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Distortion of the CMB spectrum by primeval hydrogen recombination

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Abstract. We solve the recombination equation by taking into account the induced recombinations and a physical cut off in the hydrogen spectrum. The effective recombination coefficient is parametrized as a function of temperature and free electron density and is about a factor of four larger than without the induced recombination. This accelerates the last stage of the recombination processes and diminishes the residual ionization by a factor of about 2.6. The number and energy distribution of photons issuing from the hydrogen recombination are calculated. The distortion of the cosmic microwave background (CMB) spectrum depends strongly on the cosmological parameters $\Omega,\ \Omega_b \,\mathrm{and}\, H_0$ and differs essentially from the Planck-spectrum for wavelengths $\leq 190\mu m$.

Key words: cosmology: theory – cosmic microwave background

1. Introduction

The CMB radiation in a wide frequency range has a Planck–spectrum with $T_0 = 2.728 \pm 0.002K$ (Fixsen et al. 1996). The last dramatic event that could have influenced a part of the spectrum was the recombination of the primeval hydrogen, since after it the residual ionization was very low and the radiation fields practically don't interact with the non-relativistic matter any more. The properties of the post–recombination Universe can be studied by observing the microwave background radiation. The fluctuations in the CMB are supposed to be the precursors of the largest structures observed today.

To understand the nature of the microwave background anisotropies it is necessary to have a correct picture of the recombination process itself. The problem was first studied shortly after the discovery of the CMB by Peebles (1968) and at the same time by Zel'dovich, Kurt and Sunyaev (1968). Since than several authors have adressed the problem, but no one could improve on the basic approximations used in these works. However many details were worked out. Matsuda, Sato & Takeda (1971) examined the effects of the collision processes and found that they are negligible with respect to the radiation processes. Jones & Wyse (1985) improved on the calculations to allow for the presence of non–baryonic matter. Krolik (1989, 1990) showed that two previously equally neglected scattering effects in the Ly_α line almost completely cancel each other. Sasaki & Takahara (1993) concluded that taking induced recombination into account changes the time history of the recombination at very low ionization grades, and lowers the residual ionization. They however used an approximation for the recombination cross sections and did not discuss the divergence of the sum of the recombination rates. Dell'Antonio & Rybicki (1993) followed the progress of recombination numerically, using a multilevel hydrogen atom model; all angular momentum states were treated individually up to $n = 10$. Their recombination curve was similiar to that of Jones & Wyse (1985). In an other article Rybicki & Dell'Antonio (1994) calculated the time-dependence of the Ly_α line profile in a homogenious expanding Universe. They found that the usual quasi-static approximation for the line shape is reasonable and does not cause substantial error in the solutions. They also determined the recombination history for several cosmologies. Hu, Scott, Sugiyama & White (1995) discussed the effect of physical assumptions for CMB anisotropies and on the recombination process.

Recombination, as it has been pointed out in the papers quoted above, sets in when all energy levels of hydrogen, except for the ground state are still in equilibrium with radiation. When one hydrogen atom enters the ground state, a photon with energy greater than or equal to the energy of a Ly_α transition is emitted. Photons, capable of exciting electrons in the ground state, are lost via two main routes: 1. by cosmological redshift of Ly_α photons, 2. by two–photon transition $2s \rightarrow 1s$. Of these two competing processes the first is purely cosmological, the second is atomic. Their relative importance depends on the cosmology.

By theoretical examination of the recombination process one computes the time dependence of the ionization rate, the residual ionization, the position and width of the last scattering layer and the distortion of the CMB spectrum. Though the amount of the residual ionization is important for the further evolution of the Universe (Peebles 1993, Lepp & Shull 1984), it can not be measured directly. A possible observable consequence of the hydrogen recombination in the early Universe is the distortion of the microwave background radiation spectrum. It was first calculated for the flat cosmological model by Peebles (1968). In this work we concentrate on the determination of this distortion for different cosmological models. Recently there has been a considerable observational and theoretical activity to determine the spectrum of the cosmic background radiation and carrying out galaxy counts in the far infrared/submillimeter range (Puget et al. 1996, Schlegel et al. 1997, Guideroni et al. 1997). Because the hydrogen recombination changes the CBM spectrum in the same spectral range a detailed exact recalculation of the frequency distribution of the recombination photons is important. In this paper we calculate the recombination cross sections exactly by using a continuous, physical cut off for the highly excited states of hydrogen, take into account the induced recombination and explain, why the time dependence of the recombination process is so hardly effected by different techniques and by different effective ionization curves.

The outline of the paper is as follows. In Sect. 2 we discuss the recombination process and derive the recombination equation. We give a new parametrization of the effective recombination coefficients, taking into account the induced recombination as well. In the third section we solve the recombination equation. The spectrum of the recombination photons and its dependence on the cosmological parameters is given in the fourth section, and in the fifth closing section we discuss our results.

2. The recombination process

2.1. The excited states of hydrogen

The number of recombinations in unit time can be calculated from the recombination equation as a function of the density of free electrons n_e, temperature T, and cosmological, and atomic constants. In this section we derive this equation following Peebles (1968).

At the beginning of the hydrogen recombination the helium is already completely recombined. The mass fraction of the helium is 25% of the total baryonic mass. As was remarked by Novikov and Zel'dovich (1967) the direct recombinations to the ground state are inhibited, while the new born energetic photons ionize again almost immediately when there are already some hydrogen atoms. In the following we neglect completely the direct recombinations to the ground state.

The states with principal quantum number $n = 2$ play a key role in the recombination processes. First we calculate the rate of recombinations to $n \geq 2$ for given free electron number density n_e, temperature T and n_{2s}. Here n_{nl} is the density of atoms in the state with the principal quantum number n and angular momentum quantum number l. Second, we determine the net number of $2 \to 1$ transitions in unit time by given n_{2s} and n_{1s}. The two rates must be equal, so we can eliminate n_{2s} from the calculations.

The binding energy of the $n = 2$ state is $B_2 = B_1/4 \approx 3.4\,eV$. During the recombination process there are a large number of photons with energy less than B_2, therefore the excited states of the hydrogen atoms are in thermodynamical equilibrium above the second level, i.e.

$$n_{nl}^{(\mathrm{Boltzmann})} = \frac{(2l+1)}{4} e^{-\frac{B_2-B_n}{kT}} n_2 = (2l+1) e^{-\frac{B_2-B_n}{kT}} n_{2s}. \quad (1)$$

In the case of gaseous nebulae the mean free path of low energetic photons is larger than the dimensions of the ionized region, the system is far from being in equilibrium above the $n = 2$ level in contrast to the recombining Universe.

