Astron. Astrophys. 335, 954–958 (1998)

ASTRONOMY
AND
ASTROPHYSICS

The acoustic cut-off frequency of roAp stars*

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Received 6 May 1997 / Accepted 4 April 1998

Abstract. Some of the rapidly oscillating (roAp) stars, have frequencies which are larger than the acoustic cut-off frequency determined from published stellar models which usually assume a grey atmosphere. As the cut-off frequency depends on the $T(\tau)$ relation, we have computed models and adiabatic frequencies for pulsating Ap stars with more realistic atmospheres which include a frequency dependent treatment of radiative transfer, take blanketing effects into account, and which have a better treatment of the radiative pressure. In addition, we are using opacity distribution functions specific to the atmospheric composition. With these improvements over the classical stellar models the theoretical acoustic cut-off frequency for roAp stars are increased by about 200 μHz, which brings them close to the observations.

We restrict the comparison of our computations with observations to those two 'pathological' roAp stars for which more reliable astrophysical parameters are available, HD 24712 and α Cir, and comment briefly on a third one, HD 134214. For α Cir we find models which have indeed a cut-off frequency beyond the largest observed frequency and which are well within the $T_{\rm eff}$ – $L/L_\odot$ error box. For HD 24712 only models which are hotter by about 100 K and less luminous by nearly 10% than what is actually the most probable value derived by spectroscopy would have an acoustic cut-off frequency large enough. HD 134214 fits our models best, however, the error box for $T_{\rm eff} - L/L_\odot$ is the largest of all three stars.

One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved.

Key words: stars: chemically peculiar – stars: oscillations – stars: individual: HD 24712; HD 128898; HD 134214

1. Introduction

For 5 out of 28 known rapidly oscillating magnetic chemically peculiar (CP2, Preston 1974) stars, the so-called roAp stars, the largest observed frequency exceeds the theoretical acoustic cut-off frequency, which is determined by the outermost stellar regions. Waves with frequencies larger than the cut-off frequency are not well reflected towards the resonant cavity and decay in the atmosphere with a decreasing amplitude.

It has been argued by Shibahashi and Saio (1985) that the cut-off frequency is largely influenced by the $T(\tau)$ relation which requires a careful modelling of these layers. Frequently, atmospheres in stellar models are based on an Eddington or Hopf law (e.g. Mihalas 1978) where convection is not included and radiative transfer is considered to be frequency independent (i.e. the grey case is assumed). The ATLAS9-code developed by Kurucz (1991) takes into account the effect of frequency-dependent opacities and also explicitly accounts for convection and radiative pressure. A further improvement has been implemented by us by using an opacity distribution function which is specific to the atmospheric composition of a given star.

We report here on cut-off frequencies computed for stellar models having the best atmospheres implemented which presently are available and compare them with observations.

2. Stellar models

The computation of atmosphere models alone would be sufficient to calculate the acoustic cut-off frequency. However, the measured values of luminosity are more reliable than those of surface gravity, especially with the HIPPARCOS data. We have thus computed full stellar models, with the *internal structure*, where the diffusion approximation for radiative transfer is valid, and the *atmosphere* represented by a $T(\tau, T_{\rm eff}, g)$ relation.

We started with Kurucz model atmospheres having solar composition and $\log g = 4.2$, for $T_{\rm eff}$ ranging from 7400 to 10000 K, without the "overshooting option" (Castelli 1996) and no additional contribution to line opacity by microturbulence. The approximation of a constant $\log g$ in the atmosphere in evolutionary calculations is justified as the $T(\tau, T_{\rm eff}, \log g)$ law is rather insensitive to small variations of $\log g$. Effects from a magnetic field are neglected in our stellar models.

The internal structure models were computed with the CESAM code (Morel 1993 and 1997), with the EFF equation of state, an initial hydrogen content $X = 0.7$, and a heavy-element

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* Based on HIPPARCOS data.

** Lise Meitner fellow of the Fonds zur Förderung der wissenschaftlichen Forschung abundance Z = 0.02, the OPAL95 opacities (Iglesias & Rogers 1996) and $\alpha \equiv 1.4$, the mixing-length parameter.

