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Quantitative analysis of carbon isotopic ratios in carbon stars

II. The effect of model atmosphere on the iso-intensity method

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Abstract. We discuss the analysis of $^{12}C/^{13}C$ ratios in cool carbon stars presented by de Laverny & Gustafsson (1998), who questioned the reliability of the iso-intensity method used by Ohnaka & Tsuji (1996). We show that the systematic discrepancy of $^{12}C/^{13}C$ ratios between Lambert et al. (1986) and Ohnaka & Tsuji (1996) cannot be attributed to the uncertainty of the iso-intensity method. The analysis of the iso-intensity method done by de Laverny & Gustafsson (1998) differs from that of Ohnaka & Tsuji (1996), defining the abscissa of curves of depth growth in a completely different manner. Namely, we derived the abscissa directly from model atmospheres, while they simply assumed a single excitation temperature whose value is never accurately derived. The high sensitivity of the iso-intensity method to model atmospheres, reported in their work, can be attributed to an incorrect definition of the abscissa of curves of depth growth. In fact, we show that the determination of $^{12}C/^{13}C$ ratios by the iso-intensity method is not so sensitive to model atmospheres (atmospheric structure itself and stellar parameters) as they claim, when the abscissa is properly calculated. In addition, we demonstrate that our model atmospheres can reproduce photometric and spectrophotometric observations fairly well. Therefore, their conclusion that the iso-intensity method is risky and unreliable for determining $^{12}C/^{13}C$ ratios in cool carbon stars cannot be justified.

Key words: stars: abundances – stars: atmospheres – stars: carbon – stars: fundamental parameters – stars: AGB and post-AGB

1. Introduction

The carbon isotope ratio is crucial in the understanding of the formation of carbon stars. However, the heavily line-blanketed spectra of these stars, together with the difficulty in constructing reliable model atmospheres, hamper stellar spectroscopists from obtaining reliable results. We determined $^{12}C/^{13}C$ ratios in 62 N-type carbon stars from CN lines at around 8000 Å using the iso-intensity method in Ohnaka & Tsuji (1996, OT96 hereafter), and found a discrepancy of a factor of 2 to 3 compared with the result obtained by Lambert et al. (1986, LGEH86 hereafter) for 20 stars analyzed by both authors in common. We suggested the difference of model atmospheres used in the analyses as a possible reason for the discrepancy of $^{12}C/^{13}C$ ratios. However, recent work by de Laverny & Gustafsson (1998, dLG98 hereafter) has questioned the accuracy and the reliability of the iso-intensity method. They conclude that the iso-intensity method is much more sensitive to the atmospheric structure and stellar parameters than estimated in OT96, and therefore that $^{12}C/^{13}C$ ratios in OT96 are not reliable.

In response, we will review the analysis by dLG98. We will demonstrate that their analysis has some faults which make it totally different from ours in OT96 and that their conclusion cannot be justified.

2. Spectral synthesis method

The spectral synthesis does not work to determine $^{12}C/^{13}C$ ratios for the spectra studied by OT96, as dLG98 also recognized. The most difficult part of applying the spectral synthesis in this wavelength region is that the true continuum level cannot be well defined in the observed spectra, partly because the spectral resolution of about 20 000 in OT96 is not high enough. However, dLG98 do not mention how they determined the continuum level when they compared the synthetic spectrum with the observed one. The spectrum of Fig. 1 in dLG98 seems to be normalized by the fictitious continuum which is drawn so that it travels through the highest point in the observed region. But OT96 do not claim that it is the true continuum level. It is not necessary to determine the true continuum level in order to apply the iso-intensity method unlike with the spectral synthesis method. That is one of the reasons we applied the iso-intensity method to the observed spectra.

Therefore, together with the uncertainty of molecular data, the uncertain location of the true continuum level should lead to a large uncertainty in $^{12}C/^{13}C$ ratios if they are determined by the spectral synthesis. In addition, the determination of microturbulent velocity should also be difficult, because there is actually no line free from the saturation effect in the spectral region studied. But dLG98 simply assume $V_{\rm micro}$ = 2.0 km s^{-1}

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without any detailed analysis for its determination. It should be stressed here that the spectral synthesis is quite sensitive to the adopted value of micro-turbulent velocity, especially for crowded spectra. Further, they do not mention other line broadenings such as the macro-turbulence, which has a direct effect on synthetic spectra, but should be difficult to separate from the micro-turbulence by the spectral synthesis method alone. Given these difficulties, the statement in dLG98 that the two ^{13}CN lines used by OT96 are best fitted by larger values of $^{12}\mathrm{C}/^{13}\mathrm{C}$ ratio than by the ones derived by OT96 is not necessarily correct.