The partition sum $\sum_{nl}(2l+1)n_{nl}^{(\mathrm{Boltzmann})}$ is divergent. As analysed by Hummer and Mihalas (1988) a number of effects limit the range of summation. In the considered temperature and density range the action of free protons turns out to be the most important, the action of neutral atoms are much smaller. The free protons destroy the state n with a probability $1 - w_n$, where $w_n = \exp[-(\frac{n}{n_*})^{15/2}]$, with $n_* = 1075.9\,\mathrm{cm}^{-2/5}\, n_e^{-2/15}$ (n_e in cm^{-3}). This means that the recombined electrons become unbound with $1 - w_n$ probability, before they would begin to move towards the state, corresponding to thermal equilibrium. The highly excited (n larger than $\sim n_*$) states are practically completely destroyed. The occupation numbers of the excited states in thermal equilibrium are:

$$n_{nl} = (2l+1) e^{-\frac{B_2-B_n}{kT}} n_{2s}\, w_n. \quad (2)$$

Owing to this fact, the number of electrons in bound states is finite, and thermal equilibrium between the $n \geq 2$ bound states and the continuum is possible. This approximation is good down to z $\approx$ 300. The $n = 2$ levels freeze out between the redshifts 300 and 250.

The ground state of the hydrogen atom is about 10.2 eV deeper than the $n = 2$ level. Consequently, it's occupation is greater than the occupation of the excited states together: $n_{1s} \gg \sum_{n\geq 2} n_n$. This means, that the density of free electrons plus the density of hydrogen atoms in ground state can be taken as equal to the total proton number: $n_e + n_{1s} = p$. The total baryon number (determined by Ω_b) is the sum of p and the number of baryons in the helium nuclei.

2.2. Recombination on excited states of hydrogen

The number of recombinations to the level nl and the number of ionizations from nl in unit time are

$$\alpha_{nl}\, n_e\, n_p = \alpha_{nl}\, n_e^2 \qquad \text{and} \qquad \beta_{nl}\, n_{nl} \quad (3)$$

where α_{nl} and β_{nl} are the recombination and ionization coefficients, n_p is the density of free protons. The usual definition of these quantities are

$$\alpha'_{nl} = <\sigma_{nl}^{(\mathrm{rec})} v>_{\mathrm{Maxwell}}, \quad \beta_{nl} = <\sigma_{nl}^{(\mathrm{ion})} c>_{\mathrm{Planck}}. \quad (4)$$

Here v and c are the velocities. The averages must be taken over the Maxwell–distribution for electrons and the Planck–distribution for photons.

The ionization coefficients are:

$$\beta_{nl} = (2l+1) \int c\, \sigma_{nl}^{(ion)}(\nu) \frac{8\pi\nu^2}{c^3} \frac{2}{e^{\frac{h\nu}{kT}} - 1} d\nu\,. \quad (5)$$

The relation between the recombination and ionization cross sections is: $\sigma_{nl}^{(rec)} = \sigma_{nl}^{(ion)} \frac{2\mathbf{k}^2}{\mathbf{p}^2}$, where $\mathbf{k}$ and $\mathbf{p}$ are the photon and electron momenta.

By using, instead of the electron velocity, the energy of the recombination photon in the Maxwell–distribution and expressing the recombination cross section through the ionization cross section, one arrives at

$$\alpha'_{nl}(T) \;=\; \frac{4\pi 2(2l+1)}{c^2(2\pi m_e kT)^{\frac{3}{2}}} e^{\frac{B_n}{kT}} \int_{B_n}^{\infty} \sigma_{nl}^{(ion)} (h\nu)^2 e^{-\frac{h\nu}{kT}}\, d\,h\nu, \quad (6)$$

where m_e is the electron mass. The integrals in α'_{nl} and β_{nl} are not the same. But, due to the principle of detailed balance (Milne 1924) the recombinations and ionizations in thermodynamic equilibrium must exactly cancel each other. If we let contributions from the induced recombinations modify the recombination coefficients (e.g. Mihalas 1984, Sasaki & Takahara 1993) instead of (6) we get

$$\begin{aligned}\alpha_{nl}(T) \;&=\; \frac{4\pi\, 2(2l+1)}{c^2(2\pi m_e kT)^{\frac{3}{2}}} e^{\frac{B_n}{kT}} \int_{B_n}^{\infty} \sigma_{nl}^{(ion)} \frac{(h\nu)^2}{e^{\frac{h\nu}{kT}}-1}\, d\,h\nu \\ &=\; \Big(\frac{2\pi\hbar^2}{m_e kT}\Big)^{3/2} e^{\frac{B_n}{kT}} \beta_{nl}. \end{aligned} \quad (7)$$

The term -1 under the integral in the denominator comes from the induced recombinations. Without it the principle of detailed balance would not be fulfilled.

Summing up over all n, l levels and making use of Eq. (2) the total rate of recombinations is:

$$\alpha n_e^2 \qquad \text{with} \quad \alpha = \sum_{n=2}^{\infty} \sum_{l=0}^{n-1} \alpha_{nl}\, w_n. \quad (8)$$

The total rate of ionizations from the excited states of hydrogen is:

$$\sum_{n=2}^{\infty} \sum_{l=0}^{n-1} \beta_{nl}\, n_{nl}\, w_n = \Big(\frac{m_e kT}{2\pi\hbar^2}\Big)^{3/2} e^{-\frac{B_2}{kT}} \alpha\, n_{2s} = \beta n_{2s}. \quad (9)$$

The net number of recombinations to excited levels in unit time and unit volume is the difference of these two expressions. The number density of the electrons changes, not only because of the recombination process, but also in consequence to the universal expansion. The simplest way to take this effect into account is to write on equation for the ionization rate, i.e. the ratio of free electrons to the number of free protons plus hydrogen atoms:

$$-\frac{d}{dt}\frac{n_e}{p} = \alpha\Big[\frac{n_e^2}{p} - \Big(\frac{m_e kT}{2\pi\hbar^2}\Big)^{3/2} e^{-\frac{B_2}{kT}} \frac{n_{2s}}{p}\Big]. \quad (10)$$

2.3. Depopulation of the $n=2$ level

Each $2p \to 1s$ transition liberates a Ly_α photon, which can excite the ground state, and there are always enough low energy photons to ionize from an excited state. The transition to the ground state is final, when the number of these $E_\gamma \geq B_1 - B_2$ photons is also diminished. Photons can escape from the Ly_α line either by redshift or by two–photon 2s → 1s transitions.

