Regularity in the distribution of superclusters?

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Abstract. We use a measure of clustering derived from the nearest neighbour distribution and the void probability function to distinguish between regular and clustered structures. This measure offers a succinct way to incorporate additional information beyond the two-point correlation function. Application to a supercluster catalogue by Einasto et al. (1997d) reveals no clustering in the distribution of superclusters. However, we show that this supercluster catalogue is severely affected by construction effects. Taking these biases into account we still find some indications for regularity on the largest scales, but the significance is only at the one-σ level.

Key words: large-scale structure of Universe – cosmology: theory

1. Introduction

In a recent paper Einasto et al. (1997c) report a peak in the 3D-power spectrum of a catalogue of clusters on scales of $120h^{-1}$Mpc. Broadhurst et al. (1990) observed periodicity on approximately the same scales in an analysis of 1D-data from a pencil-beam redshift survey. As is well known from the theory of fluids, the regular distribution (e.g. of molecules in a hard-core fluid) reveals itself in an oscillating two-point correlation function and a peak in the structure function respectively (see e.g. Hansen & McDonald 1986). In accordance with this an oscillating two-point correlation function $\xi_2(r)$ or at least a first peak was reported on approximately the same scale (Kopylov et al. 1988; Mo et al. 1992; Fetisova et al. 1993; Einasto et al. 1997a).

In this paper we analyze the supercluster catalogue of Einasto et al. (1997d) which was constructed from an earlier version of the cluster catalogue by Andernach & Tago (1998) using a friend-of-friends procedure. With methods based on the nearest neighbour distribution and the spherical contact distribution we can show that this supercluster catalogue is regular with 95% significance. However, taking into account the selection and construction effects, the high significance vanishes and we only find some indication for a regular distribution on large scales, showing that this supercluster catalogue is seriously af-

fected by the construction method with a friend-of-friends procedure.

This paper is organized as follows. In Sect. 2 we discuss our methods. Through some examples, we illustrate the properties of the J-function and show that it offers a concise way to incorporate information about correlations of arbitrary order. The analysis of the supercluster distribution and of a set of mock supercluster catalogues is presented in Sect. 3. We summarize our results in Sect. 4.

2. Methods

To analyze the set of points $\mathcal{X} = \{\mathbf{x}_i\}_{i=1}^{N}$, $\mathbf{x}_i \in \mathbf{R}^3$ given by the redshift coordinates[1] of the superclusters we use the spherical contact distribution $F(r)$, i.e. the *distribution function of the distances r between an arbitrary point and the nearest object in* $\mathcal{X}$. $F(r)$ is equal to the expected fraction of volume occupied by points which are not farther away than r from the objects in $\mathcal{X}$. Therefore, $F(r)$ is equal to the volume density of the first Minkowski functional as introduced into cosmology by Mecke et al. (1994). As another tool we use the nearest neighbour distribution $G(r)$, that is defined as the *distribution function of distances r of an object in $\mathcal{X}$ to the nearest other object in $\mathcal{X}$*. For a Poisson distribution the probability to find a point only depends on the mean number density $\overline{\rho}$, leading to the well-known result

$$G_{\mathrm{P}}(r) = 1 - \exp\left(-\overline{\rho}\frac{4\pi}{3}r^3\right) = F_{\mathrm{P}}(r). \tag{1}$$

Recently, van Lieshout & Baddeley (1996) suggested to use the ratio

$$J(r) = \frac{1 - G(r)}{1 - F(r)} \tag{2}$$

as a probe for clustering of a point distribution (see also Kerscher et al. 1998b). For a Poisson distribution $J(r) = 1$ follows directly from Eq. (1). As shown by van Lieshout & Baddeley (1996), a clustered point distribution implies $J(r) \leq 1$, whereas regular structures are indicated by $J(r) \geq 1$. Typical clustered structures are produced by Neyman-Scott processes (Neyman

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[1] Throughout this article we measure length in units of h^{-1}Mpc, with $H_0 = 100h$ km s^{-1} Mpc^{-1}.