3. Blending

The effect of blending is one of the most severe problems in spectral analyses of cool stars. We used the iso-intensity method, not because it is the best method, but because it is a possible compromise for the given observational data. And we reduced the effect of blending as much as possible by using central depths of lines instead of equivalent widths. As dLG98 showed in the calculation of a full spectrum and isolated lines, possible blending of the lines used in OT96 could have a direct effect on the resulting $^{12}\mathrm{C}/^{13}\mathrm{C}$ ratios. However, as shown in Fig. 13 of OT96, we collected more lines by observing a wider spectral range for TX Psc and checked the possible effects of blending. Comparison between Figs. 13a and 13b of OT96 confirms that the three ^{12}CN lines used in our standard analysis are not systematically affected by blending. And it seems that three ^{13}CN lines in Fig. 13b form the weak line part of the curve of growth, though it is also possible that all these three lines are affected by blending.

The blending problem may be less severe in the infrared region LGEH86 studied, but their work is not free from this problem, either. In fact, Ohnaka (1997) and Ohnaka & Tsuji (1998) re-analyzed the original spectra of LGEH86, which were kindly made available to us by Dr. K. H. Hinkle, and found that many fewer CO lines ($\Delta v = 2, 3$) could finally be selected for the analysis than given in LGEH86. Therefore, before their result is favored, the analysis of possible effects of blending should also be done for the analysis of LGEH86.

4. $^{12}\mathrm{C}/^{13}\mathrm{C}$ ratios from the iso-intensity method

4.1. Difference between the analysis by dLG98 and the iso-intensity method of OT96

dLG98 claim that the effect of uncertainties of model atmospheres is much larger for the iso-intensity method than discussed in OT96. It should be stressed here, however, that there is a difference of the abscissa of curves of depth growth between dLG98 and OT96.

We predicted line intensities using the weighting function method described by Cayrel & Jugaku (1963). The abscissa of curves of depth growth in our analysis is $\log(gf\Gamma_\lambda)$, while dLG98's definition is $\log(gf\lambda) - \chi\,\Theta_\mathrm{X} - \log(\kappa_\nu)$. The effect of atmospheric structure is included in the calculation of Γ_λ. In the weak line approximation, the predicted line intensity is calculated by

$$\frac{W}{\lambda} = gf\Gamma_\lambda = gf\frac{\pi e^2}{mc^2}\lambda\int_0^\infty G_\lambda(\tau_\lambda)P(\tau_\lambda)\frac{d\tau_\lambda}{\kappa_\lambda}$$
$$= gf\frac{\pi e^2}{mc^2}\lambda\int_{-\infty}^\infty G_\lambda(\tau_\lambda)P(\tau_\lambda)\tau_\lambda \ln 10\frac{d\log\tau_\lambda}{\kappa_\lambda}, \qquad (1)$$

where G_λ is the weighting function and P is the number of molecules, per gram of stellar material, at a fictitious level with statistical weight unity and with lower excitation potential χ (eV). G_λ and P are defined as follows:

$$G_\lambda(\tau_\lambda) = \frac{2}{F_\mathrm{cont}(\tau_\lambda = 0)}\int_{\tau_\lambda}^\infty \frac{dS_\lambda(t)}{dt}E_2(t)dt, \qquad (2)$$

$$P(\tau_\lambda) = \frac{P_\mathrm{mol}}{P(\mathrm{H})}\frac{1}{u(T)}10^{-\chi\theta}(1-e^{-hc/\lambda kT})\frac{1}{\mu_\mathrm{H} m_\mathrm{H}}. \qquad (3)$$

F_cont is the flux at the continuum, S_λ is the source function, E_2 is the exponential integral function, P_mol is the partial pressure of the molecule at issue, $P(\mathrm{H})$ is the fictitious pressure of hydrogen, $u(T)$ is the partition function, and μ_H is the mean molecular weight with respect to the hydrogen nucleus. Other symbols have their usual meanings.