In a stationary state the numbers of emitted and absorbed Ly_α photons are equal. Consequently, the number of photons per mode in the Ly_α resonance line is:

$$\gamma_{12} = \frac{n_{2p}}{3n_{1s} - n_{2p}} \approx \frac{n_{2p}}{3n_{1s}} = \frac{n_{2s}}{n_{1s}}. \quad (11)$$

Because of the general expansion of the Universe a frequency ν_0 is shifted in unit time by

$$\frac{\delta\nu_0}{\delta t} = \nu_0 H, \quad (12)$$

where H is the Hubble–parameter. The number of recombination photons removed from the line by redshift in unit time is:

$$\begin{aligned} R_{\rm rs} \;&=\; \frac{\delta\nu_{12}}{\delta t}\frac{8\pi\nu_{12}^2}{c^3}\big(\gamma_{12} - \gamma_{12}(\text{background})\big) \\ &=\; H\frac{8\pi\nu_{12}^3}{c^3}\big(\gamma_{12} - \frac{1}{e^{h\nu_{12}/kT}-1}\big). \end{aligned} \quad (13)$$

On the other hand, the rate of two–photon decay in the 2s states has been calculated by Spitzer & Greenstein (1951). The number of net decays in unit time is

$$R_{\rm 2phot} = A_{2s,1s}\big(n_{2s} - n_{1s} e^{-(B_1-B_2)/kT}\big), \quad (14)$$

with $A_{2s,1s} = 8.227 s^{-1}$. In thermal equilibrium the numbers of two–photon decays and the two–photon $1s \to 2s$ excitations are equal.

At this point we have four unknown quantities: $n_{1s}, n_{2s}, \gamma_{12}$ and n_e and four Eqs. (10), (11), (13) and (14); so we are left with

$$\begin{aligned} -\frac{d}{dt}\frac{n_e}{p} \;&=\Big[\frac{\alpha n_e^2}{p} - \beta\big(1 - \frac{n_e}{p}\big)e^{-(B_1-B_2)/kT}\Big] \\ &\quad \cdot \frac{8\pi\nu_{12}^3 c^{-3} H + A_{2s,1s}(p-n_e)}{8\pi\nu_{12}^3 c^{-3} H + (A_{2s,1s}+\beta)(p-n_e)}. \end{aligned} \quad (15)$$

The time dependence of p, T and H are calculated from the Friedmann equation with the density parameter $\Omega (= \Omega_b + \Omega_\gamma + \Omega_{DM})$, the baryon density parameter Ω_b and the Hubble constant $H_0 = 100\,\mathrm{h\,km\,s^{-1}Mpc^{-1}}$ ($\Lambda = 0$, the cosmological constant).

2.4. Parametrization of the recombination coefficient

To solve the Eq. (15) we need the recombination coefficient α, wich is parametrized as a function of the temperature in several ways. Peebles (1968) used $\alpha = 2.84 \cdot 10^{-11}/\sqrt{T}\mathrm{cm^3\,s^{-1}}$ on the basis of Boardman's (1964) data for the seven lowest states. Zel'dovich, Kurt and Sunyaev (1968) used $2.5 \cdot 10^{-11}/\sqrt{T}\,\mathrm{cm^3\,s^{-1}}$. Peebles (1993) using Osterbrock's (1989) data got the parametrization: $\alpha = 4.1 \cdot 10^{-10}\, T^{-0.8}\,\mathrm{cm^3\,s^{-1}}$. Recently Hummer (1994) tabulated very accurately the recombination rates. His results confirm the parametrization given by Péquignot et al. (1991) in the form of

$$\alpha(T) = 1.261 \cdot 10^{-10} \frac{T^{-0.6166}}{1 + 5.087 \cdot 10^{-3} T^{0.53}}. \quad (16)$$

Sasaki & Takahara (1993) had used the asymptotic form for the recombination cross sections which, also valid for large principal quantum numbers n, was less than 20% accurate for the important low lying levels.

When calculating the recombination coefficients without taking into account the induced recombination, the -1 falls

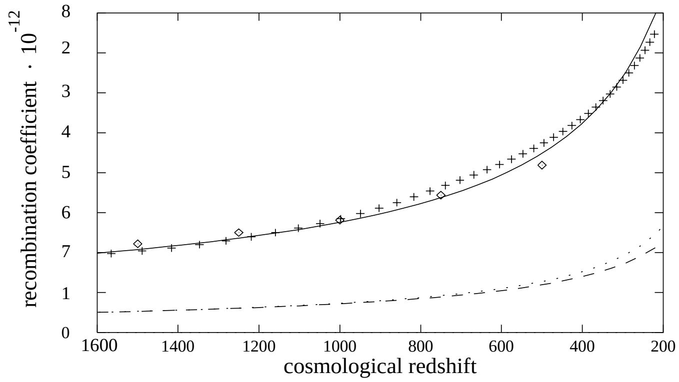

Fig. 1. Recombination coefficient as a function of redshift. Crosses: Present work (for $\Omega = 1,\ \Omega_b = 0.1,$ h = 1); Continuous line: $T^{-0.7}$ fit; Diamonds: Sasaki & Takahara 1993; Points: Eq. (16); Dashed line: Peebles 1993.

out of the denominator of the integral in (7). In this case the α_n series converges as $\log(n)/n^3$, so one can neglect the contribution of the terms with large n. If induced recombination is taken into account, the denominator under the integral in (7) goes to zero for large n, and the series of the α_n diverges as $1/n$.

Here we calculate the total recombination coefficient from (8), wherein the factor w_n, the probability that the states with principal quantum number n are not destroyed, makes the sum finite. The probability that a state is destroyed depends on the baryon density of the Universe. Consequently, the effective recombination coefficient also density dependent.

