Astron. Astrophys. 335, L89–L92 (1998)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

About the role of gravity waves in the angular momentum transport inside the radiative zone of the Sun

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Received 14 October 1997 / Accepted 23 April 1998

Abstract. The purpose of this letter is to give a simple and general view of the effect of an internal gravity wave field, on the angular momentum distribution inside the radiative zone of the Sun. It is shown that a quasi-solid rotation of the radiative zone of the Sun cannot be a direct consequence of the action of the gravity waves produced by the convective zone.

After a brief description of the properties of gravity wave, I propose a very simple model of the role played, by internal gravity waves, on the angular momentum profile of the radiative zone of the Sun.

Key words: hydrodynamics – waves – Sun: rotation – Sun: interior

1. Introduction

In the early nineties, the helioseismology data revealed a very surprising fact concerning the rotation of the Sun's interior: apparently, the radiative zone of the Sun, below the convective zone, has a quasi-solid rotation.

Schatzman (1993) was the first to attempt to solve this problem by means of gravity waves. Indeed, in atmospheric sciences, it had been known for a long time that a gravity wave is able to transport momentum (Bretherton 1969). It seems gravity waves are produced by the turbulent stresses in the convective zone of the Sun. Consequently, a gravity wave field inside the radiative zone, may play a major role on the angular momentum distribution in this region.

Later, Zahn et al. (1997) and Kumar et al. (1997), independently, arrived almost at the same conclusion as Schatzman (1993), namely that the process is very efficient and able to produce the co-rotation of the radiative zone of the Sun in 10^7 years.

These three papers are based on the work of Goldreich & Nicholson (1989), which show how, in a binary system, the rotation rate in the envelope of the massive main sequence star (MMSS) is synchronized with the companion's orbital period. Gravity waves are created by tidal torque near the boundary between the convective core and the radiative envelope of the MMSS. These gravity waves propagate into the envelope and deposit angular momentum at the surface, forcing it in co-rotation with the companion's orbital angular velocity.

Schatzman (1993), Zahn et al. (1997) and Kumar et al. (1997), have all used the same physical picture, to explain the solid rotation of the radiative zone of the Sun. Although their respective works slightly differ, they are all based on the same model, which can be summarized in a few words:
the vertical components of the phase velocity $\boldsymbol{c_\varphi}$ and of the group velocity $\boldsymbol{c_g}$ of a gravity wave are always opposite (Sect. 2). Thus, in their models where $\boldsymbol{c_\varphi}$ is directed inwards towards the core, $\boldsymbol{c_g}$ is directed outwards towards the convective zone, which is counter-intuitive. Only the prograde waves are taken into account, i.e. propagating in the same direction as the rotation and carrying a positive momentum as only these dissipate significantly. With a group velocity directed outwards, we have a flux of positive angular momentum directed outwards, i.e. a process of extraction of angular momentum from the central regions to the upper ones. So, following the same conclusions than those of Goldreich & Nicholson (1989), Schatzman (1993), Zahn et al. (1997) and Kumar et al. (1997) have concluded that the solid rotation of the radiative zone of the Sun is a direct consequence of the effect of the gravity waves.

However, one must be careful in interpreting the work done by Goldreich & Nicholson (1989), and I shall show here that it is essential to have a deeper understanding of the properties of gravity waves and of their ability to transport momentum.

Before giving a simplified picture of the effect of an internal gravity wave field on the rotation profile of the radiative zone of the Sun (Sect. 3), I am going to recall the fundamental properties of a gravity wave (Sect. 2) (see also Fritts et al. 1998) and I shall give a physical interpretation of Goldreich & Nicholson theory (Sect. 4).

2. Properties of gravity waves

Let us consider a plane parallel referential xOz. The fluid for $z > 0$ is considered stratified in density along the vertical axis z. The stratification of the fluid is characterized by N, the Brunt-Väisälä frequency. The wave is assumed to be incompressible

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and adiabatic and we use the Boussinesq approximation (the vertical wavelength is small compared to the pressure height of the fluid).

