

# Do soft gamma repeaters emit gravitational waves?

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**Abstract.** Soft gamma repeaters are identified as highly magnetized ( $B \approx 10^{14}$  Gauss) neutron stars. Magnetic stresses induce tectonic activity, and field annihilation in faults is the ultimate energy source for the observed  $\gamma$ -ray emission. As a consequence of the crustal cracking, the stored elastic energy is converted into high frequency (kHz) shear waves, that excite nonradial oscillation modes damped by gravitational wave emission. This class of objects should certainly be considered as potential sources of gravitational waves that could be detected by the present planned interferometric antennas like VIRGO or LIGO.

**Key words:** gravitational waves – stars: neutron

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## 1. Introduction

Neutron stars are certainly one of the most popular potential sources of gravitational waves (GWs) (see, for instance, Bonazzola & Marck 1994, for a recent review on astrophysical sources of GWs). Rotating neutron stars may have a time-varying quadrupole moment and hence radiate GWs, by either having a tri-axial shape or a misalignment between the symmetry and total angular momentum axes, which produces a wobble in the star motion. In the former case the GW frequency is twice the rotation frequency, whereas in the latter two modes are possible (Zimmerman 1980): one in which the GWs have the same frequency as rotation, and another in which the GWs have twice the rotation frequency (the first mode dominates by far at small wobble angles while the importance of the second increases for larger values). The merging together of two neutron stars is also a very attractive possibility to produce GWs due to the huge energy power implied in the process, and the well known dynamical behaviour of the system (Peters & Mathews 1963). The expected detection frequency of coalescence events by VIRGO has recently been reviewed by de Freitas Pacheco (1997).

Another possibility for neutron stars radiate GWs is if they oscillate in nonradial modes, a mechanism discussed already in the late sixties (Thorne & Campolattaro 1967). Neutron stars pulsating nonradially have been neglected as potential sources

of GWs in more recent studies. The reasons are the absence of a convincing mechanism to excite quadrupole modes and the weak signal expected in the process.

In a recent paper, Cheng et al. (1995) call the attention to the striking similarity, already noticed by past investigators, between the statistical behaviour of soft  $\gamma$ -ray repeaters (SGRs) and earthquakes. More than a hundred events from the source SGR 1806-20 were already detected by the satellite ICE, and from the analysis of such data, they have noticed the following statistical properties that are also observed in earthquakes: 1) power-law energy distribution (Gutenberg & Richter 1956); 2) log-symmetric waiting time distribution (see also Hurley et al. 1994a); 3) a robust correlation between waiting times of successive events; 4) a weak or no correlation between amplitudes and waiting times. The latter property distinguishes the behaviour of SGRs from other X-ray bursters, that have burst energies nearly proportional to the preceding waiting times (van Paradijs et al. 1988). If the eruption of X-bursters is related to accretion, then the reservoir must be replenished until the critical value is again attained, a condition necessary for a new outburst. On the other hand, if the reservoir is the stored strain energy in the crust of a young neutron star, this energy may not completely be depleted by quakes, which would explain the absence of correlation between waiting times and the energy of the event. In the case of the Earth, the cracking of the crust occurs when a critical stress limit is reached. The quakes generated by the cracking produce a relaxation of the stresses to values below the threshold, and a series of events with different intensities may occur, whose energy distribution follows the Gutenberg-Richter power law.

Neutron stars are believed to have solid crusts (but see Haensel 1995) and hence to develop a tectonic activity similar to the Earth. The dissipated elastic energy is sufficient to produce glitches in the rotation period of young pulsars and to induce changes in the surface magnetic field, caused by the appearance of moving tectonic platelets (Ruderman 1991a, b; Burdzyuzha et al. 1996, 1997).

If SGRs are indeed neutron stars with an important tectonic activity, we may further explore the analogy with earthquakes, considering that most of the strain energy goes into a mechanical channel, exciting normal oscillating modes. In this work the consequences of such a possibility are examined. In the proposed scenario, the main features of the soft  $\gamma$ -ray emission

may be explained, as a consequence of magnetic field annihilation in the faults produced during the cracking of the crust. The expected gravitational strain  $h_0$  due to nonradial oscillations excited by the release of elastic energy, is computed for different neutron star models, and compared with the planned sensitivity of interferometric antennas like VIRGO.