Our procedure is then the following. We calculate the first several hundred α_{nl}'s using the exact results given by Storey & Hummer (1991). For large n (≥ 200), when the recombination coefficients have already reach their asymptotic values (the difference between the exact and asymptotic value $< 0.1\%$), we use the α_n's given e.g. in Rybicki & Lightman (1979). The density dependence comes in through the w_n probabilities. The effective recombination coefficient is parametrized as a function of the temperature, T, and the electron number density, n_e, in the form

$$\alpha(T, n_e) \;=\; 1.21 \cdot 10^{-10}\, T^{-\frac{1}{2}} \cdot (\, 1 - 0.0226\, \ln(n_e\, cm^3)\ )\ \mathrm{cm}^3\ \mathrm{s}^{-1}\,. \qquad (17)$$

For a given temperature n_e and α depend on the cosmological parameters. However this dependence is mild, not larger than 10% for the usual range of parameters. We compute the $\alpha(T, n_e)$ function together with n_e by solving (15). The solution of this equation for $\mathrm{n_e}$ and $\alpha(T, n_e)$ will be given in the next section. For the sake of clarity we present our recombination rate already here (Fig. 1). The form of the $\alpha(T, n_e(T))$ function is not far from the usual $\sim T^{-0.7}$ function, but its amplitude is about four times larger than it would be without taking the induced recombination into account. Furthermore we compare our $\alpha(T)$ with (16), and with Sasaki & Takahara (1993), whereby our function is somewhat steeper than that of these authors.

Table 1. Residual ionization at $z = 220$ for five cosmolgical models. x_2 and x_1 are the residual ionizations calculated with and without taking into account the induced recombination.

Ω	1	1	0.5
Ω_b	1	0.06	0.1
h	1	1	1
x_1	$2.1359 \cdot 10^{-5}$	$3.4811 \cdot 10^{-4}$	$1.5068 \cdot 10^{-4}$
x_2	$8.3545 \cdot 10^{-6}$	$1.3565 \cdot 10^{-4}$	$5.8440 \cdot 10^{-5}$
x_1/x_2	2.5567	2.5662	2.5784

Ω	0.1	0.3
Ω_b	0.1	0.01
h	0.75	0.5
x_1	$9.9243 \cdot 10^{-5}$	$2.2974 \cdot 10^{-3}$
x_2	$3.7740 \cdot 10^{-5}$	$8.6741 \cdot 10^{-4}$
x_1/x_2	2.6297	2.6486

3. The solution of the recombination equation

We solved the Eq. (15) for recombination numerically using Eqs. (17) and (9). The results with and without taking into account the induced recombination are compared.

3.1. The residual ionization

At $z \sim 200$ $\mathrm{H_2}$ formation becomes possible. Since free electrons and protons serve as catalysts for the formation of molecular hydrogen (Peebles 1993), the value of the ionized fraction at this epoch is a very important.

At about $z \sim 500$ β becomes much smaller than $\mathrm{A_{2s,1s}}$. It is independent of the cosmology, because both β and $\mathrm{A}_{2s,1s}$ are atomic quantities. In this case the fraction in Eq. (15) is one. The second term in the square bracket is also small, so instead of the Eq. (15) for $x = n_e/\mathrm{p}$ we can write

$$\frac{dx}{dz} = \frac{\alpha p}{zH}x^2 = \mathrm{const}\frac{\sqrt{\Omega}}{\Omega_b h}x^2(1 - 0.0226\ln p\, x). \qquad (18)$$

When the induced recombinations are not taken into account the density dependent term does not appear and the residual ionization scales with $\sqrt{\Omega}\Omega_b^{-1}h^{-1}$. The logarithmic term contains another combination of the cosmological parameters, therefore the scaling law is mildly violated. The residual ionizations at $z = 220$ are shown in Table 1. When the induced recombination is taken into account the residual ionization is reduced by a factor of about 2.6. Small deviations from this factor are due to the violation of the scaling law. The total number of ions is proportional to $x\,\Omega_b$, so it is almost independent of the baryon density.

We refined the previous treatment (Sasaki& Takahara 1993) of the induced recombination by using the exact recombination cross sections and applying a physical cutoff in the summation on the hydrogen states. This causes a 10 - 20% change in the residual ionization.

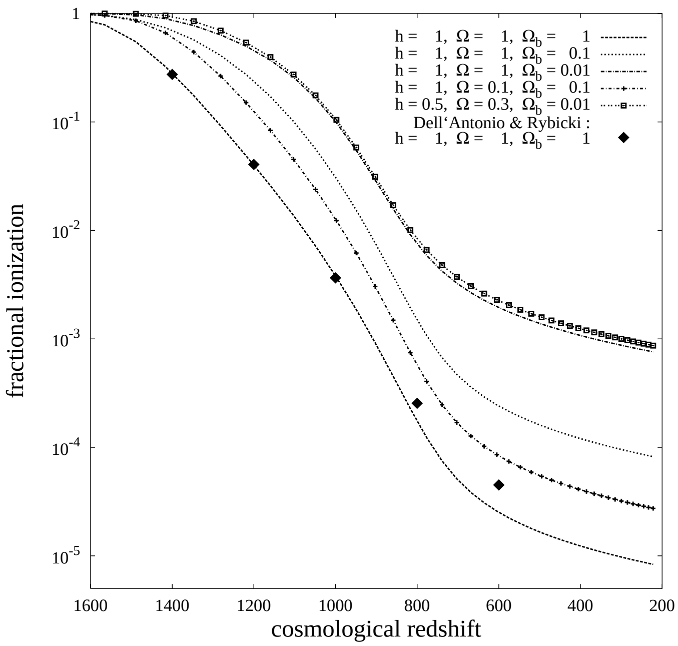

Fig. 2. Recombination history for different cosmological models.

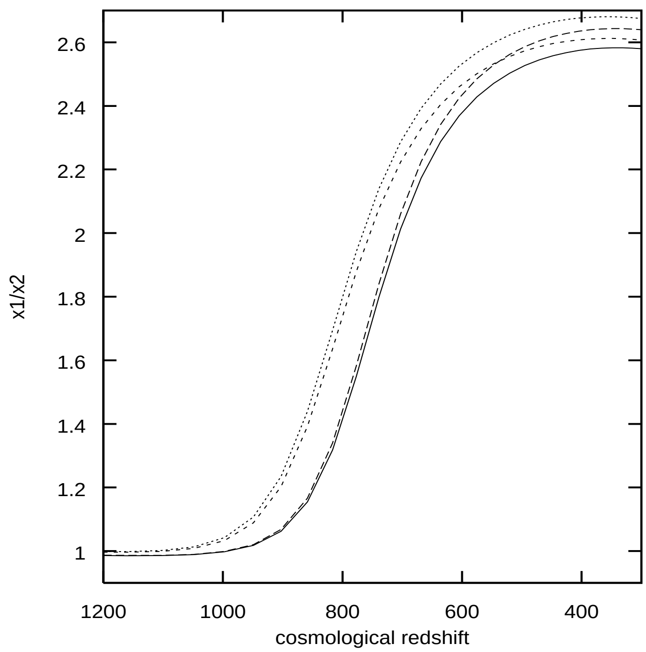

Fig. 3. Ratio of the fractional ionization calculated with (x_2) and without (x_1) taking induced recombination into account. From right to left:
a) Ω = 1, Ω_b = 1, h = 1
b) Ω = 0.1, Ω_b = 0.1, h = 1
c) Ω = 1, Ω_b = 0.01, h = 1
d) Ω = 0.3, Ω_b = 0.01, h = 0.5

3.2. *The time dependence of the fractional ionization*

The recombination history for different sets of comological parameters is shown in Fig. 2. The quotient of the fractional ionization at different cosmologies grows with time and approximately follows the x $\sim \sqrt{\Omega}\Omega_b^{-1}h^{-1}$ scaling law. We compare our result with Dell'Antonio and Rybicki (1993), who did not take induced recombinations into account. For corresponding parameters our recombination curve runs significantly under their curve, because the last phase of the recombination processes is accelerated by the induced recombinations.

