Astron. Astrophys. 335, 969–972 (1998)

ASTRONOMY AND ASTROPHYSICS

The effect of interior magnetic field on the modified Urca process and the cooling of neutron stars*

Y.F. Yuan[1,2] **and J.L. Zhang**[2]

[1] CCAST (World Laboratory) P.O. Box 8730, Beijing 100080, P.R. China
[2] Center for Astrophysics, University of Science and Technology of China, Hefei 230026, Anhui, P.R. China
(yuan@cfa.ustc.edu.cn; jlz@cfa.ustc.edu.cn)

Received 10 March 1998 / Accepted 25 March 1998

Abstract. The effect of a strong magnetic field on the modified Urca process in neutron star cores is considered in this paper. It is found that this effect is significant when the interior magnetic field is greater than 10^{18} G, especially at some resonant states. In these situations, the strong magnetic field will significantly increase the rate of neutrino emission and influence the cooling of neutron stars. Comparing with the observational data, we provide information about the order of the strength of interior magnetic field for the first time. The strength we obtained is about $\sim 10^{19}$ G.

Key words: dense matter – stars: magnetic fields – stars: neutron

1. Introduction

The thermal evolution of a neutron star may provide information about the interior or outer layer of the star, for instance, determining the equation of state and determining whether quark state, hyperons, pion or koan condensation exists, whether the direct or modified Urca process dominates, whether the nucleons are Cooper pairing (see Page & Baron 1990, Haensel & Gnedin 1994 and references therein), whether the star has an accreted envelope (Potekhin, Chabrier and Yakovlev 1997), and so on.

Having considered the symmetry energy of nucleons, Lattimer et al. (1991) showed that a sufficiently large proton fraction in interior occurs, so the direct Urca reaction which lead to a very rapid cooling of neutron star happens:

$$n \to p + e^- + \bar{\nu}_e, \quad p + e^- \to n + \nu_e. \tag{1}$$

This happens only when the density ρ is several times larger than the standard nuclear density $\rho_0 = 2.8 \times 10^{14}\ \mathrm{g\ cm^{-3}}$. In the cores of not too massive stars ($M < (1.3-1.4)\, M_\odot$, depending on the model of dense matter), the direct Urca process is forbidden because of violation of momentum conservation. Chiu and Salpeter (1964) therefore proposed modified Urca reactions

$$n + n \to n + p + e^- + \bar{\nu}_e, \quad n + p + e^- \to n + n + \nu_e, \tag{2}$$

where the bystander particle (a neutron) is present to absorb momentum. The neutrino cooling rate in the process (2) was calculated by Bahcall and Wolf (1965), Friman and Maxwell (1979) and Maxwell (1987). Because of large phase-space reduction due to nucleon degeneracy, this rate is strongly suppressed. At temperature T $\sim 10^9$ K the rate of the modified process is about five orders of magnitude lower than that of the direct process. Actually Eq. (2) represents the neutron branch. The second branch is the proton branch which was put forward by Itoh and Tsuneto (1972) and recently reanalyzed by Yakovlev and Levenfish (1995). Contrary to the result of Maxwell (1987), the later authors showed that this branch is as efficient as the neutron one.

It is presumed that the interior magnetic field strength could reach $\sim 10^{18}$ G according to the scalar virial theorem which is based on Newtonian gravity (Lai & Shapiro 1991). However, the matter density in the core is so high that the effect of general relativity is significant. Consequently, the interior magnetic field strength is expected to be further increased above 10^{18} G (Bandyopadhyay, Chakrabarty & Pal 1997). Lai and Shapiro (1991) have considered the influence of a strong magnetic field on some simple non-equilibrium processes, including β- decay and inverse β - decay. In this paper we study the effect of a high magnetic field on the modified Urca process in neutron star cores. Because the nuclear composition of the neutron star central layers is not known with certainty (Shapiro & Teukolsky 1983), we will consider the magnetic n-p-e equilibrium model. Chakrabarty et al. (1997) found that the nuclear matter in beta equilibrium converts into a stable proton-rich matter for magnetic field $\sim 10^{20}$ G (the critical magnetic field for protons) and beyond, hence the direct Urca process happens. Since the magnetic field we consider here is below $\sim 10^{20}$ G, we may neglect the influence of the strong magnetic field on the nuclear matter in neutron star cores as the first approximation.