## 2. $\gamma$ and GW emission from SGR-like sources

### 2.1. The $\gamma$ -ray emission

Neutron stars are able to develop tectonic activity (see, for instance, Ruderman 1991c and references therein). Such an activity will probably depend on age, rotation period and torques acting on the star, among other physical variables. Rotation discontinuities (glitches) are often observed in pulsars with mean ages around 0.7-1.0 Myr (Shemar & Lyne 1996) and, within 3 kpc from the sun, only about 2% of the pulsar population displays a glitch activity. On the other hand, the number of SGRs inside the Galaxy is unknown, and presently only four sources are included in such a class of objects: SGR 0526-66, SGR 1806-20, SGR 1900+14 and SGR 1814-13. The first was suggested to be associated with the supernova remnant N49 in the Large Magellanic Cloud, whereas the following two sources are located in the galactic plane. SGR 1806-20 may be associated to the supernova remnant G10.0-0.3, with an angular distance of about  $7^\circ$  from the galactic center. Distance estimates are quite uncertain, ranging from 5 to 15 kpc (Wallyn et al. 1995). The last one was only recently proposed as a new member of this class (IAUC 6743). The small number of sources detected presently and the possible association with supernova remnants suggest that the soft  $\gamma$ -ray emission phase has a short lifetime ( $10^2$  -  $10^3$  years) and that the major tectonic activity is strongly suppressed on such a timescale. The subsequent activity will produce quakes of lesser intensity, which could be related to the glitch phenomena.

SGRs behave quite differently from other high energy transient sources. Their typical photon energy is about 10-30 keV and the burst characteristics are quite similar from one event to another. In a first approximation, spectra can be represented by a black-body. In this case, if the temperature is constrained within the above interval, then the different observed values of the fluence require variations of the emitting surface (Fenimore et al. 1994).

The scenario to be explored has essentially been developed in the past years (Alpar et al. 1984, Ruderman 1991a). We assume that neutron stars have a superfluid core and a solid crust with a typical thickness  $L$  of about 1 km. The neutron superfluid fills the space between the crustal lattice nuclei, forming a quasi-parallel array of quantized vortex lines, which are expected to be pinned to those nuclei constituting the crustal lattice (Alpar et al. 1984).

The solid crust as soon as formed, is under stresses and the maximum shear stress that the crust can bear before cracking is (Ruderman 1991b)

$$S_{max} \approx \frac{L}{R} \mu \theta_{max} \approx 2 \times 10^{26} \left( \frac{\theta_{max}}{10^{-2}} \right) \text{ erg.cm}^{-3} \quad (1)$$

where  $R$  is the neutron star radius ( $\approx 10$  km),  $\mu$  is the lattice shear modulus and  $\theta$  is the dimensionless strain. The numerical value corresponds to a typical crust density of about  $5 \times 10^{13}$  g.cm $^{-3}$ . On the other hand, it is possible that the stellar core protons form a type II superconductor and, in this case, the core magnetic field would be organized into an array of quantized flux tubes ending at the base of the crust. The average field of these magnetic flux tubes is about  $B_q \approx 10^{15}$  Gauss. The flux tubes do not interpenetrate the superfluid vortices and, as the core vortex lines move out away from the spin axis, the magnetic flux tubes are forced to move with them (Srinivasan et al. 1990). The motion of these magnetic flux tubes generates stresses on the base of the crust of the order of