In Fig. 3 we show the ratio of fractional ionization calculated with (x_2) and without (x_1) taking induced recombination into account. In the first part of the process the induced free–bound transitions and the numerical value of the effective recombination coefficient have little effect on the course of the process, because the depopulation of the $n = 2$ states is much slower than the recombination to the excited states, the thermal equilibrium corresponding to the occupation of $n = 2$ is always maintained. Down to $z \approx 1000$ the x_1/x_2 ratio is one. Between $z \approx 1000$ and $z \approx 500$ the change owing to induced recombinations depends on the cosmology. Later, when $z \leq 500$, especially when the baryon density Ω_b is small, the depopulation of $n = 2$ states is faster than the recombination to the excited states and this last process determines the net recombination rate. At this stage the induced recombinations play an important role. As can be seen in Fig. 3, the x_1/x_2 ratio depends hardly on the cosmological parameters.

3.3. *The last scattering surface*

From the $n_e(z)$ function one can estimate the probability that a CMB photon has not been scattered since a given redshift z. Instead of this probability, usually its differential is calculated. This quantity determines the probability density that the radiation was last scattered at z and is expressed as $g(z) = e^{-\tau}\, d\tau/dz$. Here τ is the Thomson scattering optical depth: $\tau = -\int_0^z c\sigma_T n_e(z)\frac{dt}{dz}dz$, and σ_T is the Thomson cross section. The parameters, determining the position and width of this layer are given in Table 2. These results are in reasonable agreement with those of White, Scott & Silk (1994). This occurs because the last Thomson scattering happens early on, when the induced recombinations don't play any role.

Table 2. Position and width of the last scattering surface, calculated for different sets of cosmological parameters.

Ω	1	1	1	0.1	0.1
Ω_b	1	0.1	0.01	0.1	0.1
h	1	1	1	1	0.5
$z_{\rm rec}$	1032.0	1043.6	1120.0	1028.4	1020.1
Δz	85.7	90.8	134.3	85.0	83.8

4. The spectrum of the recombination photons

4.1. *The time variation of the photon spectrum*

The evolution of the photon spectrum during recombination can be calculated from the continuity equation in frequency space

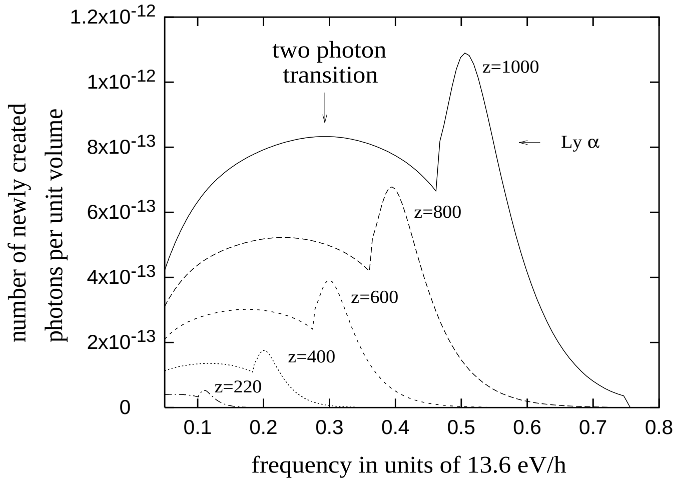

Fig. 4. Spectra of recombination–photons at different redshifts for $\Omega = 1, \Omega_b = 0.1, h = 1$.

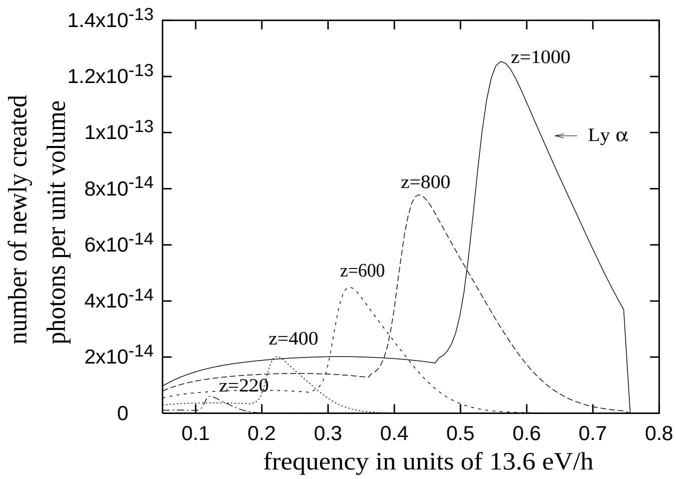

Fig. 5. Spectra of recombination–photons at different redshifts for $\Omega = 1, \Omega_b = 0.01, h = 1$.

(Peebles 1968):

$$\frac{\partial}{\partial t}\left(\frac{\nu\, n(\nu,t)}{p(t)}\right) = H(t)\,\nu\,\frac{\partial}{\partial \nu}\left(\frac{\nu\, n(\nu,t)}{p(t)}\right) + \frac{\nu\; Q(\nu,t)}{p(t)}\,. \quad (19)$$

$n(\nu,t)$ is the number density of photons with frequency ν, and $Q(\nu,t)$ is the net rate of production of photons per unit volume and unit frequency interval. The Planck–spectrum fulfills this equation without the source term, so (19) is also true for $n'(\nu,t) = n(\nu,t) - n_{\rm Planck}(\nu,t)$. Here we introduce new dependent variables with the definition

$$g(\nu,t) = \frac{\nu}{p(t)}\, n'(\nu,t) \quad (20)$$

and define the independent variables x, τ as

$$x = \ln\frac{h\nu}{B_1} \quad \text{and} \quad \tau = \int^t H(t')dt'. \quad (21)$$

Instead of the Eq. (19) we have

$$\left(\frac{\partial}{\partial \tau} - \frac{\partial}{\partial x}\right) g(x,\tau) = \frac{B_1\, e^x}{2\pi\hbar\, H(\tau)\, p(\tau)}\, Q(x,\tau). \quad (22)$$

In the absence of sources the spectrum moves to the left (with “velocity” 1) in the $(\tau\,,x)$ plane.