& Scott 1958) which have been used to model the distribution of galaxies. Regular structures are seen in a periodic, or a crystalline arrangement of points. In a statistical sense, and opposed to clustering, regular ("ordered") structures are also seen in liquids. Qualitatively one may explain the behaviour of $J(r)$ as follows:

- In a clustered distribution of points $G(r)$ increases faster than for a random distribution of points, since the nearest neighbour is typically in the close surroundings. $F(r)$ increases more slowly than for a random distribution, since an arbitrary point is typically inbetween the clusters. These two effects give rise to $J(r) \leq 1$.
- On the other hand, in a regular distribution of points, $G(r)$ increases more slowly than for a random distribution of points, since the nearest neighbour is typically at a finite characteristic distance (e.g. in the case of a crystal). $F(r)$ increases faster, since the typical distance from a random point to a point on a regular structure is smaller. These two effects cause $J(r)$ to be greater than unity.
- $J(r) = 1$ indicates the borderline between clustered and regular structures.

2.1. Gaussian approximation

For a stationary point distribution $1 - F(r)$ is equal to the probability that no galaxy is inside a sphere $\mathcal{B}_r$ with radius r:

$$F(r) = 1 - \rho_0(\mathcal{B}_r), \tag{3}$$

with $\rho_0(\mathcal{B}_r)$ being the void probability function (White 1979). Similarly, $1 - G(r)$ is equal to the probability that there is an object at $\mathbf{x} \in \mathcal{X}$ *and* that there is no other object inside the sphere $\mathcal{B}_r(\mathbf{x})$ centered on $\mathbf{x}$:

$$G(r) = 1 - \frac{\rho_1(\mathbf{x} \mid \mathcal{B}_r(\mathbf{x}))}{\overline{\rho}} \rho_0(\mathcal{B}_r(\mathbf{x})). \tag{4}$$

$\rho_1(\mathbf{x} \mid \mathcal{B}_r(\mathbf{x}))$ is the density that we observe a point at $\mathbf{x}$ under the condition that $\mathcal{B}_r(\mathbf{x})$ is empty[2]. Therefore we obtain (Sharp 1981):

$$J(r) = \frac{\rho_1(\mathbf{x} \mid \mathcal{B}_r(\mathbf{x}))}{\overline{\rho}}. \tag{5}$$

Following Stratonovich (1963) we can express the conditional density ρ_1 in terms of the n-point correlation functions, the normed cumulants, ξ_n (see also White 1979):

$$\rho_1(\mathbf{x} \mid \mathcal{B}_r(\mathbf{x})) = \overline{\rho} - \sum_{n=1}^{\infty} \frac{(-\overline{\rho})^{n+1}}{n!} \times$$

$$\times \int_{\mathcal{B}_r(\mathbf{x})} \mathrm{d}^3 x_1 \dots \int_{\mathcal{B}_r(\mathbf{x})} \mathrm{d}^3 x_n \, \xi_{n+1}(\mathbf{x}, \mathbf{x}_1, \dots, \mathbf{x}_n). \tag{6}$$

A Gaussian approximation, i.e. $\xi_n = 0$ for $n > 2$, yields

$$J(r) \approx 1 - \overline{\rho} \, 2\pi \int_0^r \mathrm{d}s \, s^2 \xi_2(s). \tag{7}$$

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[2] Assuming a stationary point distribution we can choose $\mathbf{x}$ to be the origin.
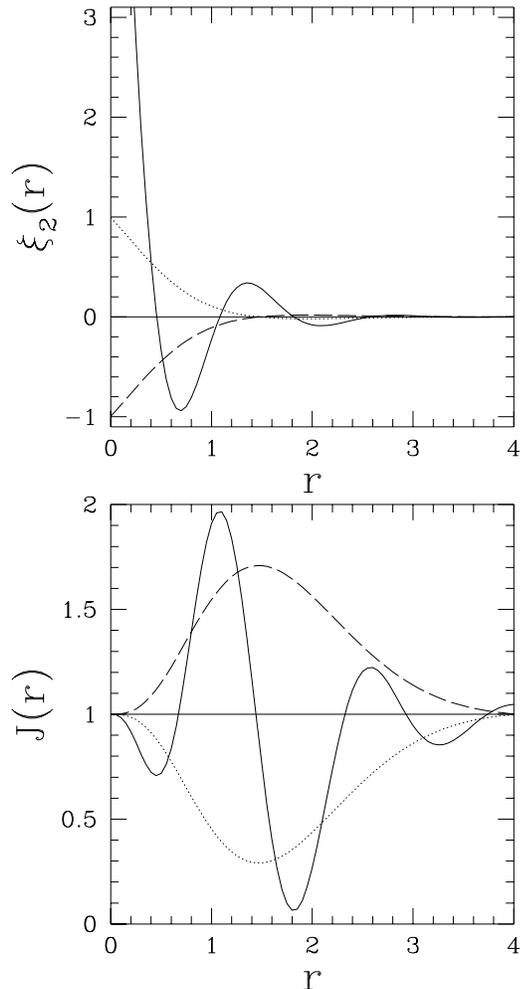