$$S_{mag} \approx \frac{BB_q}{8\pi} \quad (2)$$

and cracking may occur if  $S_{mag} \geq S_{max}$ , corresponding to a critical crustal field  $\mathbf{B}$  greater than  $5 \times 10^{12} \left( \frac{\theta_{max}}{10^{-2}} \right)$  Gauss. The critical field depends on the adopted value for the strain  $\theta_{max}$ , a parameter rather uncertain in the case of the crystal lattice that constitutes the crust of neutron stars. The melting temperature of the lattice is about  $3 \times 10^9$  K, and the actual temperature is probably much less. In this case, assuming that laboratory experiments on Li crystals can be extrapolated to the high pressures present in neutron star crusts,  $\theta_{max} \approx 10^{-2}$ , although smaller values have been suggested in the literature (Smoluchowski & Welch 1970; Ruderman 1991b). Here, it is assumed that the crust structure is characterized by a  $\theta_{max} = 10^{-2}$ , implying that rather high magnetic fields are required to produce significant tectonic effects. In the present scenario, SGRs would be associated to highly magnetized neutron stars. Similar scenarios have already been invoked to explain the  $\gamma$ -ray burst phenomenon (Vietri 1996).

As the magnetic stresses develop, the crust can be deformed or cracked. In case of cracking, magnetic field annihilation in the fault reduces the local stresses, heats the medium and produces a sudden increase in the local photon emission. Equating the Stefan energy flux to the rate of magnetic energy per unit area dissipated in the fault, a rough estimate of the resulting temperature can be obtained

$$T = 17.6 \left( \frac{L}{1 \text{ km}} \right)^{1/4} \left( \frac{1 \text{ s}}{t} \right)^{1/4} \left( \frac{B}{5 \times 10^{12} \text{ G}} \right)^{1/2} f_M^{1/4} \text{ keV} \quad (3)$$

where  $f_M$  is the fraction of the magnetic energy dissipated in the event.

The total emitted photon energy corresponds essentially to the field annihilation in the fault, namely

$$E_\gamma = 1.2 \times 10^{42} f_S \left( \frac{B}{5 \times 10^{12} \text{ G}} \right)^2 f_M \text{ erg} \quad (4)$$

where  $f_S$  is the fractional area of the star surface corresponding to faults.

In order to exemplify, let us assume a neutron star with an initial field  $\mathbf{B} = 10^{14}$  Gauss. If 10% of the initial magnetic energy is converted into heat, then from eq. (3) the resulting temperature is about 44 keV, consistent with the observed values. The total emitted energy is  $E_\gamma \approx 5 \times 10^{39} \left( \frac{f_S}{10^{-4}} \right)$  erg. A

comparison with data requires the knowledge of the source distance and the relative area of the faults. One should emphasize that the  $\gamma$ -ray emission in our model is **not isotropic**, alleviating energy requirements. We shall return to this problem later.

## 2.2. The GW emission

In the preceding section, we have shown that under certain circumstances, the crust of a neutron star cracks due to magnetic stresses. A fraction of the initial magnetic energy is annihilated and released as high energy photons. The stored elastic energy is also released in the event, being converted into shear vibrations with frequencies in the kHz regime (Blaes et al. 1989). These waves are able to excite nonradial modes, which will be damped by GW emission.

Considering the one-parameter approach of Baym & Pines (1971), the energy of a rotating neutron star with a solid crust is

$$E = E_0 + \frac{1}{2}I\Omega^2(1 - \varepsilon) + W\varepsilon^2 + B(\varepsilon_0 - \varepsilon)^2 \quad (5)$$

where  $\varepsilon$  and  $\varepsilon_0$  are respectively the actual stellar oblateness and the reference oblateness, corresponding to the strain-free situation;  $W$  measures the gravitational potential of the star and  $B$  measures the stored elastic energy. If  $\varepsilon \leq \theta_{max}$ , oblateness reducing crustquakes are possible. According to Pandharipande et al. (1976), rotating neutron stars may have  $B$  values around  $10^{48}$  -  $10^{49}$  erg, depending on the adopted equation of state. Thus, stars with initial rotation periods of about 8 ms may have rotation induced stresses corresponding to a stored elastic energy of about  $10^{45}$  erg.

Pulsar glitches may give another hint concerning the stored elastic energy. If glitches are associated with tectonic activity, then the observed variation of the angular velocity is related to the released elastic energy by (Baym & Pines 1971)

$$\Delta E_{el} \approx W\theta_{max} \frac{\Delta\Omega}{\Omega} \quad (6)$$

where  $W$  is the gravitational potential energy of the star.

