)$ (obtained by differentiating Eq. (46) with respect to r) related to galaxies of Table 1. As in F-models we normalized the density profiles by imposing

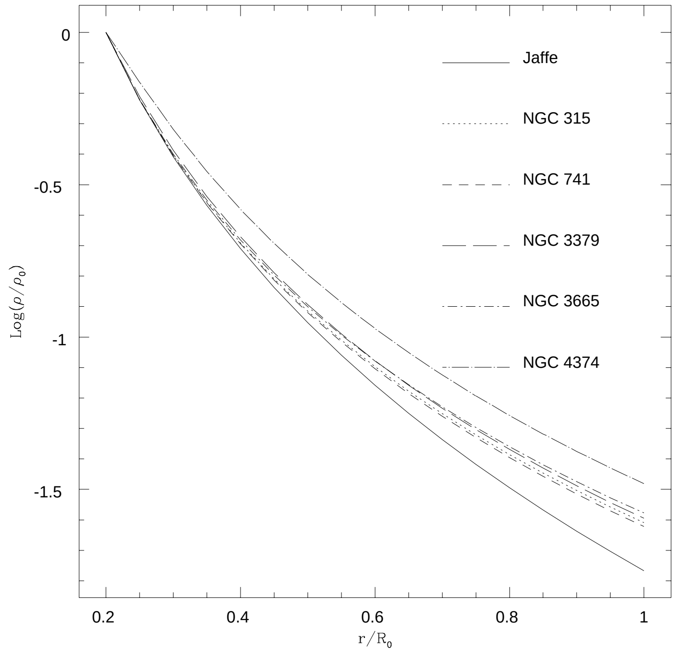

Fig. 5. Mass density profiles in M3 model related to the galaxies of Table 1 together with the Jaffe luminous mass density profile.

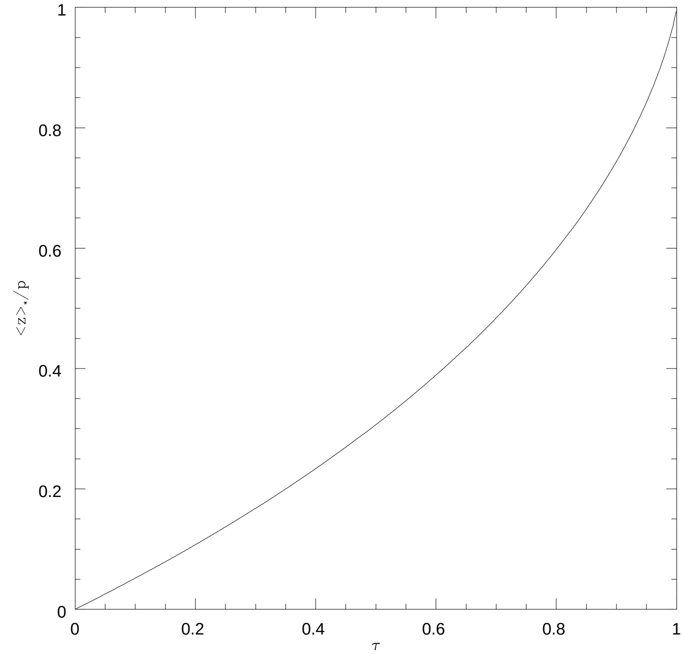

Fig. 6. Metallicity in the stars ($\frac{<z>_*}{p}$) as a function of τ, obtained from Eq. (31)

the condition $\rho_T(0.2) = \rho_{Lum}(0.2)$. We observe that all galaxies have a density profile a little less steep than the Jaffe profile. In any case there is a very good agreement between our total mass density profiles and the luminous density profile.

A model similar to that presented here (M3) was considered by Ciotti et al. (1995). In their work they found that a wind model produces metallicity gradients smaller than those shown before. However, in their work, they assumed two different hypothesis with respect to M3 model:

1. they considered a uniform distribution of matter whereas here is shown that a good agreement with observational data requires a Jaffe distribution of matter;
2. they assumed that the metallicity in the gas is proportional to the thermal energy in the gas injected by supernovae at the onset of the galactic wind. This is in contrast with our relations. In fact Eq. (34) states that $E_{th_{SN}}(r, s_{GW}) \propto s_{GW}$ and Eq. (20) states that $z \propto ln\left(\frac{g_0(x,t)}{g_0(x,t)-\alpha s_{GW}}\right)$. Therefore the relation between the gas metallicity and its thermal energy is not simply linear.

Previous results confirm that also a radially dependent galactic wind time is able to reproduce abundance gradients and mass density profiles in agreement with observational data (similarly to models involving dissipative processes).

It is interesting to find in the M3 model the relation between the abundance gradient and the ratio $\frac{M_s}{M}$ (i.e. the ratio between the mass remaining in the galaxy after the gas is removed by a galactic wind and the initial mass). In fact in Larson (1974b) and Carlberg (1984) models the abundance gradient depends very strongly on mass loss. The logarithm abundance gradient within a massive model galaxy is -0.5 and it flattens toward zero with increasing mass loss, namely in lower mass galaxies (see Fig. 3 of Larson 1974b). Also in the M3 model we have the same behaviour although the dependence between the abundance gradient and the mass loss is weaker. Moreover in this case we can only discuss qualitatively the previous behaviour. In M3 model we have

$$\frac{M_s}{M} = \frac{\int 4\pi x^2 g_0 f(x)\tau(x)dx}{\int 4\pi x^2 g_0 f(x)dx} \simeq \int \tau(x)dx \qquad (48)$$

where the last equality holds because $f(x) \simeq x^{-2}$ as we saw before. From Eq. (48) we observe that small $\frac{M_s}{M}$ ratios imply small values of τ in the galaxy.

Therefore from Fig. 6 we can conclude that small $\frac{M_s}{M}$ ratios imply small metallicity gradients because the function $\frac{<z>_*(\tau)}{p}$ is flatter for small τ than for higher ones. However the effect is not very considerable (on the contrary to what happens in dissipative models) because the function in Fig. 6 is almost linear (i.e. its second derivative is positive but not very high).

5. Conclusions

We found a relation between matter and metallicity distribution in two different contexts (F-models and GW-models) on the basis of the idea that the gas velocity (F-models) or the time (t_{GW}) when the gas leaves the galaxy (GW-models) depends both on the matter distribution (because of the gravitational attraction), and, of course, on the time that the gas spends in a given region. On the other hand this time is connected with the metallicity in that region.

Main results found are:

1. the mechanism proposed is able to produce metallicity gradients in good agreement with observational data. This result confirms the results obtained by numerical calculation (Martinelli et al. 1998).
2. F-models produce metallicity gradients small, unless the value of the parameter n is about -0.9. At this regard it will be interesting to use a more complicate expression for the gas velocity, with dependence on time, in such a way to allow radial inflow at the beginning and an outflow later. This is more realistic and can produce, probably, a metallicity larger than the yield in the central region of the galaxies (in fact, in this case, the assumptions of the theorem T(8) of Edmunds (1990) are not verified).
3. Both contexts produce density profiles related to the total mass in agreement with the Jaffe distribution (namely in agreement with the observed brightness profiles) confirming the result that dark matter in elliptical galaxies is not very influent within the effective radius (it should be distributed in a diffuse halo). In particular in GW-models the relation between matter and metallicity does not depend neither on the star formation rate nor on the initial mass distribution.

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