Fig. 1. Above we see the two-point correlation function $\xi_2(r)$ of an anticorrelated point distribution (dashed), a correlated point distribution (dotted), and of a process mimicking the cluster distribution (solid); r in arbirary units. Below the corresponding $J(r)$ with $\overline{\rho} = 1$ in the Gaussian approximation are depicted.

In Fig. 1 we show several two-point correlation functions $\xi_2(r)$, satisfying the normalization condition $\int_0^{\infty} \mathrm{d}s \, s^2 \xi_2(s) = 0$, and the corresponding $J(r)$ in the Gaussian approximation. From the third example (solid lines in Fig. 1) we see that a point distribution which is correlated (clustered) on small scales may show anticorrelation on large scales, such that $J(r) < 1$ for small and $J(r) > 1$ for large r. Two simple examples of a correlated distribution with $\xi_2(r) > 0$, and of an anticorrelated distribution with $\xi_2(r) < 0$ clearly show the expected $J(r) < 1$ and $J(r) > 1$ respectively.

2.2. Beyond the Gaussian approximation

According to Eqs. (5) and (6) $J(r)$ depends on correlations of arbitrary order. Therefore, a Gaussian approximation to $J(r)$ may be misleading. We illustrate this with two different point distributions: a Poisson (i.e. random) distribution of points, and points given by the model of Baddeley & Silverman (1984).

Both point distributions exhibit the same two-point characteristics, but the example of Baddeley & Silverman (1984) is regular by construction. To generate a realization of the point distributions by Baddeley & Silverman (1984) we divide the unit square into 20×20 cells and randomly place 0, 1, or 10 points into each cell, with a probability of $1/10$, $8/9$, and $1/90$ respectively. In Fig. 2a and b we display these point distributions with 388 points in a square. By visual inspection the set of points given by Baddeley & Silverman (1984) shows a regular structure. Larger voids are only seen in the Poisson distribution. We obtain $J(r) \geq 1$ for the regular point set, clearly distinguishable from the $J(r) = 1$ for the Poisson distributed points (Fig. 3). Both have the same two point correlation function $\xi_2(r) = 0$ by construction. No scale can be deduced from the two-point correlation function $\xi_2(r)$ as seen in Fig. 3. Since the number density and $\xi_2(r)$ are equal in both point sets, the difference in $J(r)$ results from high-order correlations only. $J(r)$ and its variance diverge near the intrinsic scale $r = 1/20$ of this specific regular point distribution, since $F(r)$ approaches unity.

If we estimate $F(r)$ from one realization of a point process, $F(r)$ becomes unity when r becomes larger than the radius of the biggest empty sphere which fits inside the sample. Similarly, we get $G(r) = 1$ for r larger than the largest distance between neighbouring objects. Therefore the estimate of the J-function from a single point set becomes undetermined beyond these radii. Since $J(r)$ is a global measure we are still able to detect regular structures, as global features of a point pattern, from the $J(r) \geq 1$ for radii r below the intrinsic scale, here $1/20$.

As discussed in Bedford & van den Berg (1997), $J(r) = 1$ does not necessarily imply Poisson distributed points. The morphological measure $J(r)$ was used by Kerscher et al. (1997) to investigate large scale fluctuations in the galaxy distribution and earlier by Sharp (1981) to test a hierarchical model for galaxy clustering.

Since all real astronomical catalogues are spatially limited we have to use edge-corrected estimators as detailed in Kerscher et al. (1997). The rationale behind these estimators is to use only the points whose possible nearest neighbours are contained in the sample window. With these estimators we neither make any assumptions about the exterior of our sample, nor do we use weighting schemes. However, our analysis of the supercluster sample is restricted to a radial distance of at most $60\text{-}70h^{-1}\text{Mpc}$.