For the Crab pulsar  $\frac{\Delta\Omega}{\Omega} \approx 10^{-8}$  and, from the above equation  $\Delta E_{el} \approx 10^{43}$  erg. The glitches observed in the Vela pulsar correspond to relative variations in angular velocity two orders of magnitude higher, implying  $\Delta E_{el} \approx 10^{45}$  erg. However, this theory predicts recurrence periods for glitches of the Vela pulsar of the order of  $10^3$  up to  $10^4$  yr, which are few orders of magnitude higher than the observed value ( $\approx 2.8$  yr). This is because one assumes that the crust is under stresses rotationally induced, but if high magnetic fields as discussed above, instead of rotation, are the origin of stresses, then crustquakes are still a possibility to explain macro-glitches, ( see, for instance, Blanford 1995). It is worth mentioning that the pulsar monitoring program at Jodrell Bank (Shemar & Lyne 1996), have evidenced that nine in fourteen objects display glitches with amplitudes  $\frac{\Delta\Omega}{\Omega} \approx 10^{-6}$ , comparable to those observed in the Vela pulsar. Therefore, if such an activity is tectonic in origin, values of about  $10^{45}$  erg for the energy released are not unusual, if magnetic fields are indeed the main origin of stresses.

Here we assume such a possibility, and we explore the consequences of a given event associated to an energy release  $E = 10^{45}$  erg, supposing that such an energy is essentially channeled into nonradial pulsation modes.

Under these conditions, the expected signal-to-noise ratio for a gravitational detector, if both source and antenna have a quadrupole beam pattern is (Thorne 1987)

$$\left(\frac{S}{N}\right)^2 = 3 \langle F^2 \rangle \int_0^\infty \frac{\langle |\tilde{h}^2(\nu)| \rangle d\nu}{S_h(\nu)} \quad (7)$$

where  $F$  is the detector beam pattern factor,  $\tilde{h}(\nu)$  is the Fourier transform of the gravitational strain amplitude and  $S_h(\nu)$  is the noise spectral density (in  $\text{Hz}^{-1}$ ) of the detector. In the above equation, averages are performed over the relevant angles defining the detector geometry and the source orientation.

If we use the results by Thorne (1969), the gravitational strain amplitude can be written as

$$h(t) = h_0 e^{i(\omega_n t - \frac{t}{\tau_n})} \quad (8)$$

where  $h_0$  is the initial amplitude,  $\omega_n$  is the angular frequency of the  $n$ -mode and  $\tau_n$  is the corresponding damping timescale. The initial amplitude can easily be related to the total energy  $E$  through the equation

$$h_0 = 2 \left( \frac{GE}{c^3 \tau_n} \right)^{1/2} \frac{1}{r \omega_n} \quad (9)$$

where  $G$  is the gravitational constant,  $c$  is the velocity of light and  $r$  is the distance to the source.

The S/N ratio can now be derived calculating the Fourier transform of Eq. (8) and substituting into Eq. (7). One obtains, after integration

$$\frac{S}{N} = 1.37 \times h_0 \left[ \frac{\tau_n}{S_h(\nu_n)} \right]^{1/2} \quad (10)$$

The numerical factor was obtained using the explicit dependence of the beam pattern  $F$  on angles appropriate to laser beam detectors (Forward 1978; Rudenko & Sazhin 1980).

If we impose  $\left(\frac{S}{N}\right) = 3$  (the confidence level usually required), Eq. (10) allows an estimate of  $h_0$ . In fact, the choice of a specific neutron star model and the sensitivity of the detector are also required, since the damping timescale and the noise spectral density appear explicitly in Eq. (10). The model is defined by adopting an equation of state, and by fixing the stellar mass. Once these parameters are established, proper frequencies and damping timescales can be computed. Concerning the noise spectral density  $S_h(\nu)$ , we adopt the planned sensitivity for the interferometric laser beam antenna VIRGO (A. Brillat 1997, private communication). In the present calculations, the pulsating properties of neutron star models as computed by Lindblom & Detweiler (1983), are adopted. The expected gravitational amplitude was computed for three different models based on distinct equations of state, and three different equilibrium configurations for each model.