3. Cut-off frequencies

We make the following assumptions for computing the cut-off frequencies. For low-degree modes (which are the only modes observable for stars different from the Sun), the displacement is essentially vertical, so that the horizontal component can be neglected. Consequently, we consider only radial modes and, because we investigate modes of high radial orders, we adopt the Cowling approximation, i.e. neglect the perturbation of the gravitational potential.

Acoustic waves are reflected towards the interior and are well trapped, if the angular velociy ω is smaller than the acoustic potential V, whereas if ω is larger, the mode propagates into the atmosphere dissipating mechanical energy which decreases the mode amplitude. The *cut-off frequency*, $\nu_{\rm M}$, is the maximum value of the potential in the outermost stellar layers above which modes propagate outwards (see, e.g., Unno et al, 1989).

According to Vorontsov & Zarkhov (1989), the potential for radial modes can be written as:

$$V_1^2 = N^2 - \frac{c}{2}\frac{\mathrm{d}}{\mathrm{d}r}(cf) + \frac{c^2}{4}f^2, \quad (1)$$

where $f = 2/r + N^2/g - g/c^2 - (1/2c^2)(\mathrm{d}c^2/\mathrm{d}r)$ and N is the Brunt-Väisälä frequency. Another formulation is proposed by Gough (1986):

$$V_2^2 = c^2/(4H_\rho^2)\,(1 - 2\mathrm{d}H_\rho/\mathrm{d}r) \quad (2)$$

where $H_\rho = -(\mathrm{d}\ln\rho/\mathrm{d}r)^{-1}$ is the density scale height. Vorontsov & Zarkhov (1989) use the acoustical depth as the dependent variable, while Gough (1986) uses the radius. These different eigenfunctions give slightly different forms of V. In the approximation of an isothermal atmosphere, Eqs. (1) and (2) reduce to:

$$V_3^2 = \omega_{\rm c}^2 = \frac{c^2}{4H_\rho^2}. \quad (3)$$

Rapid density variations in the medium lead to an abrupt rise in the potential which constitutes a reflecting barrier to waves. But to some extent, waves penetrate and may be even tunnelling outwards (see e.g., Christensen-Dalsgaard and Frandsen 1983, Balmforth & Gough 1990). The thickness of the potential barrier, which determines the reflexion efficiency, is a function of the outer boundary conditions and of the potential V. The concept of the acoustic cut-off frequency is well defined only in the case of the isothermal atmosphere, and implies the existence of a temperature minimum where the potential is the largest. There was so far no clear evidence of such a temperature minimum in Ap stars, but Simon & Landsman (1997) have perhaps recently discovered a corona in those stars, which would establish the existence of a potential barrier. We shall hereafter assume that a temperature minimum exist and identify the cut-off frequency with the potential at this point.

We denote $\nu_{\rm M}^{(1)}$, $\nu_{\rm M}^{(2)}$, and $\nu_{\rm M}^{(3)}$ the cut-off frequencies derived from expressions (1), (2) and (3) respectively. Table 1 gives the values of the cut-off frequencies $\nu_{\rm M}^{(1)}$, $\nu_{\rm M}^{(2)}$ and $\nu_{\rm M}^{(3)}$ for a model of 1.8 $M_\odot$ as a function of age. The cut-off frequency scales as the characteristic frequency $\Omega_g = (\mathrm{G}M/R^3)^{1/2}$ and decreases along the main sequence (Shibahashi, 1991). The three expressions of the potential lead to a different thickness of the potential barrier and therefore a different efficiency of the wave reflexion (see e.g. Gabriel, 1992), which reflects the problem of the definition of the acoustic cut-off frequency concept. They yield however approximately the same values of the acoustic cut-off frequency. The cut-off frequency computed with these improved model atmospheres is about 8.5% larger than that derived from classical stellar models. Nonadiabatic effects and magnetic field are expected to affect our results, but probably not by such a large amount.

We will focus our discussion in the following sections on those two roAp stars, HD 24712 and HD 128898, for which we have more reliable mass estimates due to the availability of HIPPARCOS parallaxes.