The weighting function represents the contribution of each layer throughout the atmosphere. Therefore, the excitation effect is taken into account in a more sophisticated way than in dLG98. Here we should be careful in defining the excitation temperature. Plotting $\log\Gamma_\lambda$ against χ exhibits that $\log\Gamma_\lambda$ can approximately be written as $\log\Gamma_\lambda(0) - \chi\langle\theta_\mathrm{ex}\rangle$, where $\langle\theta_\mathrm{ex}\rangle$ can be considered as the reciprocal excitation temperature. The important point is that $\langle\theta_\mathrm{ex}\rangle$ can uniquely be evaluated for all lines with different excitation potentials, if the atmospheric structure is given. Namely, $\langle\theta_\mathrm{ex}\rangle$ should not be different from line to line in the weak line approximation. To make this point clear, consider the determination of excitation temperatures from observed line intensities. Observationally, the excitation temperature can be determined from weak lines by plotting $\log(W/gf\lambda)$ against χ: the gradient of the plot gives the excitation temperature. This means that the excitation temperature can uniquely be determined for all the weak lines. Strictly speaking, the gradient changes slightly with excitation potential, but as will be shown in Fig. 2, the change of the gradient is quite minor.

On the other hand, dLG98 defined the abscissa as $\log(gf\lambda) - \chi\,\Theta_\mathrm{X} - \log(\kappa_\nu)$. They evaluated Θ_X and $\log(\kappa_\nu)$ at the layer of $\tau_\mathrm{line} = 0.1$. But the abscissa of curves of growth should not be calculated in such a way, because the excitation temperature is almost uniquely determined for all lines with any excitation potential – not for an individual line – in the weak line approximation. The abscissa of curves of growth is calculated in the weak line approximation, and that is why the saturation effect appears as flattening of curves of growth. Moreover, we computed Γ_λ directly from model atmospheres as shown above, while dLG98 simply assumed a single value of Θ_X to compute the abscissa. This is not so different from the so-called coarse analysis used in the 1950's. The only difference is in using model atmospheres to ascertain the temperature at $\tau_\mathrm{line} = 0.1$, but they offer no justification for estimating Θ_X in such a way. This difference of the abscissa makes the re-construction of curves of depth growth by dLG98 something totally different from the iso-intensity method used by OT96.

In fact, incorrect definition of the abscissa by dLG98 obstructs their analysis of the accuracy and the reliability of the iso-intensity method. It means that the effect of effective temperature errors is not correctly estimated by dLG98. It is rather natural that they found a discrepancy in the estimate of the uncertainty of $^{12}C/^{13}C$ ratios resulting from uncertainty regarding effective temperature. In OT96, we have already estimated the effect of the uncertainty of effective temperature on the resulting $^{12}C/^{13}C$ ratios by calculating Γ_λ. The uncertainty of $^{12}C/^{13}C$ is about 0.02 dex for $\Delta T_{\rm eff} = 200$ K in N-type carbon stars, for which we assumed C/O = 1.1. As for the effect of metallicity, it is possible that their error estimate is also influenced by their erroneous way they calculate line intensities. But this effect should be examined by the proper application of the iso-intensity method.

We assumed C/O = 1.1 for all the N-type carbon stars, while the determination of C, N, and O abundances should be done prior to that of $^{12}C/^{13}C$ ratio. dLG98 argue that the adoption of C/O ratios derived by LGEH86, which are characterized by moderate carbon enrichment (C/O < 1.1) for about half of their program stars, would lead to higher sensitivity of the resulting $^{12}C/^{13}C$ ratios to other stellar parameters. However, our re-analysis of their original spectra of TX Psc, BL Ori, and V Aql (Ohnaka 1997, Ohnaka & Tsuji 1998) reveals that carbon stars are more carbon-rich than believed, at least for these three stars. In fact, C/O ratios in TX Psc, BL Ori, and V Aql are 1.1, 1.3, and 2.5, respectively, while the results of LGEH86 for these stars are C/O = 1.027, 1.039, and 1.25, respectively. Though the discrepancy of C/O ratios is also under investigation, this result means that the argument of dLG98 against our assumption of C/O = 1.1 for N-type carbon stars cannot necessarily be justified.

4.2. *Effect of the atmospheric structure*

We suspected that the discrepancy of $^{12}C/^{13}C$ ratios between LGEH86 and OT96 might be attributed to the difference of the atmospheric structures represented by different models used in the analyses. The best way to examine this possibility is to analyze $^{12}C/^{13}C$ ratios using the iso-intensity method in which the abscissa is correctly calculated, but with a model used in LGEH86. Dr. P. de Laverny kindly provided us with an original model of LGEH86, and we calculated Γ_λ from models used in LGEH86 and OT96. The stellar parameters are $T_{\rm eff}$ = 3000 K, $\log g$ = 0.0, C/O = 1.1, [O/H] = 0.0, and [Fe/H] = 0.0. A micro-turbulent velocity of 3.0 km s^{-1} is assumed and the effect of turbulent pressure is included in the model of OT96, while $V_{\rm micro}$ = 2.0 km s^{-1} is assumed and the turbulent pressure is neglected in LGEH86. Since the model of LGEH86 is labeled by an optical depth of the continuum at 5000 Å, we converted it into a scale of an optical depth of the continuum at 1.04 μm, by which our model is labeled. In Fig. 1, we show a comparison between two models. The two models are in agreement in the layers shallower than $\log \tau(1.04\mu{\rm m}) = -2$, while the difference of the temperature structures amounts up to 400∼500 K in the deeper layers around $\log \tau(1.04\mu{\rm m}) = 0$[1].