When the right hand side of (22) differs from zero, the solution can be written in integral form as:

$$g(x,\tau) = \int_x^{x+\tau} \frac{B_1\, e^{x'}}{2\pi\hbar,\, H(\tau')\, p(\tau')}\, Q(x',\tau')dx', \quad (23)$$

with $\tau' = x + \tau - x'$.

4.2. *The spectrum of photons issuing from the recombination*

From the solution of Eq. (15) we know the time dependence of the free electron density $n_e(t)$. Using the Eqs. (14) and (13) the net two–photon transition rate and the Ly_α redshift rate can be calculated. The spectrum of the photons emerging from the $2s \to 1s$ transition can be taken from Spitzer & Greenstein (1951). The appropriately normalized two–photon spectrum in terms of the variable x is:

$$\Phi(x) = 0.7081 e^x \psi\left(\frac{4}{3}e^x\right). \quad (24)$$

The $\int_{-\infty}^{x_{12}} \Phi(x)dx = 2$.

For our purposes the frequency distribution of the Ly_α photons can be approximated by a delta function on the ν axis, so

$$Q(x,\tau) = R_{2ph}\Phi(x) + R_{rs}e^{x-x_{12}}\delta(x - x_{12})\,. \quad (25)$$

4.3. *The dependence of the spectrum on cosmological parameters*

With $n_{1s}(t)$, the ratio of Ly_α redshift rate to the two–photon decay rate can be calculated from Eqs. (13) and (14). This ratio depends on the cosmology. For the $2 \to 1$ transition neglecting the -1 in the denominator of Eq. (13) and using the numerical values, one arrives at

$$R = \frac{R_{rs}}{R_{2phot}} = \frac{8\pi\nu_{12}^3 c^{-3} H}{A_{2s,1s} n_{1s}} = \frac{1}{1-x}\;\frac{632.92}{(1+z)^{3/2}}\,\frac{\Omega^{1/2}}{\Omega_b h}. \quad (26)$$

The redshift of the new–born photons is given by Eq. (22) in terms of the variables x and τ. The spectrum at different redshifts is shown in Fig. 4 ($\Omega = 0.1$, $\Omega_b = 0.1$, $h = 1$) and Fig. 5 ($\Omega = 1$, $\Omega_b = 0.01$, $h = 1$). When the baryon density is low, the 2s $\to$ 1s transition plays a minor role.

These results disagree with Dell’Antonio & Rybicki (1993), who state that the energy distribution of the photons emitted by the $2s \to 1s$ transition rather strongly peaked at $\nu = n_{\mathrm{Ly}_\alpha}/2$ and the $2s \to 1s$ transitions give no more than 1% difference in the free electron densities and the line strength. Our results (Fig. 4 and Fig. 5) show that the spectrum of $2s \to 1s$ photons is broad (see also Spitzer & Greenstein 1951). Though for certain cosmologies the contribution of these photons is small, but as it is demonstrated in Fig. 6, for other set of parameters this