Table 1. Acoustic cut-off frequencies $\nu_{\rm M}^{(1)}$, $\nu_{\rm M}^{(2)}$ and $\nu_{\rm M}^{(3)}$ in μHz (Eqs. 1,2 and 3) as a function of age (in Myr) for models with 1.8 $M_\odot$ and Z = 0.02.

age	$\log(T_{\rm eff})$	$\log(L/L_\odot)$	$\nu_{\rm M}^{(1)}$	$\nu_{\rm M}^{(2)}$	$\nu_{\rm M}^{(3)}$
100	3.915	1.018	3116	2879	3008
200	3.913	1.031	2985	2702	2876
300	3.911	1.042	2864	2989	2768
400	3.916	1.010	2726	3009	2674
500	3.903	1.066	2581	2653	2476
600	3.898	1.077	2426	2330	2378
800	3.880	1.099	2073	1879	2004
1000	3.858	1.112	1650	1661	1642
1100	3.841	1.116	1328	1330	1322

Table 2. List of roAp stars for which the largest published frequency is larger than the cut-off frequency determined from standard stellar models.

HD	Observed ν_{max} (μHz)	Ampl. mmag	Comment
6532	2 402	0.77	Kurtz et al. (1996)
24712	2 807	0.20	Kurtz et al. (1989)
128898	2 566	0.12	Kurtz et al. (1994)
134214	2 950	3.40	Kurtz et al. (1991)
203932	2 838	0.17	Martinez et al. (1990)

4. Comparison with observations

For about 5 out of 28 known roAp stars, the largest published frequency (see Tab. 2 and references therein) exceeds the expected theoretical acoustic cut-off frequency determined from standard stellar models (with a grey atmosphere). We do not consider here roAp stars for which the highest observed frequency probably is a harmonic of their nonlinear oscillation (HD 83368 (Kurtz et al. 1993), HD 101065 (Martinez & Kurtz 1990), HD 137949 (Kurtz et al. 1991), and HD 161459 (Martinez et al. 1991)).

The photometric and spectroscopic properties of the roAp star **HD 24712** with $T_{\rm eff} = 7250 \pm 150$ K (Ryabchikova et al. 1997), $\log(L/L_\odot) = 0.91 \pm 0.04$ (based on $\pi_{\rm HIPPARCOS} = 0\farcs02041 \pm 0\farcs00084$, a bolometric correction of - 0.085 (Schmidt-Kaler 1982) and neglecting interstellar extinction), can be reproduced with a model of 1.63 $M_\odot$, $Z = 0.02$ and an age of about 900 Myr. A stellar model hotter by about 100 K, less luminous by nearly 10%, and less evolved by 100 Myr would have a cut-off frequency in agreement with the largest observed frequency. However, such a model is compatible only with the lower left corner of the error box (see Fig. 1a).

We have also computed models with appropriate age for **HD 128898** (α Cir). For the first time, a Kurucz model atmosphere was calculated with an opacity distribution function specific to the composition of α Cir (Piskunov & Kupka 1997). Stellar models with 1.93 $M_\odot$, $Z = 0.03$ and an age of 400 Myr, fit the observed values (Kupka et al. 1996) of $T_{\rm eff} = (7900 \pm 200)$ K and $\log(L/L_\odot) = 1.11 + 0.01/-0.02$. The luminosity is based on $\pi_{\rm HIPPARCOS} = 0\farcs06097 \pm 0\farcs00058$, a bolometric correction of - 0.12 (Schmidt-Kaler 1982), neglecting interstellar extinction. The computed acoustic cut-off frequency is compatible with the largest observed frequency (see Table 2) and we can therefore conclude, that no discrepancy may exist for this roAp star between theoretical and observed cut-off frequencies.

However, the cut-off frequency depends on the model input parameters and one must therefore account for uncertainties inherent to observations and modelling. For example, there are problems with the photometric calibration of fundamental parameters of CP stars. Models of different mass and age can fit the same star in the H-R diagram within the error box. For HD 24712, e.g., the cut-off frequency varies from 2725 μHz (model with 1.60 $M_\odot$ and 800 Myr) which is only 80 μHz short of the largest observed frequency, to an even lower value of 2294 μHz (1.63 $M_\odot$, 1100 Myr). For α Cir, a model with 1.90 $M_\odot$ and 600 Myr gives $\nu_{\rm M}^{(1)} = 2329\,\mu$Hz which would be clearly smaller than the largest observed frequency.