For the present step, we assume that the fluid is at rest. A wave solution of the form

$$e^{i(k_h x + k_v z - \omega t)} \tag{1}$$

is introduced in the linearized equations of hydrodynamics (Holton 1992, Lighthill 1978) and we obtain the dispersion relation

$$\omega^2 = N^2 \frac{k_h^2}{k_h^2 + k_v^2} \tag{2}$$

where $\boldsymbol{\kappa} = (k_h, k_v)$ is the wave vector and ω the frequency.

Let us now consider a gravity wave ($\omega < N$) produced at the origin ($x = z = 0$).

i) there are only four possible directions of propagation for $\boldsymbol{\kappa}$ which gives the same ω

$$* \, k_h > 0, k_v > 0 \tag{3}$$

$$* \, k_h > 0, k_v < 0 \tag{4}$$

$$* \, k_h < 0, k_v > 0 \tag{5}$$

$$* \, k_h < 0, k_v < 0 \tag{6}$$

then gravity waves are very anisotropic. Without loss of generality, we now concentrate on the case:

$$k_h > 0, k_v < 0$$

ii) $\omega(k_h, k_v)$ is an homogeneous function of order 0; so by Euler theorem

$$\boldsymbol{\kappa}.\boldsymbol{c_g} = 0 \tag{7}$$

then, the direction of the propagation of the energy is parallel to equiphases, whose equations are:

$$k_h x + k_v z = constant$$

iii) the radiation condition imposes that $\boldsymbol{c_g}$ originates from the source (here, the origin), and is directed outward.

iv) the incompressibility condition being

$$\boldsymbol{\kappa}.\boldsymbol{v}' = 0 \tag{8}$$

where $\boldsymbol{v}' = (u', w')$ is the velocity perturbation due to the wave, the energy of the wave is confined on the equiphase going through the origin and we have in this case a positive momentum flux $\overline{u'w'} > 0$, where $\overline{(\,)}$ denotes an average in the x direction.

v) Using the linearized equations, we easily show that (Bretherton 1969)

$$\rho\overline{u'w'} = \frac{k_h}{\omega}\overline{c_{gv}E} \tag{9}$$

where $\boldsymbol{c_g} = (c_{gh}, c_{gv})$ and E the energy density of the wave.

This last equation may be understood by
(vertical momentum flux)=$\frac{k_h}{\omega}$(vertical energy flux).
As $\overline{u'w'} > 0$ because $\boldsymbol{v}'$ is parallel to $\boldsymbol{c_g}$, we see by Eq. (9) that
if $c_{gv} > 0$ then $k_h > 0$
if $c_{gv} < 0$ then $k_h < 0$
therefore, with our choice $k_h > 0$, c_{gv} must be positive too, i.e. in the opposite direction of the vertical component of $\boldsymbol{\kappa}(k_v < 0)$.

So we can give the conclusion of this elementary analysis:

* the energy is going away from the source at the group velocity $\boldsymbol{c_g}$ on a way confined on the equiphase going through the source.
* the phase goes at right angle of the energy and the vertical components of the phase and group velocities are opposite.
* a momentum flux is associated with the propagation of the wave and is

$$F_M = \rho\overline{u'w'} = \frac{k_h}{\omega}F_E \tag{10}$$

F_E is the vertical energy flux.

It is important to note that the sign of F_M is the same as the one of k_h, so the momentum is carried by the wave at the group velocity in the k_h direction (Lighthill 1978).

3. A simple model for the angular momentum transport in the Sun

I now use all this results in order to get a picture of what might happen in the Sun. For sake of simplicity, we still assume a 2D plane parallel description. This may seem to be too simplistic but it does not affect the result dramatically.

The convective zone is in the region $z < 0$, the radiative zone in the region $z > 0$ (with z increasing towards the core) and we suppose that the waves are generated at $z = 0$. As in Sect. 2, the fluid for $z > 0$ is stratified, and we use the Boussinesq approximation. First, we suppose that there is no dissipation process of the waves, the waves are said “conservative”. But in contrast with Sect. 2, we now assume a stationary flow

$$\boldsymbol{U}(z) = \alpha \, z \, \boldsymbol{e_x} \tag{11}$$

parallel to the x axis. α is a positive constant and $\boldsymbol{e_x}$ is the unit vector in the x direction. We have chosen a flow velocity null at the locations of the sources ($z = 0$) and growing towards to the center. This is to simulate what we should expect inside the radiative zone of the Sun, when angular momentum transport is not efficient.