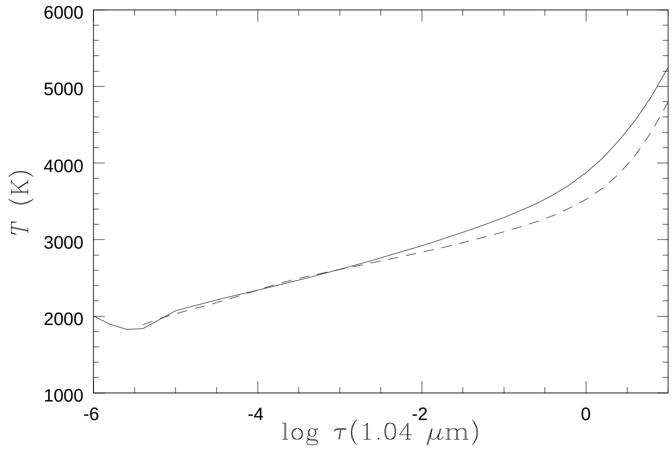

Fig. 1. Comparison of model atmospheres of LGEH86 (dashed line) and OT96 (solid line) with the same set of parameters, which are $T_{\rm eff}$ = 3000 K, $\log g$ = 0.0, C/O = 1.1, [O/H] = 0.0, and [Fe/H] = 0.0. $V_{\rm micro}$ = 3.0 km s^{-1} is assumed and the effect of turbulent pressure is included in the model of OT96, while $V_{\rm micro}$ = 2.0 km s^{-1} is assumed and the turbulent pressure is neglected in LGEH86

Table 1. The effect of the difference of atmospheric structures on resulting $^{12}C/^{13}C$ ratios is estimated by $\log \Gamma_\lambda(\chi = 0\,{\rm eV}) - \log \Gamma_\lambda(\chi = 1.2\,{\rm eV})$. The stellar parameters of the models are given in the legend to Fig. 1

Model	$\log \Gamma_\lambda(\chi = 0\,{\rm eV}) - \log \Gamma_\lambda(\chi = 1.2\,{\rm eV})$
OT96	2.043
LGEH86	2.053

We calculated Γ_λ with these two models and found that $\langle\theta_{\rm ex}\rangle$, the gradient of $\log \Gamma_\lambda$ against χ, which plays a decisive role in the determination of $^{12}C/^{13}C$ ratios in the iso-intensity method, is quite insensitive to the atmospheric structure.

Fig. 2 shows the plot of $\log \Gamma_\lambda$ against χ for these two model atmospheres. Table 1 gives the effect of $\log \Gamma_\lambda$ on resulting $^{12}C/^{13}C$ ratios, which can be estimated by $\log \Gamma_\lambda(\chi = 0\,{\rm eV}) - \log \Gamma_\lambda(\chi = 1.2\,{\rm eV})$, because the excitation potentials of ^{12}CN and ^{13}CN lines used in OT96 are about 1.2 eV and 0 eV, respectively. The effect of the difference of the atmo-

[1] However, in the comparison of these two models in OT96 (see Fig. 19 in OT96), the difference of the temperature structures between two models is from 400 to 500 K even in shallow layers. For example, Fig. 19 in OT96 shows that the difference of the temperature at the layer of $\log P_g = 1$ is about 500 K. The optical depth of this layer is $\log \tau(1.04\mu{\rm m}) = -3.0$ in our model. This may be because the model of LGEH86 shown in Fig. 19 of OT96 is not a model with $T_{\rm eff}$ = 3000 K. Since LGEH86 do not give a clear description of model parameters in the figures of their models, we inferred stellar parameters from their figures. The model of LGEH86 shown in Fig. 19 of OT96 might have been a model with $T_{\rm eff}$ = 2800 K.