A similar situation exists for **HD 134214** which, unfortunately, has a considerably larger error box due to the HIPPARCOS parallax of only 10.92±0.89 marcsec and a $T_{\rm eff}$ which could be estimated only from photometric indices.

Evolutionary tracks and lines of constant cut-off frequency for Kurucz models are plotted in Fig. 1a together with the observational error boxes. Models with $Z = 0.02$ were investigated for HD 24712 and HD 134214, and models with $Z = 0.03$ for HD 128898. Errors on the mass and age determination are about 0.02 $M_\odot$ and 200 Myr for HD 24712, and 0.02 $M_\odot$ and 100 Myr for HD 128898.

Asteroseismology potentially is a powerful tool for determining the evolutionary status of stars via the frequency separation

$$\nu_0 = (2 \int_0^R (dr/c))^{-1} \sim \nu_{n,\ell} - \nu_{n-1,\ell}$$

(see e.g. Shibahashi 1991, and Kurtz & Martinez 1993). If ν_0 can be measured, a more reliable estimate of the evolutionary status can be derived than with our classical approach, because

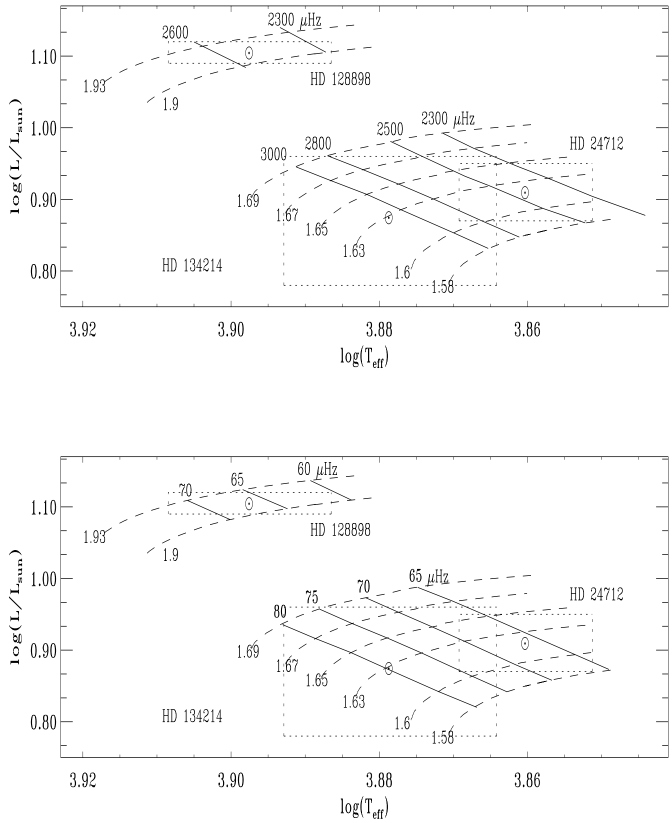

Fig. 1. a (upper panel): HR diagramme for stars with 1.58 $M_\odot$ to 1.69 $M_\odot$ for $Z = 0.02$ (age up to 1000 Myr), and with 1.90 and 1.93 $M_\odot$ for $Z = 0.03$ (age up to 700 Myr) (dashed lines). The roAp stars HD 24712, HD 134214 and HD 128898 are indicated by circles and error boxes. Full lines are lines of constant cut-off frequency $\nu_{\rm M}^{(1)}$ for the Kurucz models, for 2300, 2500, 2800 and 3000 μHz for $Z = 0.02$, and for 2300, 2500 and 2600 μHz for $Z = 0.03$, **b** (lower panel) and lines of constant frequency spacing $\nu_0 = (2 \int_0^R (dr/c))^{-1}$ for the same models, from 80 to 65 μHz for $Z = 0.02$, and from 70 to 60 μHz for $Z = 0.03$. For HD 24712 the frequency splitting $\nu_0 = 68\,\mu$Hz (Kurtz et al. 1989), and 50 μHz (Kurtz et al. 1994) for HD 128898.

no bolometric correction (determined for chemically 'normal' stars) and interstellar extinction (with large local differences) have to be used. Fig. 1b shows lines of constant frequency spacing ν_0 for the same models as for Fig. 1a.