3. Results

3.1. The supercluster sample

We use the supercluster sample compiled by Einasto et al. (1997d) with 220 superclusters out to $z = 0.12$. The sample was generated with a friend-of-friends algorithm using a linking length of $24h^{-1}\text{Mpc}$ from an earlier version of the cluster sample by Andernach & Tago (1998) and includes all superclusters of at least two member clusters. A detailed discussion of the sample is given in Einasto et al. (1997d). We limit our analysis to a region within galactic latitude $|b| > 20°$ and a maximum radial distance of $330\ h^{-1}\text{Mpc}$. As directly suggested by the sample geometry we perform our analysis separately for the
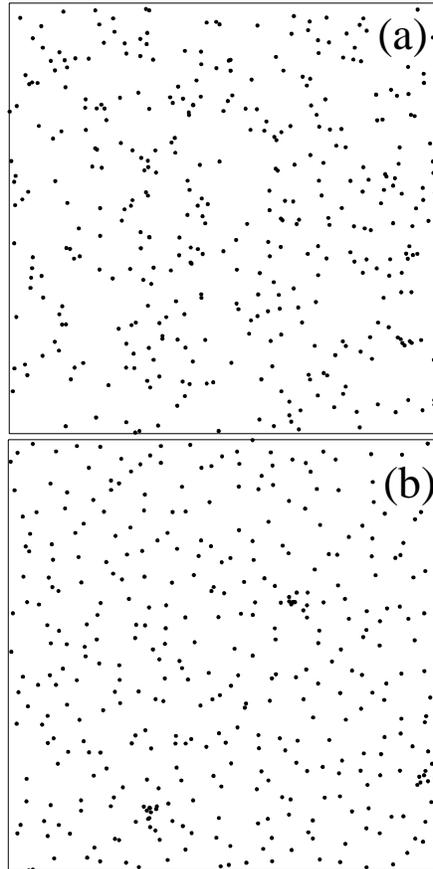

Fig. 2a and b. In plot **a** randomly distributed points and in plot **b** points from realization of the regular example by Baddeley & Silverman (1984) are displayed.

northern and southern parts (in galactic coordinates). 95 superclusters enter into the northern part (mainly Abell sample) and 116 superclusters into the southern part (mainly ACO sample). The selection effects are modeled with an independent radial and angular selection function (see Einasto et al. 1997a).