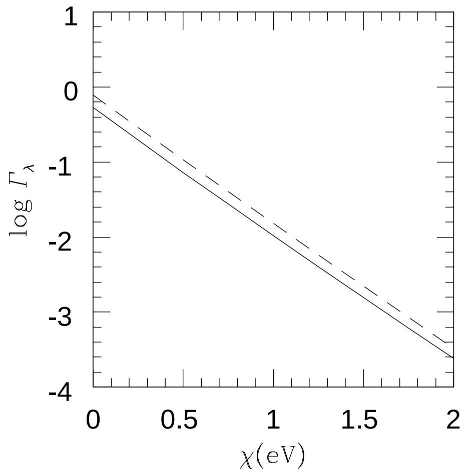

Fig. 2. $\log \Gamma_\lambda(\chi)$ calculated from the models of LGEH86 and OT96 shown in Fig. 1. The solid line is calculated from the model of OT96, while the dashed line from that of LGEH86

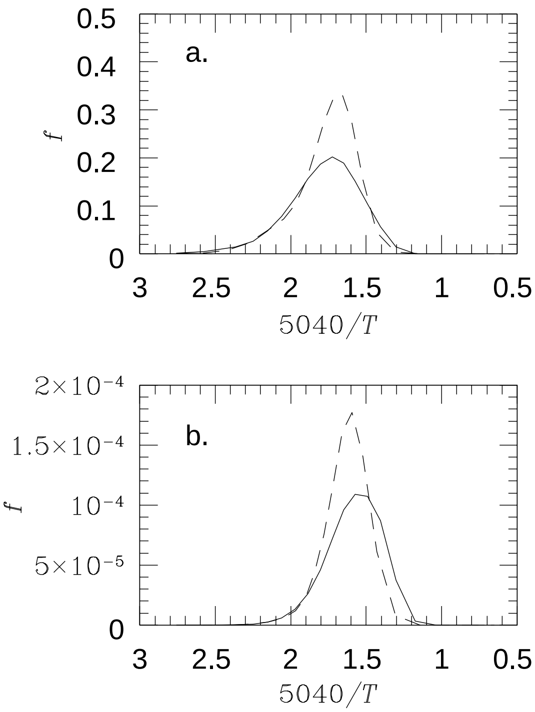

Fig. 3a and b. Contribution functions for a fictitious line of the CN red system at 8000 Å, calculated with the models of LGEH86 and OT96, are plotted against the reciprocal temperature of each layer. In each panel, the solid line represents the contribution function with the model of OT96, while the dashed line represents that with the model of LGEH86. **a** An excitation potential of 0.0 eV is assumed. **b** An excitation potential of 2.0 eV is assumed

spheric structures has turned out to be only 0.01 dex. It could be expected that $\langle\theta_{\rm ex}\rangle$ would be different for different model atmospheres. To examine the reason for the insensitivity of $\langle\theta_{\rm ex}\rangle$ to the atmospheric structure, we show in Fig. 3a and b the plot of contribution functions against $5040/T$ for the two models. The contribution function is the integrand of Eq. (1) and defined as:

$$f(\tau_\lambda) = \frac{\pi e^2}{mc^2}\lambda G_\lambda(\tau_\lambda)P(\tau_\lambda)\tau_\lambda \frac{\ln 10}{\kappa_\lambda}. \tag{4}$$

The temperature of the layer at which the contribution function has its maximum can also be considered as the excitation temperature. In fact, the gradients of $\log \Gamma_\lambda(\chi)$'s in Fig. 2 are between 1.6 and 1.7 for both of LGEH86 and OT96 models. On the other hand, Fig. 3a and b clearly shows that the reciprocal temperatures at the maxima of the contribution functions also range from 1.6 to 1.7. This agreement means that our definition of the excitation temperature is consistent. Moreover, as Fig. 3a and b illustrates, the temperatures at the maxima of the contribution functions are almost the same for the two models. Namely, the excitation temperatures for the two models are almost the same, despite the difference of the atmospheric structures. Therefore, the effect of the atmospheric structure is too small to explain the discrepancy of $^{12}C/^{13}C$ ratios of a factor of 2 to 3. Of course, this argument should not be taken too far, because we performed the calculation of Γ_λ above only for the two models. However, this calculation suggests that the difference of the atmospheric structures is quite unlikely to be a cause of the discrepancy of $^{12}C/^{13}C$ ratios between LGEH86 and OT96. This conclusion was reached by dLG98 in spite of improper application of the iso-intensity method, while we arrived at this same conclusion, applying the method appropriately.