The dispersion relation of gravity wave is, as before,

$$\tilde{\omega} = \omega - k_h U = N \frac{|k_h|}{(k_h^2 + k_v^2)^{\frac{1}{2}}} \tag{12}$$

where ω is the absolute frequency in the inertial reference frame and $\tilde{\omega}$ the intrinsic frequency, i.e. the frequency in the frame of the mean flow.

Since the medium is time independent, ω is constant, and because the medium is homogeneous in x direction, k_h is independent of z, so $\tilde{\omega}$ will change with depth z according to

$$\tilde{\omega}(z) = \omega - k_h \alpha z = N \frac{|k_h|}{(k_h^2 + k_v^2)^{\frac{1}{2}}} \quad (13)$$

and to each wave is attached a momentum flux

$$F_M = \frac{k_h}{\omega - k_h \alpha z} F_E \quad (14)$$

It can be shown that, except when the wave reaches the critical layer, at $z_c = \omega/(k_h\alpha)$, F_M is constant for stationary, conservative and linear waves (Eliassen & Palm 1961). Now, all the waves we consider have a positive vertical group velocity (inward) and so k_v is negative.

i) case where $k_h > 0$ (prograde wave): from Eq. (13), as the wave propagates inwards, the intrinsic frequency $\tilde{\omega}$ is Doppler-shifted toward zero and $|k_v|$ stretches to ∞. When the wave arrives at the depth z_c, it cannot propagate further inwards.

ii) case where $k_h < 0$ (retrograde wave): as the wave propagates inwards, the intrinsic frequency $\tilde{\omega}$ is Doppler-shifted toward N and $|k_v|$ stretches to zero. When the wave arrives at the depth $z_R = -(N - \omega)/(k_h\alpha)$, the sign of k_v and c_{gz} changes and the wave is reflected.

However, as the waves propagate inside the Sun, they are damped by radiative dissipation (Press 1981), so the momentum flux is no longer a constant. Let us consider now two waves with the same ω but with opposite k_h:

for the prograde wave, $\tilde{\omega} \to 0$ and $|k_v| \to \infty$, so the dissipation process is enhanced and the positive momentum carried by it is rapidly lost and given to the flow. So, the flow will accelerate.

in contrast, as the retrograde wave propagates, $\tilde{\omega}$ increases and $|k_v| \to 0$, the dissipation process is less efficient and a small part of the negative momentum is deposited into the flow. The wave keeps a great part of its negative momentum and continues to propagate inwards.

Before analyzing the effect of such waves on the profile of the flow in the radiative zone, we suppose that the energetic spectrum of emission of waves is symmetric with respect to the sign of k_h, so that there are on the average, as many prograde waves as retrograde waves; and we assume that there are no reflected waves from the interior (they are completely damped before $\tilde{\omega} = N$), i.e. all retrograde waves end by dissipating.

The description given above, shows that, the net momentum flux is zero, so the radiative zone cannot accelerate or decelerate as a whole. In the uppers layers, there will be a net acceleration of the fluid. But, at deeper and deeper layers, the positive momentum flux decreases rapidly because the prograde waves are disappearing, by dissipation or critical layer effect. Then, only the retrograde waves are left, which by dissipation, set negative momentum into the fluid. We must therefore expect a deceleration of the deep layers.

Thus, as a net result, gravity waves, generated at the bottom of the convective zone, extract momentum from the inner layers and transport it to the uppers ones, with no net external flux.

However, it seems to be quite difficult to explain the quasi-solid rotation of the radiative zone of the Sun, by this very simple model. Indeed, this analysis is valid for a profile of the flow given by Eq. (11): the velocity profile is growing linearly from the regions of the sources (z=0) towards the centrals regions; but as this profile is going to change by the effect of the waves, there is a feed-back on the propagation of these waves which, via the Doppler-shift effect, will lay momentum at other depths, and so on. A prediction of the velocity profile of the radiative zone, in presence of a gravity wave field is therefore very complicated by this feed-back effect and is beyond this short analysis. In fact, with such forcing of waves at the top of the radiative zone, the flow profile may be stationary or may be oscillating like the "quasi-biennial oscillation" (Plumb & McEvan 1978) of the equatorial stratosphere of the Earth. Particularly, a quasi-solid rotation may be a highly unstable profile because the least deviation of the velocity of the flow from this profile will be amplified by the Doppler-shift effect, which enhances the deposit of momentum where the intrinsic frequency decreases.