3.2. Regular structures?

In Fig. 4 the values of $J(r)$ for the supercluster distribution are plotted together with the average and 1σ-error of 99 realizations of a (pure) Poisson distribution with the same sample size and geometry. A $J(r)$ larger than unity, as expected for a regular distribution of the points (see Sect. 2), is clearly seen. $J(r)$ for both parts is above one, lying outside the 1σ-range of the Poisson distribution on scales from $15h^{-1}\text{Mpc}$ to $50h^{-1}\text{Mpc}$. The kink in $J(r)$ at $r = 45h^{-1}\text{Mpc}$ in the southern part and at $55h^{-1}\text{Mpc}$ in the northern part indicates the typical scale on which the nearest supercluster is situated. This agrees with the median distance to the nearest poor supercluster of $45h^{-1}\text{Mpc}$ as estimated by Einasto et al. (1997d). As discussed in Sect. 2 $J(r)$ becomes unreliable on scales beyond $60h^{-1}\text{Mpc}$ (see also Fig. 4). With a (nonparametric) Monte-Carlo test, as described by Besag & Diggle (1977), we conclude with 95% confidence
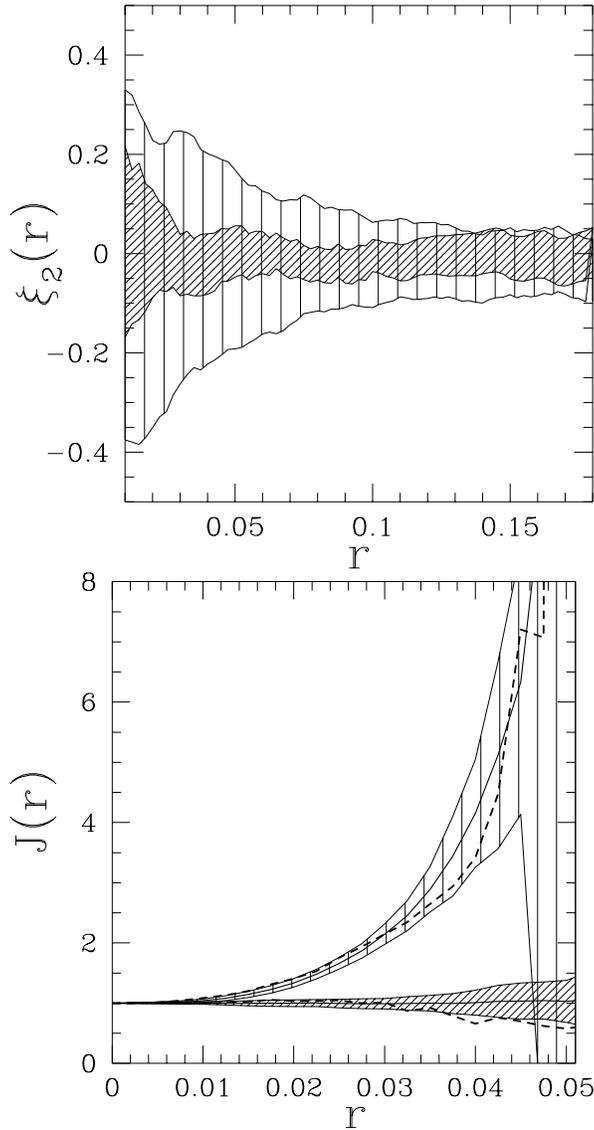
Fig. 3. The plots show the two-point correlation function $\xi_2(r)$ on top, and $J(r)$ at the bottom for the random points (dark shaded) and for the regular example by Baddeley & Silverman (1984) (light shaded). The areas correspond to the 1σ-error estimated from fifty realizations. The dashed lines display the $J(r)$ for the point distributions shown in Fig. 2a and b; r is in units of the side length of the square.

that the superclusters given by Einasto et al. (1997d) are not compatible with Poisson distributed points with the same number density. We will see that this is *not* decisive, since up to now we did not include selection and construction effects.

To test the influence of the selection effects and the construction process we generate 99 "mock supercluster catalogues". We start with Poisson distributed points within a sphere of $370h^{-1}$Mpc incorporating the radial and angular selection effects of the galaxy cluster catalogue given by Einasto et al. (1997a); then we apply a friend-of-friends procedure with linking length of $24h^{-1}$Mpc to identify the mock "superclusters". As seen in Fig. 5, using a friend-of-friends algorithm, we generate an empty sphere with radius of at least $24h^{-1}$Mpc around
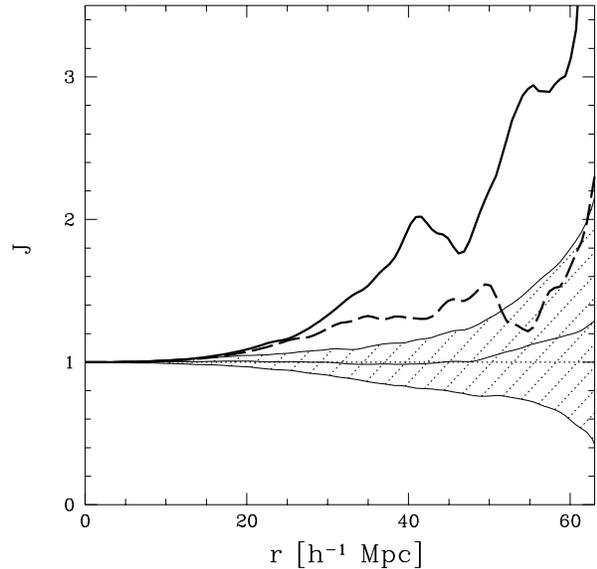

Fig. 4. $J(r)$ for the northern part (dashed), the southern part (solid), and a Poisson sample (shaded area) are displayed. The curves are smoothed with a triangular kernel with a total width of $3h^{-1}$Mpc.

each supercluster center, introducing an artificial anticorrelation, leading to $J(r) > 1$. Therefore, the regularity seen in the northern part of the supercluster sample up to scales of $60h^{-1}$Mpc is at least partly an artifact of the construction. Still the southern part shows a $J(r)$ above the mean $J(r)$ of the mock superclusters, mostly outside the 1σ-range, but a definite statement with a significance of 95% (roughly 2σ) is no longer possible.