5. Tests of model atmospheres

Though the previous section shows that the determination of $^{12}C/^{13}C$ ratios using the iso-intensity method is insensitive to the atmospheric structure to some extent, it is of course desirable to test the validity of the model atmospheres used in the analysis. OT96 tested the model atmospheres by Na I D lines and H_2 line (see Figs. 6 and 20 in OT96). However, our re-analysis of the original spectra of LGEH86 for TX Psc, BL Ori, and V Aql (Ohnaka 1997, Ohnaka & Tsuji 1998) reveals that the H_2 1-0 $S(0)$ line at 2.2 μm may be contaminated by the contribution of an extra-component above the photosphere. For example, the radial velocity of the H_2 line in V Aql shows a distinct deviation from the photospheric value derived by high excitation lines of the CO second overtone bands and coincides with radial velocities of low excitation lines of the CO first overtone bands. These low excitation lines of the CO first overtone bands exhibit intensity anomalies: That is, the observed absorption is much stronger than predicted from the final model, computed with the carbon and oxygen abundances derived from weak lines. Such intensity anomalies imply the existence of a warm molecular envelope above the photosphere. In fact, Tsuji (1988) suggested such an extra-component for M giants, based on high-resolution infrared spectra, and it has recently been confirmed, based on infrared spectra acquired with the Infrared Space Observatory (ISO) (Tsuji et al. 1997). The radial velocity deviation of the H_2 line means that it is contaminated or mainly formed in the warm molecular envelope. Though H_2 lines have been considered for

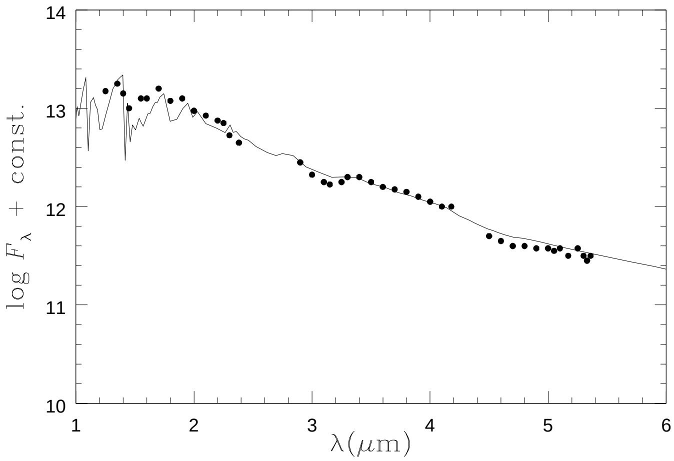

Fig. 4. Comparison between the emergent flux from our final model for TX Psc (solid line) and the photometric observation in Johnson et al. (1985) (filled circles)

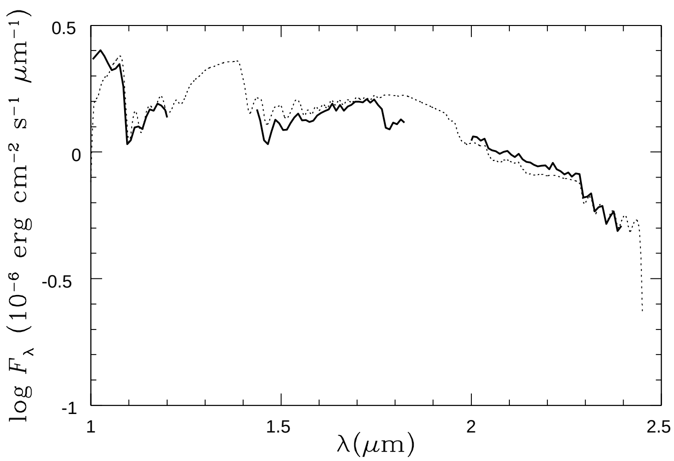

Fig. 5. Comparison between the synthetic spectrum calculated with our final model (dotted line) and the spectrum observed by Lázaro et al. (1994) for TX Psc (solid line)

use as a test of model atmospheres, this result shows that they are not necessarily appropriate for such a purpose. Na I D lines might also be affected by the non-photospheric contribution, since the excitation potentials are zero.

Instead, spectral energy distributions (SED's) and synthetic spectra covering a wide spectral range serve as tests of model atmospheres, and our model atmospheres can reproduce photometric and spectrophotometric observations fairly well.

In Fig. 4, we show a comparison between the predicted emergent flux and the photometric observation in Johnson et al. (1985) for TX Psc. The final model atmosphere for this star was computed with the carbon and oxygen abundances determined in a self-consistent way (Ohnaka 1997, Ohnaka & Tsuji 1998), and the emergent flux was computed from that final model. The match is quite fair, though it is rather poor for the strong absorption feature due to CN around 1.5 μm.