The effect of a very strong magnetic field on the modified Urca process is considered in Sect. 2. Cooling curves of neutron

Send offprint requests to: Y.F. Yuan, Center for Astrophysics, University of Science and Technology of China, Hefei 230026, China

* This work is supported by the National Natural Science Foundation and National Climbing Programme on Fundamental Researches

stars and their comparison with the observational data are presented in Sect. 3. The conclusion and discussions will be given in Sect. 4.

2. Modified Urca process in strong magnetic fields

Throughout this paper, we consider the magnetic fields beyond $B_{\rm c}$, where $B_{\rm c} = m_{\rm e}^2 c^3/\hbar e = 4.414\times 10^{13}$ G is the critical magnetic field strength by the relation $\hbar(eB/m_{\rm e}c) = m_{\rm e}c^2$. Hence, we must use the relativistic Dirac equation for the electrons, which yields energy eigenstates

$$E_n = \left[c^2 P_z^2 + m_{\rm e}^2 c^4 \left(1 + n\frac{2B}{B_{\rm c}}\right)\right]^{1/2}, \tag{3}$$

where quantum number n is given by

$$n = n_{\rm L} + \frac{1}{2} + \sigma, \tag{4}$$

for the Landau level $n_{\rm L} = 0, 1, 2, ...$ and spin $\sigma = \pm\frac{1}{2}$ (For reference, see Mészáros 1993). The usual sum over electron states (per unit volume) in a zero field is

$$\sum_{\rm e} \rightarrow \frac{2}{h^3}\int {\rm d}^3 p = \frac{1}{\pi^2\lambda_{\rm e}^3}\int \left(\frac{p}{m_{\rm e}c}\right)^2 {\rm d}\left(\frac{p}{m_{\rm e}c}\right), \tag{5}$$

which should be replaced as follows when the magnetic field is nonzero,

$$\sum_{\rm e} \rightarrow \sum_n \frac{eB}{h^2 c} g_n \int {\rm d}p_z = \frac{B/B_{\rm c}}{(2\pi)^2\lambda_{\rm e}^3}\sum_n g_n \int {\rm d}\left(\frac{p_z}{m_{\rm e}c}\right), \tag{6}$$

where $\lambda_{\rm e} = \hbar/m_{\rm e}c$ is the electron Compton wavelength. Notice that the cyclotron frequency of protons is about 1836 ($m_{\rm p}/m_{\rm e}$) times smaller than the electron cyclotron frequency. However, as shown by Lai and Shapiro (1991), the values of n_m for electrons and protons are essentially the same for n-p-e system because of charge neutrality, here n_m is the upper limit of the summation over n in Eq. (6). Therefore, we cannot ignore magnetic field effects on the proton of state when n_m is small.

In the following calculations, we ignore the contribution of the proton branch of the modified Urca process in a magnetic field because the proton number density is about ten percent of neutron one in neutron star cores. According to Friman and Maxwell (1979), Yakovlev and Levenfish (1995) ($\hbar = c = k_{\rm B} = 1$), the neutrino energy production rate ε_0 of the neutron branch in zero magnetic field is given by

$$\varepsilon_0 = \frac{1}{2(2\pi)^{14} s} T^8 AIS \sum_{\rm spins} |M|^2, \tag{7}$$

$$A = 4\pi\left[\prod_{j=1}^{5}\int d\Omega_j\right]\delta\left(\sum_{j=1}^{5} p_j\right), \tag{8}$$

$$I = \int_0^\infty dx_\nu x_\nu^3 \left[\prod_{j=1}^{5}\int_{-\infty}^{+\infty} dx_j f_j\right]\delta\left(\sum_{j=1}^{5} x_j - x_\nu\right) \tag{9}$$

$$S = \prod_{j=1}^{5} p_{{\rm F}_j} m_j^*, \tag{10}$$

where p_j (j=1-4) are the nucleon momenta and p_5 is the electron momentum, $p_{{\rm F}_j}$ are the corresponding Fermi momenta of particles, $|M|^2$ is the squared matrix element of the first process (2), s=2 is a symmetry factor for that process, T is the temperature of the equilibrium system, $x_\nu = p_\nu/T$ is the dimensionless momenta of neutrino and $x_j = v_{{\rm F}_j}(p - p_{{\rm F}_j})/T$ are those of other particles, here $v_{{\rm F}_j}$ is the Fermi velocity and m_j^* is an effective mass and $f_j = [exp(x_j) + 1]^{-1}$ is the Fermi-Dirac function. The integrations in A and I are standard and yield (e.g. Shapiro & Teukolsky 1983)

$$A = \frac{2\pi(4\pi)^4}{p_{{\rm F}_n}^3}, \quad I = \frac{11513}{120960}\pi^8. \tag{11}$$