4. Discussion and conclusion

We have shown that the statistical properties of the supercluster distribution as given by Einasto et al. (1997d) are seriously affected by the construction with a friend-of-friends procedure. This is not astonishing since the linking length of $24h^{-1}$Mpc is already one fifth of the claimed regularity scale of $120h^{-1}$Mpc. The distinction between regular and clustered point patterns with the J-function is unambiguous for a homogeneous and isotropic point distribution. In such a case the borderline is given by $J(r) = 1$. Our procedure for generating mock supercluster samples, where we start with a Poisson sample, include the selection effects, and redo the supercluster identification with a friend-of-friends algorithm in the same way as for the real cluster sample, results in a $J(r) \neq 1$ even though we started from a Poisson distribution. Although it is plausible, that such mock-supercluster samples describe the borderline between clustered and regular structures, there is no proof of this assertion. The apparent regularity in the northern part of the sample can be explained as a result of these construction effects, whereas the southern part still shows a trend towards regular structures outside the 1σ-range.

The results for the oscillating two-point correlation function of galaxy clusters and, correspondingly the peak in the
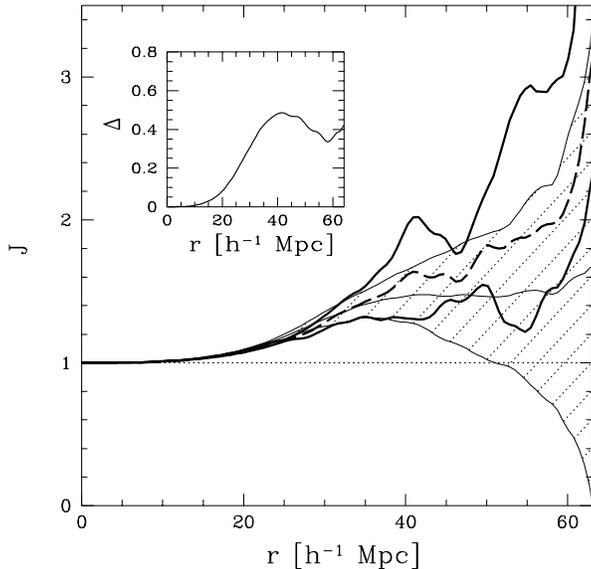
Fig. 5. The lower solid line is the result for the northern part, the upper solid line the result for the southern part and the dashed line the result for the whole sample. The shaded area marks the 1σ-range estimated from 99 mock supercluster realizations. The inset plot shows the difference Δ between the mean values of $J(r)$ for the pure Poisson process and the mock superclusters samples. All curves are smoothed with a triangular kernel with a total width of $3h^{-1}$Mpc.

power spectrum were obtained with estimators using weighting schemes and boundary corrections, which rely heavily on the assumption of homogeneity. Up to now there is no reliable way to prove this from the three-dimensional distribution of galaxies and clusters. There are some hints that the universe reaches homogeneity on scales above several hundreds of h^{-1}Mpc (see the discussions by Guzzo 1997 versus Sylos Labini et al. 1998). We adopted a conservative point of view and used estimators which do not make any assumptions about the distribution of superclusters outside the sample window. In Sect. 2 we showed that the J-function can be estimated from one point set only for scales smaller than the radius of the largest void. Therefore, we do not reach the claimed regularity scale at $120h^{-1}$Mpc. Still, the measure $J(r)$ gives us information about global properties of the supercluster distribution, in our case a tendency towards regular structures.

We analyzed the distribution of clusters of the more recent redshift compilation by Andernach & Tago (1998) with the J-function. We found the expected clumping of galaxy clusters, as indicated by $J(r) \leq 1$. Qualitatively, the $J(r) \leq 1$ may be explained with a $\xi_2(r) > 0$ and the Gaussian approximation in Eq. (7). This clumping out to scales of $40h^{-1}$Mpc is confined mainly to the interior of the superclusters. Isolated "field" clusters were not included in the supercluster sample but may contribute to the correlation seen up to scales of $50h^{-1}$Mpc in the cluster samples. One hierarchical level higher, the supercluster centers themselves show a tendency towards regular structures. Again this can be explained qualitatively with the Gaussian approximation (see Fig. 1). A theoretical example il-

lustrating such a hierarchical property is given by Neyman-Scott processes (Neyman & Scott 1958): In such a process the overall distribution of points shows correlation (i.e. $\xi(r) > 0$ for small r), but the cluster centers of these points are distributed randomly by construction.