However, a synthetic spectrum based on our final model for TX Psc can reproduce spectrophotometric observations. In Fig. 5 we show the synthetic spectrum computed from the final model, together with the observational data of Lázaro et al. (1994). Lines of the CO first and second overtone bands, the C_2 Phillips system, and the CN red system are included in the calculation of the synthetic spectrum. As the figure demonstrates, the agreement between the calculation and the observation is quite good.

The photometric and spectrophotometric observations are in low spectral resolution, and it is impossible to check $^{12}C/^{13}C$ ratios by comparing the synthetic spectra with the observations. But the agreements shown above are favorable evidence for the validity of our model atmospheres. The slight disagreement between the observed and the predicted fluxes in Figs. 4 and 5 might be attributed to the temporal variation of the strength of molecular absorption features, which is clearly seen in other observations of Lázaro et al. (1994). As the next section mentions, measurements of angular diameters of carbon stars suggest the temporal variation of the effective temperature. In order to test model atmospheres more self-consistently, synthetic spectra should be calculated with effective temperatures determined at the same epoch with spectrophotometric observations.

Table 2. Comparison of effective temperatures determined using the IFM and angular diameters. A.D: effective temperatures determined based on the angular diameters by Dyck et al. (1996), IFM: effective temperatures determined based on the IFM by OT96, $F_{\rm BOL}$(OT96): effective temperatures determined with the bolometric fluxes used in OT96 and the angular diameters by Dyck et al. (1996). For Y Tau, UU Aur, X Cnc, and AQ Sgr, Dyck et al. (1996) determined the effective temperatures based on the angular diameters measured by Quirrenbach et al. (1994) and re-evaluated bolometric fluxes

Star	$T_{\rm eff}$(A.D.) (K)	$T_{\rm eff}$(IFM) (K)	$T_{\rm eff}$($F_{\rm BOL}$(OT96)) (K)
AQ And	3362 ± 394	2970	2818
V Aql	3328 ± 480	2790	2657
WZ Cas	3140 ± 193	3160	3090
RV Cyg	2784 ± 97	2709*	—
V460 Cyg	3200 ± 157	3230	3112
W Ori	3177 ± 306	2980	2770
RT Ori	2839 ± 292	3100	2724
Z Psc	3240 ± 239	3145*	—
TX Psc	2921 ± 60	3080	2814
Y Tau	2787 ± 169	2880	2616
UU Aur	2899 ± 82	3010	2728
X Cnc	2653 ± 102	2910	2641
AQ Sgr	3234 ± 257	3100*	—

*: $T_{\rm eff}$ are derived by the $(J-L)_0$–$T_{\rm eff}$ relation, which is based on the $T_{\rm eff}$'s determined directly from the IFM.

6. Determination of effective temperature

We determined effective temperatures of 33 program stars using the Infrared Flux Method (IFM) in OT96. For four stars, effective temperatures were derived also using angular diameters in order to check the result by the IFM. As Table 6 of OT96 shows, the effective temperatures derived by the IFM and angular diameters are in agreement within the limits of uncertainty of both methods, though the IFM tends to give higher effective temperatures.

Recently Dyck et al. (1996) determined effective temperatures of 15 carbon stars using angular diameters, based on new interferometric observations at the K-band, at which the effect of scattering is minor as compared with the optical region. Table 2 provides a comparison of the effective temperatures determined by Dyck et al. (1996) and those by OT96. Since the bolometric fluxes used in Dyck et al. (1996) are not necessarily in good agreement with those used in OT96, we also calculated effective temperatures with the bolometric fluxes used in OT96 and the angular diameters by Dyck et al. (1996). Such values are given in the fourth column of Table 2. For RV Cyg, Z Psc, and AQ Sgr, OT96 did not determined the effective temperatures directly from the IFM, but from the $(J-L)_0$–$T_{\rm eff}$ relation. Therefore, we do not give any values in the fourth column of Table 2 for these three stars. Though the effective temperatures of OT96 tend to be higher than those given in the fourth column, they are in agreement within the range of accuracy of both methods except for RT Ori. But even if we adopt the effective temperatures given in the fourth column, subsequent changes on $^{12}{\rm C}/^{13}{\rm C}$ ratios are quite minor for N-type stars. As Table 9 of OT96 shows, the uncertainty of $\Delta T_{\rm eff} = 200$ K only leads to a difference of 0.02 dex in $^{12}{\rm C}/^{13}{\rm C}$ ratios.