Taking the nucleon-nucleon interactions into account, Friman and Maxwell (1979) obtained

$$\varepsilon_0 = (7.4\times 10^{20})(\rho/\rho_0)^{2/3} T_9^8 \ {\rm ergs\ cm^{-3}\ s^{-1}}. \tag{12}$$

The previous studies demonstrated that the matrix elements for the neutron decay and electron capture reactions when magnetic field is nonzero are the same as those when magnetic field is zero (Fassio-Canuto 1969, Canuto & Ventura 1977, Lai & Shapiro 1991). We therefore need only consider the effect of a magnetic field on phase-space integration of the charged particles. Hence the energy loss rate of the modified Urca process in a magnetic field is written as

$$\varepsilon = \varepsilon_0 \cdot R, \tag{13}$$

where ε_0 is the rate in zero magnetic field, while R is the factor which describes the effect of the magnetic field on the modified Urca process. This factor can be written as follows according to Eqs. (8) and (9):

$$R = \frac{J}{I\cdot A}, \tag{14}$$

$$J = 4\pi\sum_{\rm e} f(x_{\rm e})\sum_{\rm p} f(x_{\rm p})\int\prod_{j=1}^{3} d\Omega_j \int_0^\infty dx_\nu \cdot x_\nu^3 \left[\prod_{j=1}^{3}\int dx_j f(z_j)\right]\delta\left(x_\nu - \sum_{j=1}^{5} z_j\right)\delta\left(\sum_{j=1}^{5} p_j\right). \tag{15}$$

After integrating over x_j with j=1-3, we have

$$J = \frac{1}{2}B_{\rm e}^*\left(\frac{m_{\rm e}c}{p_{\rm F_e}}\right)^2 \sum_n g_n \int_{-\infty}^{+\infty} {\rm d}\left(\frac{p_z^e c}{kT}\right) f(x_{\rm e}) \cdot\frac{1}{2}B_{\rm p}^*\left(\frac{m_{\rm p}c}{p_{\rm F_p}}\right)\sum_m g_m \int_{-\infty}^{+\infty} {\rm d}\left(\frac{p_z^{\rm p} c}{kT}\right) f(x_{\rm p}) \cdot H(x_{\rm e} + x_{\rm p}), \tag{16}$$

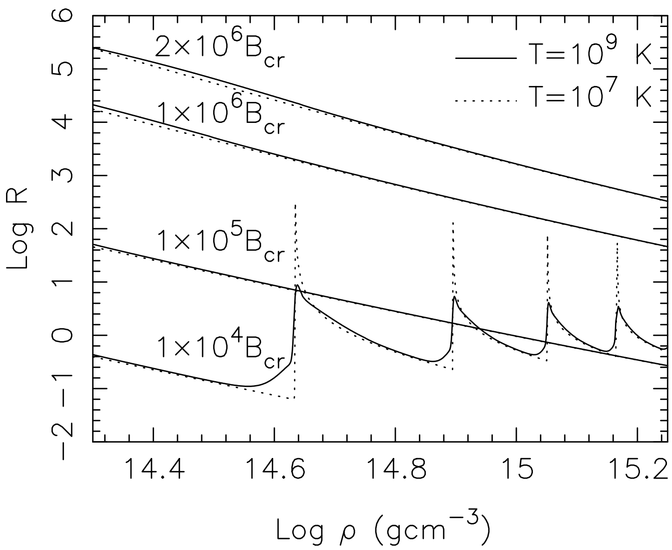

Fig. 1. Ratio R versus the density of neutron star cores under the influence of different interior magnetic field strengths

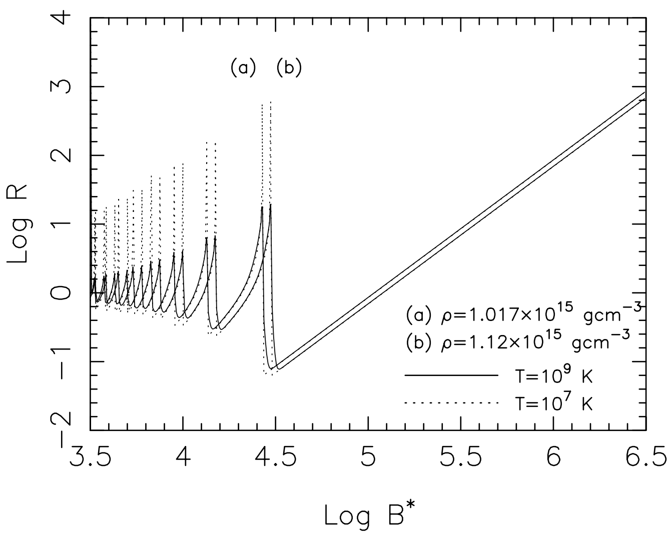

Fig. 2. Ratio R versus the interior magnetic field strength B^* (in units of B_{cr}) in two neutron star models