Unlike the two-point correlation function the J-function incorporates information stemming from high order correlations. Our example in Fig. 3 illustrates, that a regular structure detected unambigously with the J-function may not be visible in an analysis with the two-point correlation function $\xi_2(r)$ alone.

Another problem is the fluctuations between the northern and southern parts of the sample. This may be attributed to the different selection effects entering the Abell and ACO parts of the sample, probably due to the different sensitivity of the photo plates used. However, in the case of the IRAS 1.2 Jy galaxy catalogue such fluctuations were shown to be real on scales of $200h^{-1}$Mpc (Kerscher et al. 1998a). Also, Zucca et al. (1997) find from the ESP survey, that at least in the southern hemisphere the local density is below the mean sample density out to $140h^{-1}$Mpc. If we assume that the fluctuations decrease on scales above $200h^{-1}$Mpc, the finding of regular structures on such large scales is a great challenge to the standard scenarios of structure formation by gravitational instability, starting from Gaussian initial density fluctuations. Implications of these regular structures for the standard scenarii of structure formation are discussed in Einasto et al. (1997b) and Szalay (1997).

References

Andernach, H., Tago, E.: 1998, In: *Proc. Large Scale Structure: Tracks and Traces, Potsdam, Germany* (Singapore), Müller V., Gottlöber S., Mücket, J.P., Wambsgans, J. (eds.), World Scientific, in press, astro-ph/9710265
Baddeley, A.J., Silverman, B.W., 1984, Biometrics 40, 1089
Bedford, T., van den Berg J., 1997, Adv. Appl. Prob. 29, 19
Besag, J., Diggle, P.J., 1977, Appl. Statist. 26, 327
Broadhurst, T.J., Ellis, R.S., Koo, D.C., Szalay, A.S., 1990, Nature 343, 726
Einasto, J., Einasto, M., Frisch, P. et al., 1997a, MNRAS 289, 801
Einasto, J., Einasto, M., Frisch, P. et al., 1997b, MNRAS 289, 813
Einasto, J., Einasto, M., Gottlöber, S. et al., 1997c, Nature 385, 139
Einasto, M., Tago, E., Jaaniste, J. et al., 1997d, A&AS 123, 119
Fetisova, T.S., Yu.Kuznetsov, D., Lipovetskii, V.A. et al., 1993, Astron. Lett. 19(3), 198
Guzzo, L., 1997, New Astronomy 2(6), 517
Hansen, J.P., McDonnald, I.R., 1986, *Theory of simple liquids*, Academic Press, New York and London
Kerscher, M., Schmalzing, J., Buchert, T., Wagner, H. 1998a, A&A 331, 1

Kerscher, M., Pons-Bordería, M.J., Schmalzing, J., et al., 1998b, in preparation

Kopylov, A.I., Yu.Kuznetsov D., Fetisova T.S., Shvartsman V.F.: 1988, In: *Large Scale Structure of the Universe*, Audouze, J.A. et al. (ed.), IAU, pp. 129

Mecke, K.R., Buchert, T., Wagner, H., 1994, A&A 288, 697

Mo, H.J., Deng, Z.G., Xia, X.Y. et al., 1992, A&A 257, 1

Neyman, J., Scott, E.L., 1958, J. R. Stat. Soc. 20, 1

Sharp, N., 1981, MNRAS 195, 857

Stratonovich, R.L., 1963, *Topics in the theory of random noise* Vol. 1, Gordon and Breach, New York

Sylos Labini, F., Montuori, M., Pietronero, L., 1998, Physics Rep. 293, 61

Szalay A.S., 1997, In: *Proc. of the 18th Texas Symposium on Relativistic Astrophysics* (New York), Olinto, A. (ed.), AIP

van Lieshout, M.N.M., Baddeley A.J., 1996, Statist. Neerlandica 50, 344

White, S.D.M., 1979, MNRAS 186, 145

Zucca, E., Zamorani, G., Vettolani, G. et al., 1997, A&A 326, 477