One concern about the determination of effective temperatures of SC stars is, however, the effect of the absorption of CS at around 3.9 μm, because it can have an effect on the L'-band (3.7 μm) flux used in the IFM. Recently Aoki et al. (1998) have analyzed infrared spectra of N- and SC-type carbon stars acquired with the ISO, and identified the strong absorption of CS at around 3.9 μm for 3 SC stars. They suggest that the effective temperatures of SC stars determined by OT96 should be lowered by about 150 – 180 K, if the effect of the CS absorption is taken into account. As demonstrated in Table 9 of OT96, the uncertainty of $\Delta T_{\rm eff} = 200$ K would lead to a difference of 0.1 dex in $^{12}{\rm C}/^{13}{\rm C}$ ratios for SC stars. Therefore, the correction of effective temperatures discussed in Aoki et al. (1998) would lead to an increase of $^{12}{\rm C}/^{13}{\rm C}$ ratios in SC stars by slightly less than 0.1 dex.

The effective temperatures derived from angular diameters are almost independent of uncertainties of model atmospheres, but there are still uncertainties about this method such as the errors in evaluating bolometric fluxes and the effect of scattering in angular diameter measurements. For example, comparison of bolometric fluxes for AQ Sgr, RT Cap, and TW Oph derived by Dyck et al. (1996) and Lázaro et al. (1994) shows significant disagreement. In addition, temporal variations of angular diameter are reported for Y Tau (Schmidtke et al. 1986) and TX Psc (Quirrenbach et al. 1994).

Simultaneous observations of angular diameter and bolometric flux should be carried out to obtain more accurate or actual effective temperatures, and to check the consistency between effective temperatures from the IFM and angular diameters. The analysis of $^{12}{\rm C}/^{13}{\rm C}$ ratios should also be done with spectra acquired at the same epoch.

7. $^{12}{\rm C}/^{13}{\rm C}$ determination of LGEH86

Though LGEH86 analyzed high-resolution infrared spectra where line density is lower than in the optical region, the analysis cannot be free from blending. As Sect. 3 mentions, Ohnaka (1997) and Ohnaka & Tsuji (1998) re-analyzed the spectra and found that the number of CO lines finally selected for the analysis is many fewer than given in LGEH86. The effect of blending should also be examined for the analysis of LGEH86 in the same way that dLG98 did for the iso-intensity method.

It should not be concluded that LGEH86's result is more reliable, despite their claims that their analysis is rather insensitive to model atmospheres and that the values from the CN red system, the CO first and second overtone lines are consistent. Even if $^{12}{\rm C}/^{13}{\rm C}$ ratios were derived from ^{12}CN and ^{13}CN lines with similar equivalent widths as dLG98 claim, it does not mean that the effect of model atmospheres could be canceled as long as ^{12}CN lines with high excitation potentials and ^{13}CN lines with low excitation potentials are used.

8. Concluding remarks

We have reviewed the analysis of dLG98, and demonstrated that they do not properly discuss the accuracy and the reliability of the result of OT96. The higher sensitivity of the iso-intensity method to model atmospheres, reported in dLG98, can be attributed to the way in which they calculate the abscissa of curves of depth growth. As Sect. 4 shows, their method of analysis differs significantly from ours and they are discussing something other than the iso-intensity method. Therefore, the discrepancy of $^{12}{\rm C}/^{13}{\rm C}$ ratios between LGEH86 and OT96 cannot be attributed to the inaccuracy of the iso-intensity method. Moreover, the determination of $^{12}{\rm C}/^{13}{\rm C}$ ratios using the iso-intensity method has been shown to be insensitive to model atmospheres to some extent, if it is properly applied.

The effective temperature scale in OT96 tends to be higher than that derived using angular diameters, but is in agreement within the bounds of accuracy of both methods for most of the program stars. However, more consistent observational data – that is, simultaneous observations of angular diameter and bolometric flux – could give more reliable effective temperatures of carbon stars.

Given the difference between dLG98 and OT96 about how to apply the iso-intensity method, the conclusion of dLG98 that the $^{12}{\rm C}/^{13}{\rm C}$ ratios derived by OT96 are not reliable can no longer be justified. At the same time, however, our suspicion that the difference of model atmospheres can explain the discrepancy of $^{12}{\rm C}/^{13}{\rm C}$ ratios has turned out to be not correct, either. We are re-analyzing the original spectra of LGEH86, and more thorough and detailed investigation into the discrepancy of $^{12}{\rm C}/^{13}{\rm C}$ ratios will be possible. For example, the lines finally selected for the analyses should be compared, to check the effect of blending. Moreover, the relatively low line density of the infrared spectra provides us with an opportunity to apply the iso-intensity method to the CN red system lines, in addition to the standard analysis using equivalent widths or synthetic spectra. Such investigations are expected to bring the controversy on $^{12}\mathrm{C}/^{13}\mathrm{C}$ ratios in carbon stars to an end.

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