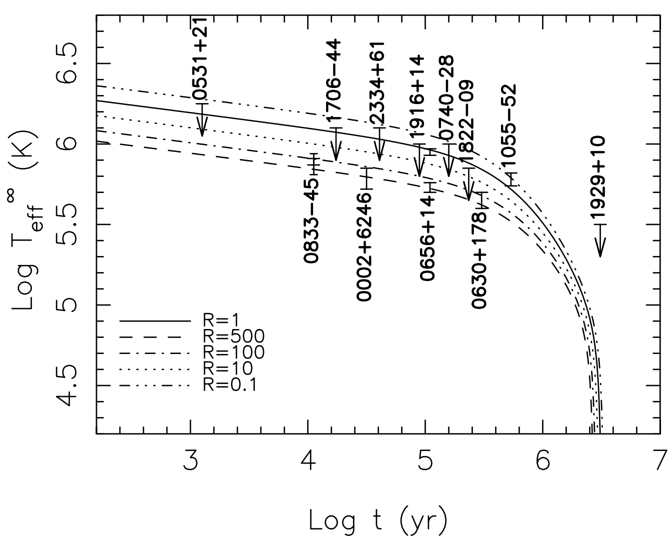

Fig. 3. Cooling curves of neutron stars based on the standard neutrino cooling. The zero magnetic field case corresponds to the solid line

where

$$H(x) = \frac{1}{2}\int_0^\infty \mathrm{d}s \cdot s^3 \cdot \frac{\pi^2 + (s-x)^2}{\mathrm{e}^{s-x}+1} \quad . \tag{17}$$

$B_{\rm e}^*$ and $B_{\rm p}^*$ in Eq. (16) are the magnetic field strengths in units of the critical magnetic field for electrons and protons, respectively.

Fig. 1 shows the ratio R as a function of density ρ when T=10^9 K and T=10^7 K, respectively. The saw-teeth shape of the curves is due to the fact for B$\neq 0$, we have more available phase space for electrons at given density, which causes the decay rate to increase. For the same reason, a nonzero magnetic field decreases the inverse β-decay rate (Lai & Shapiro 1991), the competition of these two reaction rates therefore causes the energy loss rate to oscillate with the increase of density. The stronger the magnetic field is, the larger the amplitude is. But the tendency is to increase the reaction rate of the modified Urca process in the range of the core density. It is also evident from Fig. 1 that the temperature only influences the reaction rate when the electrons are in resonant states at which the electrons begin to fill the higher Landau levels.

3. Cooling curves of neutron stars

In order to compare our results with the observational data, we calculate the cooling curves of neutron stars based on some approximations as follows: the first is the isothermal core approximation which assumes that the internal temperature T is constant within the star core with $\rho \geq \rho_{\rm b} = 10^{10}\ {\rm g\,cm^{-3}}$ (Glen & Sutherland 1980). Second, the effect of the magnetic field in the envelope of neutron stars on the relation between the internal temperature T and its effective temperature $T_{\rm eff}$ can be neglected (Page 1995). Hence, in order to get the internal temperature T, we apply the fitting formula for $T_{\rm eff}$ as a function of T of Potekhin, Chabrier and Yakovlev (1997) for a pure iron envelope. Third, we assume that the magnetic fields in neutron star cores are uniform and straight. To illustrate the main essence of the magnetic field on the neutron star cooling, finally, we neglect the superfluidity of neutron (Levenfish & Yakovlev 1996), and the internal heating (Cheng, Chau, Zhang & Chau 1992).