The observed value $\nu_0 = 68\,\mu$Hz for HD 24712 (Kurtz et al. 1989) is consistent with our classically determined error box and indicates an effective temperature which should be larger by about 100 K than what was obtained spectroscopically. There is a serious problem for α Cir, because the observed value of ν_0 is 50 μHz (Kurtz et al. 1994) which cannot be reconciled with the spectroscopically determined effective temperature and/or the luminosity derived via the HIPPARCOS parallax. One has to stress, however, that the amplitudes for the overtone oscillations relative to the mode with the largest amplitude are very small and of the order of only a few 0.1 mmag and ν_0 might have been incorrectly determined.

5. Conclusion

We have shown that the major impact for modelling pulsation frequencies close to the cut-off frequency comes from the inclusion of a frequency-dependent treatment of radiative transfer and of blanketing effects, as well as from a better calculation of the radiative pressure in the model atmospheres. For the parameter space discussed in this paper, such model atmospheres merged with stellar models derived from the CESAM code increase the cut-off frequency by about 8.5 % relative to the value derived from the grey Hopf $T(\tau)$ relation.

However, when investigating a large parameter space in mass, age and metallicity, it appears to first order to be sufficient to compute stellar models with a simple grey (e.g. Hopf –) atmosphere which is fast, and to approximate the effects of a better treatment of the atmosphere on the acoustic cut-off frequency by increasing this frequency by about 8.5%. This factor is in agreement with speculations of Shibahashi & Saio (1985) and of Matthews et al. (1990, 1996) that a steeper than solar temperature gradient would increase the cut-off frequency and hence bring theoretical results closer to the observations.

For two roAp stars with probably the best available mass and luminosity estimates, HD 24712 and α Cir, we find models with parameters in agreement with the observational error box which have a theoretical cut-off frequency larger than the largest observed frequency and hence are in agreement with observations. One may thus speculate that the old controversy about a mismatch between observed largest frequencies and theoretical cut-off frequencies of roAp star models is resolved.

We have assumed solar abundances for our models except for α Cir, which, however, has an abundance pattern that does not significantly change the model atmosphere from one with solar abundances. HD 24712, on average, is even less peculiar than α Cir. Assuming that Z = 0.03 reflects only a surface composition for α Cir and Z = 0.02 would be the better choice for a stellar model, one can expect a smaller mass to fit α Cir and a smaller cut-off frequency as deduced from Fig. 1. For other roAp stars with a more peculiar abundance pattern, the deviation from model atmospheres with solar abundances or models with a scaled heavy element abundance will be larger (Gelbmann 1997). Abundant rare-earth elements, through blanketing effects, could decrease the surface temperature and thus increase the cut-off frequency. This frequency might also be affected by a chemical composition gradient (Vauclair & Dolez 1990). Finally, improving CP2 star pulsation models requires also to account for the magnetic field (Dziembowski & Goode 1996), non-adiabatic and NLTE-effects. Unfortunately, NLTE atmospheres with a treatment of line blanketing as sophisticated as in our models (ATLAS9 plus opacity distribution functions specific to the composition), are not available in the foreseeable future, at least for the main elements contributing to the opacity in the upper atmospheric regions.

Acknowledgements. We are indebted to the referee, Wojciech Dziembowski and to Hiromoto Shibahashi for their detailed remarks which helped improving the manuscript, as well as to Maurice Gabriel, Douglas Gough and Günter Houdek for their constructive comments. We are grateful to Claude van't Veer, Claude Mégessier and Rodney Medupe for stimulating discussions. We also thank Jørgen Christensen-Dalsgaard and Nikolai Piskunov for their useful comments.

This research was done within the working group *Asteroseismology-AMS*. Computing resources and financial support for this international collaboration were provided by the Fonds zur Förderung der wissenschaftlichen Forschung project *S 7303-AST* (Lise Meitner Fellowship granted for NA, project *M00323-AST*), and Digital Equipment Corp. (Europe External Research Program, project *STARPULS*). NA is grateful to PPARC (UK) for financial support.

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