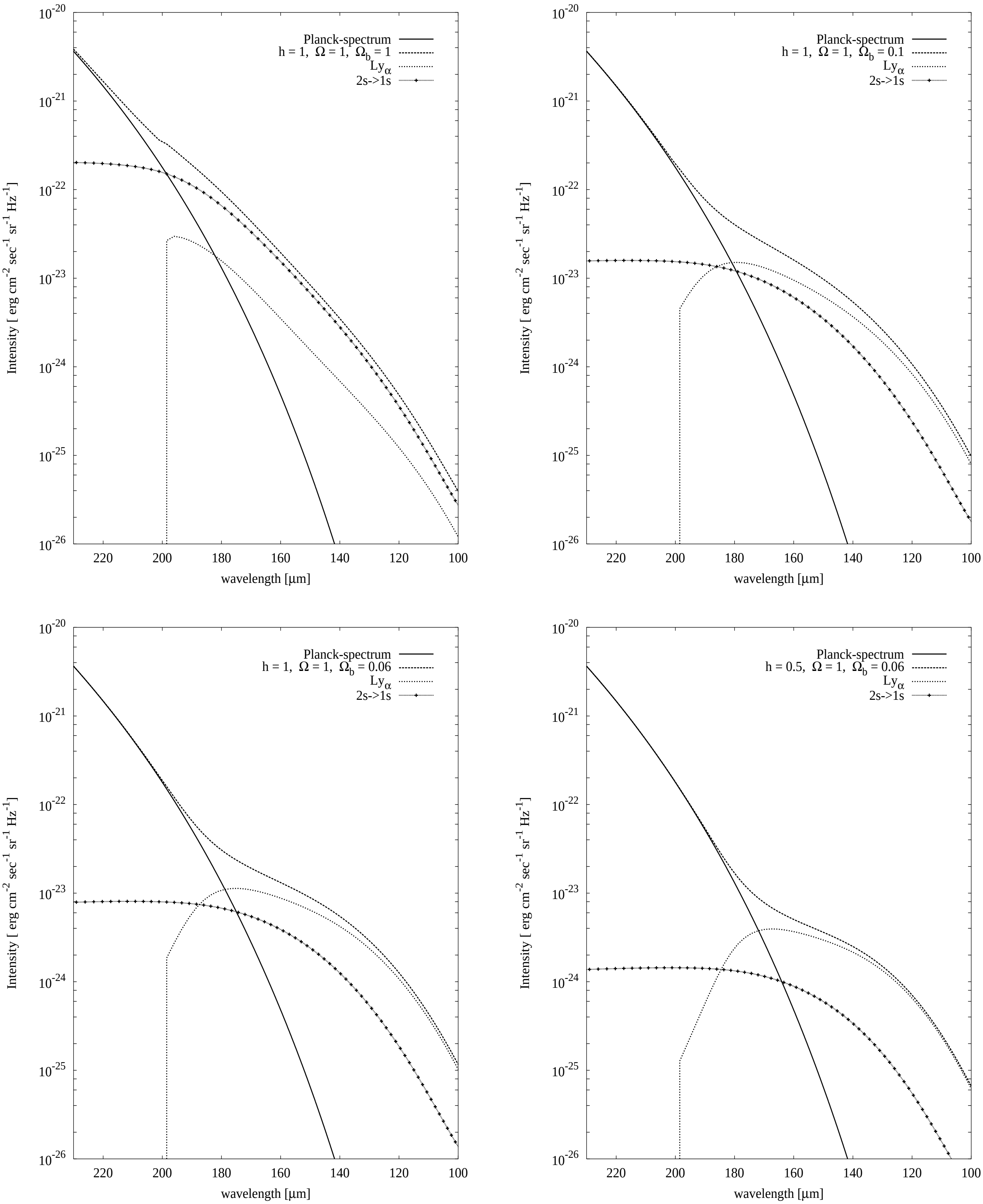

Fig. 6. Contribution of the Ly$_\alpha$ Photons and the $2s \rightarrow 1s$ two–photon transitions to the distortion of the CMB for different cosmological models.

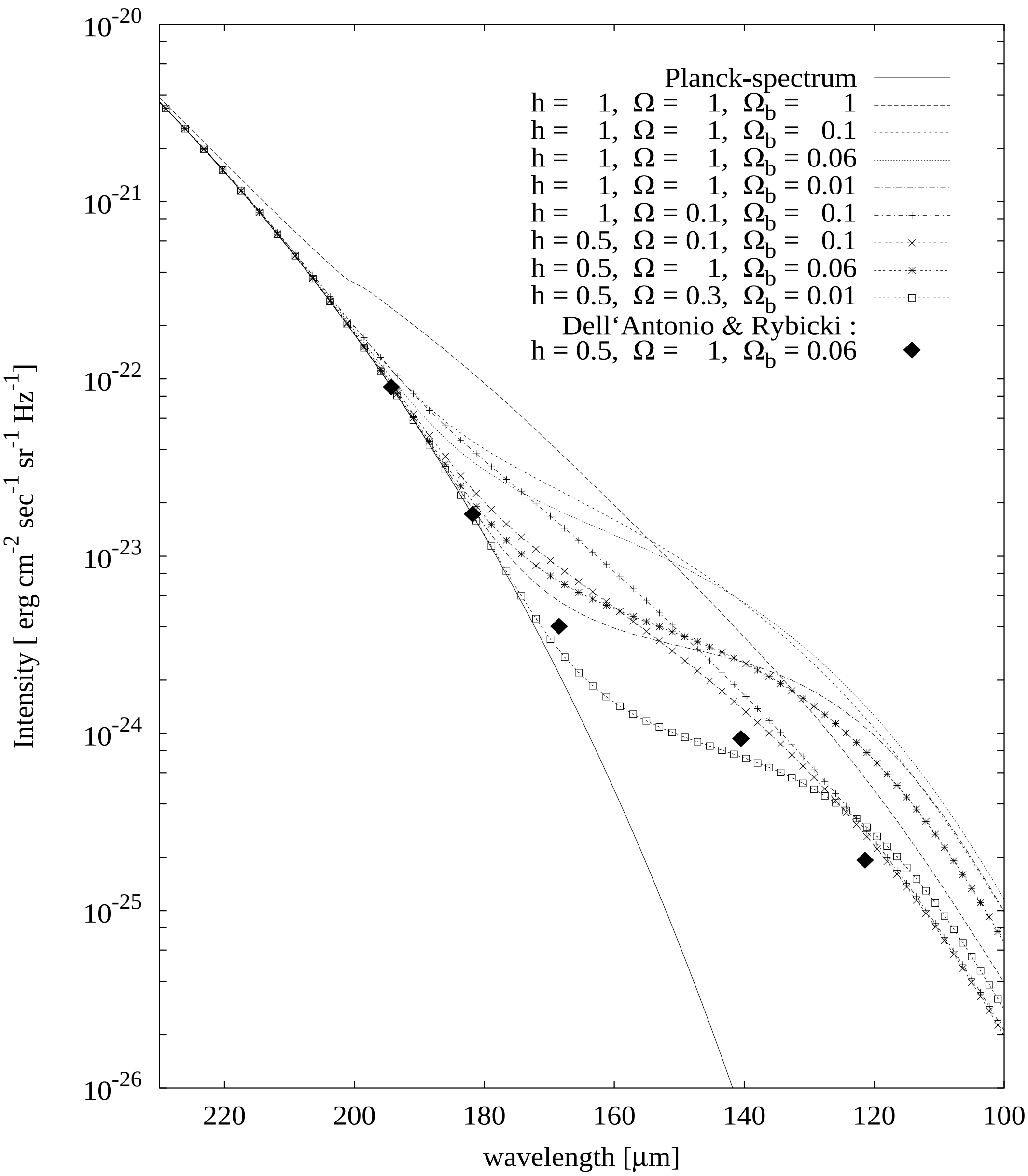

Fig. 7. CMB Spectra for different cosmological models.

contribution is quite considerable. In the models with high Ω and low Ω_b the two–photon transitions are negligible, but for $h^2\Omega_b \geq 0.1$ they are more important, and for the flat, high Ω_b model they are dominant. In spite of the above mentioned disagreement our recombination history curve (Fig. 2) agrees well with that of Dell'Antonio & Rybicki (1993), because by computing this function they took into account also the 2s $\rightarrow$ 1s transitions.

Fig. 7 shows a part of the CMB radiation spectrum for different cosmological parameters, where the distortion due to recombinations is the most important. The hydrogen recombination begins when in the background radiation there are less energetic photons than hydrogen atoms. Consequently, the spectrum of the photons issuing from the recombination has a maximum near the Planck-curve. The other maximum in this spectrum corresponds to the two–photon transitions, is longwards from the first maximum at $h\nu_{\max,2} = (B_2 - B_1)/2 = 5.1$ eV, and is under the Planck–curve. The $2s \rightarrow 1s$ photons influences the distortion in two ways. First, the short wavelength part of their spectrum is above the Planck-curve and give a direct contribution to the distortion (Fig. 6). Second, the number of recombinations are given by the number of hydrogen atoms. If a considerable part of recombinations happens by two–photon decays, there are less redshifted Ly$_\alpha$ photons, the amplitude and shape of their spectrum changes.

For given Ω_b and h there are more photons above the Planck–curve when both Ω and R are large. When Ω and h are given, the spectrum with the larger Ω_b lies above all the others. For a fixed value of Ω and Ω_b the spectrum with larger h is larger.