With regard to the neutron star models, we examine the neutron stars with mass $M \sim 1.4 M_\odot$, the central density $\rho = 1.017\times10^{15}\ {\rm g\,cm^{-3}}$ or $\rho = 1.12\times10^{15}\ {\rm g\,cm^{-3}}$ (Wiringa & Fiks 1988). Both models are typical cases of standard neutrino cooling. Our results are presented in Fig. 2 and Fig. 3. The $T_{\rm eff}^\infty$ in Fig. 3 is the effective temperature measured by a distant observer. The observational data on neutron star surface temperature are based on Table 1 of Page (1997). In Table 1, we present the strength of the magnetic field on the neutron star

Table 1. Surface and internal magnetic fields of neutron stars

PSR	Log $T_{\rm eff}^{\infty}$ (K)	B (10^{19}G)	B_s (10^{12}G)
1055-52	5.74-5.82	≤ 0.4	1.1
0833-45	5.87-5.91	3.0-15.0	3.4
	5.81-5.94	4.8-7.9	
0002+6246	5.72-5.85	≥ 5.6	7.0
0656+14	5.93-5.97	6.7-14.0	4.7
0630+178	5.60-5.70	3.4-14.0	1.6

surface which is derived from the observations (Taylor, Manchester & Lyne 1993) and that of the magnetic fields in neutron star cores which is obtained from Fig. 2 and Fig. 3 for case (a). If we believe that the larger the strength of the internal magnetic field, the larger that of the outer one, it is consistent with our prediction that the strength of magnetic field on PSR 1055-52 surface is less than that of others in table one. Therefore, we draw the conclusion that we could present the information on the magnetic field in neutron star cores from measuring the temperatures of neutron stars, which will benefit to understand the creation and evolution of the magnetic field of neutron stars. The strength is about 10^{19} G in our scenario.

4. Conclusion and discussions

In the interior of a neutron star, the magnetic field may be so high that the electrons only reside in the lowest Landau level, so the magnetic field can significantly influence the modified Urca process. Then the effect of a strong magnetic field must be accounted for when considering the cooling of neutron stars. In this paper, we study the effect of a strong magnetic field on the modified Urca process and on the cooling curves of neutron stars. We find that whether the strong magnetic field can increase or decrease the reaction rate of the modified Urca process depends on the strength of the interior field and the density of the neutron star cores. Comparing the cooling curves with the observational data, we find a way to present the information on the interior magnetic field. The strength we obtained is about 10^{19} G for many not so old neutron stars.

In the above discussions, the interaction between the nucleons was neglected in deriving the equation of state. Careful considerations should be based on the relativistic mean field theory (Serot & Walecka 1986), and more detailed results will be present in the future. The difference between them, however, may not be great, and the conclusions we draw cannot be changed dramatically.

References

Bahcall J.N., Wolf R.A., 1965, Phys. Rev. 140B, 1452

Canuto V., Ventura J., 1977, Fund. Cosmic Phys. 2, 203

Bandyopadhyay D.,Chakrabarty S.,Pal S., 1997, Phys. Rev. Lett. 79, 2176

Chakrabarty S., Bandyopadhyay D.,Pal S., 1997, Phys. Rev. Lett. 78, 2898

Cheng K.S., Chau W.Y., Zhang J.L., Chau H.F., 1992, ApJ 396, 135

Chiu H.Y., Salpeter E.E., 1964, Phys. Rev. Lett. 12, 413

Fassio-Canuto L., 1969, Phys. Rev. 187, 2141

Friman B.L., Maxwell O.V., 1979, ApJ 232, 541

Glen G., Sutherland P., 1980, ApJ 239, 671

Haensel P., Gnedin O. Y., 1994, A&A 290, 458

Itoh N., Tsuneto T. 1972, Progr. Thoret. Phys. 48, 1849

Lai D., Shapiro S.L., 1991, ApJ 383, 745

Lattimer J.M., Pethick C.J.,Prakash M., Haensel P., 1991, Phys. Rev. Lett. 66, 2701

Levenfish K.P.,Yakovlev D.G., 1996, Astron. Lett. 22, 56

Maxwell O.V., 1987, ApJ 316, 691

Page D., Baron E., 1990, ApJ 354, L17 (Erratum in 1991, ApJ 382, L111)

Page D., 1997, ApJ 479, L43

Page D., Sarmiento A., 1996, ApJ 473, 1067

Potekhin A.Y., Chabrier,G., Yakovlev D.G., 1997, A&A 323, 415

Shapiro S.L., Teukolsky S.A.,1983, Black Holes, White Dwarfs, and Neutron Stars, Wiley-Interscience, New-York

Serot B.D., Walecka J.D., 1986, Adv. Nucl. Phys. 16, 1

Taylor J.H., Manchester R.N., Lyne A.G., 1993, ApJS 88, 529

Wiriga R.B.,Fiks V., 1988, Phys. Rev. C, 1010

Yakovlev D.G.,Levenfish K.P., 1995, A&A 297,717