4. A case of quasi-solid rotation

We now come back to the Goldreich & Nicholson model to give an example of a quasi-solid rotation produced by means of gravity waves.

There is a companion which is spinning around a massive main sequence star at a slower rate than the rotation of the MMSS. The deformation of the convective core due to the tidal torque propagates slower than the medium of the MMSS, i.e. have a negative velocity relative to the fluid, and so generates a retrograde gravity wave. In fact, in this case, the gravity wave is like a topographic wave or lee wave (Holton 1992, chap 9) which is a gravity wave created by a wind on a hump. In the reference frame of the companion, the hump is at rest (it is the deformation but not the fluid), and the wind is the fluid of the MMSS's interior. Because the pressure is higher on the upwind slope than on the downwind ones, a net force is exerted on the hump. By reaction, a drag acts on the wind. If the conditions for the existence of a gravity wave are filled, the braking doesn't act at the location of the hump, but is transmitted in the uppers layers by the means of the lee wave.

So, the lee gravity wave transports a momentum outwards at the positive group velocity c_g (so going away the source) and in the opposite direction of the wind.

We say that the gravity wave has a negative momentum, so there is an upward flux of negative momentum (Bretherton 1969). This is a retrograde wave because it is characterized by a negative horizontal phase velocity relative to the mean flow.

The intrinsic frequency of the gravity wave is given by

$$\tilde{\omega} = -kr(\Omega - \Lambda) \quad (15)$$

(by choice, all frequencies are positive) k is the horizontal wave number (negative), r is the distance from the center, $(\Omega - \Lambda)$ is the angular speed of the MMSS with regards to the companion (positive).

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By conservation of the number of horizontal wavelengths, we have

$$kr = k_c r_c \quad (16)$$

where the subscript c means the top of the core.

$$k_c = -\frac{2\pi}{\lambda_c} \quad (17)$$

where λ_c is the horizontal wavelength at the top of the core, $\lambda_c = \pi r_c$, so we find

$$\tilde{\omega} = 2(\Omega - \Lambda) \quad (18)$$

Thus, now, we can calculate the flux of angular momentum

$$F_\Omega = r\frac{k}{\tilde{\omega}}F_E = -\frac{F_E}{\Omega - \Lambda} < 0 \quad (19)$$

The momentum transported by the lee wave is negative (in the opposite direction of the rotation of the envelope) and is carried outwards at the vertical group velocity. By the dissipation of the waves, this moment is gradually set in the upper layers, slowing down the envelope. As soon as a layer is in co-rotation, $\Omega - \Lambda = 0$, the wave has reached the critical layer where $\tilde{\omega} = 0$. The wave cannot propagate upper and consequently deposits its remaining momentum at this location. So, the critical layer goes down slowly, training the co-rotation between the companion and the outer envelope.

5. Conclusions

The present analysis shows with simple physical arguments how an internal gravity wave field can play a crucial role on the redistribution of angular momentum inside the stable zone of the Sun. As we have shown, even if extraction of angular momentum from the central regions towards the outer ones is possible, the evolution of the flow profile may be highly complex, and the explanation of a co-rotation of the radiative zone of the Sun, seems to be quite difficult to obtain. The fundamental difference between the evolution of the radiative zone of the Sun and the one of the envelope of the MMSS, is that, in the first case, prograde and retrograde waves are produced nearly symmetrically even though in the second case, only retrograde waves (with negative momentum) are produced.

In contrast with what has been done previously, this analysis shows that co-rotation is not a straightforward consequence of the momentum transport by gravity waves. A crude description is not sufficient.

A more complete treatment, which must take into account the effects of sphericity, rotation, stellar loss of angular momentum and stochastic generation, is required. (in progress)

Acknowledgements. I gratefully thank Evry Schatzman who introduced me to the subject and for helpful discussion. I thank Marie-Jo Goupil for her help in writing this letter and Yin-Ching Lo for his encouragements. During the revision of this letter, I learnt that Gough D. (1997) has made the same remarks concerning this subject.

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