In Fig. 7 we compare our spectrum with that of Dell'Antonio & Rybicki (1993). In the 120 μm $< \lambda <$ 170 μm range there are about a factor of 2–3 more photons as seen by comparing our curve with the diamonds. That may be caused by the different technique in computing the recombination process. It seems us somewhat arbitrary to handel the 10 lowest level in a different way as the others. Burgess (1958) used this method to compute the recombination spectrum in nebulae. In the case of nebulae this method gives good results, because there is no thermal equilibrium above the n = 2 states, the electrons recombine on the low lying states and cascade down.

5. Summary and discussion

Background photons stimulate recombination processes and that give an important contribution to the effective recombination coefficient. The resulting recombination coefficient is about a factor of four larger than calculated without the induced recombinations (Fig. 1) and depends, besides the temperature, on the free electron (proton) density as well. In contrast to the recombining Universe in a gaseous nebulae with the same temperature and free electron density the recombination coefficient is given by (16), because there are no photons that should stimulate recombination on excited states. The photons, except the Ly_α photons, emerging from recombination in the nebulae leave the nebula without interaction.

For any given principal quantum number the α_{nl} recombination coefficients depend considerably on the angular momentum quantum number l. There is a peak around $l = 4$ and α_{nl} vanishes when l is high. Transitions between states with high l's are slow, which could influence the relative occupation of s and p states. Since this last effect is probably small (see Hu et al. 1995), we have completely neglected it here.

In the main part of the recombination process the free–bound transitions are more rapid than the depopulation of the $n = 2$ state and the thermodynamic equilibrium is maintained by reionization. Consequently, the course of the recombination process does not depend on the details of the $\alpha(T, n_e)$ function. Because of that, authors, using quite different effective recombination coefficients, come up with similiar time dependence for recombination. When, however, the free electron density is already low, the depopulation of the $n = 2$ states is more rapid than the recombinations of the $n \geq 2$ states and the speed of the recombination process is determined by the free–bound transitions of the excited states and the induced recombinations are important. As a consequence, our ionization curve (Fig. 2) is for small ionization grades steeper than that of Dell'Antonio & Rybicki (1993), who did not take this effect into account.

The last scattering of the CMB photons happens, with high probability, in the first phase of the recombination, when the number of free–bound transitions to the excited states is large. The induced recombinations and the numerical value of the effective recombination coefficient do not influence the position of the last scattering layer. Our results agree with those of White, Scott & Silk (1994).

On the other hand the residual ionization is a quantity determined by the details of the recombinations of the excited states at low n_e. At $z < 500$, whether induced recombination is taken into account or not, the fractional ionization is proportional to $\sqrt{\Omega}\Omega_b^{-1}h^{-1}$. However, because of the induced recombinations, the residual ionization is reduced by a factor of about 2.6. The exact value of this factor depends on the cosmological parameters. A consequence of the low residual ionization and the higher number of photons in the tail of the CMB spectrum is that the final abundance of molecular hydrogen will be much smaller than that estimated by Lepp & Shull (1983).

The hydrogen recombination gives contribution to the CMB spectrum in the $\lambda <$ 190 μm range. The amplitude and form of this distortion depends on the number of photons issuing from the recombination, and their distribution between the two decay way, the $2s \to 1s$ two–photon mode and the redshifted Ly_α mode. The induced recombinations only in the last phase influence the recombination process, when the ionization grade already very low. This gives a very small contribution to the distortion of the spectrum.

The contribution of helium recombination to the photon spectrum was discussed by Lyubarsky & Sunyaev (1983) and by Fahr & Loch (1991). According to their results the recombination of He sets in at a redshift of about four times larger than that for hydrogen recombination. But the photon energy is also four times larger and the photons emerging from helium recombination could also distort the CMB spectrum in the same frequency range as the H–recombination. However, as it was pointed out by Peebles (1995), at those redshifts where He II recombines there is already a trace of recombined hydrogen, which can absorb some of the photons created during helium recombination. Thus direct recombination into the ground state is possible for helium and the Saha formula is a good approximation to describe the helium recombination history. The optical depth due to ionization between z_1 and z_2 is

$$\tau = \int_{z_1}^{z_2} \sigma_{ion} n_{1s} \frac{dr}{dz} dz \, . \qquad (27)$$

The redshift of the helium recombination is z_1, the cross section for hydrogen ionization is σ_{ion} [$\approx (B_1/h\nu)^{8/3}, \mathrm{with}\, \nu(z) = \nu(z_1)\frac{z}{z_1}$], and the number density of hydrogen atoms, as computed from the Saha formula, is $n_{1s}(z)$. The distance to the radiation source at the moment of emission, $r(z_1)$, is calculated from the Friedmann equation. The integral reaches 1 at about $z_2 \approx 1900$. Consequently, in contrast to the results mentioned above, helium recombination does not leave any trace on the CMB spectrum.

As is discussed in Hu et al. (1995), the difference between the temperature of the photons and the kinetic temperature of the electrons has an effect below z $\approx$ 200. We follow the recombination process only down to z = 220, so this effect is neglected.

The possible $ns \to 1s$ and the $nd \to 1s$ two–photon transitions could also have some importance. The number of bound electrons in the $n \geq 3$ states is of the same order of magnitude that for n_{2s}. The two–photon decay probabilities for these states are smaller than for n_{2s}. Moreover some of these photons will have more energy than $B_2 - B_1 = 10.2$ eV. However, the contribution of these transitions is not entirely negligible, they can change the balance between the two routes, redshift and two–photon decay, for the elimination of energetic photons and change the spectrum of the recombination photons. This correction will be discussed in detail elsewhere.

If there were some energy input prior to the hydrogen recombination and a "y-distortion" of the spectrum (Zel'dovich & Sunyaev 1969) it would be easier to observe the consequences of H–recombination (Lyubarsky & Sunyaev 1983). However, the observations permit a very low upper limit for the y-distortion (Mather et al. 1994, Fixsen et al. 1996).

The distortion seems to occur at wavelengths where other sources give considerable contributions. By doing the calculations we had hoped that the zodiacal forground emission, the dust and molecular emission from the interstellar medium would be reliably modeled, subtracted and the calculated spectral distortion would be measurable. Recently that substraction has been done (Schlegel, Finkbeiner & Davis 1997) and it turned out that in the considered wavelength range the measured upper limit of the background is at least two order of magnitude higher as the spectrum calculated here. As it was pointed out by Guideroni et al. (1997), it could originate from early starlight scattered by dust.

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