The first column of Table 1 gives the model number, following the same notation as Lindblom & Detweiler (1983). In

**Table 1.** Quadrupole Pulsational Properties of Neutron Stars

Model	$M/M_{\odot}$	R(km)	$\nu_0(kHz)$	$\tau_0(s)$	$h_0$	$r_{max}$
O	0.55	12.33	1.36	1.30	$8.9 \cdot 10^{-23}$	1.2
O	1.57	12.84	1.72	0.22	$2.7 \cdot 10^{-22}$	0.8
O	2.38	11.58	2.20	0.16	$4.2 \cdot 10^{-22}$	0.5
N	0.57	13.24	1.25	1.55	$7.5 \cdot 10^{-23}$	1.4
N	1.45	13.82	1.51	0.30	$2.0 \cdot 10^{-22}$	1.0
N	2.56	12.27	2.08	0.18	$3.7 \cdot 10^{-22}$	0.5
M	0.49	17.46	0.88	4.67	$3.3 \cdot 10^{-23}$	2.7
M	1.44	15.79	1.33	0.40	$1.6 \cdot 10^{-22}$	1.2
M	1.76	11.91	2.24	0.14	$4.6 \cdot 10^{-22}$	0.4

particular, model N is based on Walecka's equation of state, whereas the others correspond to softer equations of state. The second column gives the stellar mass, and the three other give respectively the radius, the frequency of the fundamental quadrupole mode and the corresponding damping timescale. The resulting gravitational strain amplitude  $h_0$  calculated from Eq. (10), which is essentially a characteristic of the considered detector, is given in the sixth column of Table 1. We may now turn to the source properties which affect the value of  $h_0$ , explicitly given by Eq. (9). Since the fundamental frequency and the damping timescale are determined by the model, the maximum distances (in kpc) which may be probed by the detector in each case are given in the last column of Table 1.

### 3. Discussions

In this work a scenario is proposed to explain the mechanism responsible for the observed  $\gamma$ -ray emission from SGRs. The magnetic field topology in the crust, if the neutron star has a superconducting core, may lead to field annihilation between platelets or in the faults generated by the cracking of the crust. Neutron stars with initial magnetic fields of about  $10^{14}$  Gauss are required to heat the medium to temperatures up 30-40 keV. However, the present model predicts a total  $\gamma$ -ray emission not higher than  $10^{39}$  erg, which is by far smaller than that derived for the "1979 March 5" event, produced by SGR 0526-66. If this source is associated with the supernova remnant N49 in the LMC, then the energy released in soft  $\gamma$ -rays would be about  $10^{44}$  erg. Such a huge amount of energy is comparable, in our picture, to the total amount of magnetic energy initially stored in the crust. This is certainly a difficulty for the model (and for any other model with a different energy reservoir), which can only be relaxed if SGR 0526-66 is considerably nearby. It should be emphasized that, by different arguments, Fenimore et al. (1994) have also raised some doubts about the localisation of SGR 0526-66 in the LMC.

The amount of elastic energy stored before the cracking of the crustal lattice, is comparable to the magnetic field energy stored in the crust. Therefore such a reservoir of energy would have the same difficulties to explain the "March 5" event, if the source would indeed be localized in the LMC. Moreover, most of the elastic energy is converted into high frequency shear waves,

following the work by Blaes et al. (1989), and these waves will likely excite the normal oscillating modes of the star.

Under these conditions, the first point to be noticed is that quadrupole modes will be damped by GW emission. The frequencies of these modes are in the kHz range, and they are included in the wideband sensitivity curve of laser beam interferometers like the french-italian antenna VIRGO, or the american detector LIGO. The eventual detection of GWs from these sources would represent an important tool for the diagnosis of the neutron star interiors, since frequencies and damping timescales depend on the equation of state, as well as on the stellar mass. Inspection of Table 1 shows that the oscillation frequency increases and the damping time decreases with the mass of the star, irrespective of the equation of state. Low mass stars produce larger gravitational amplitudes and can be seen more deeply in the Galaxy. The reason is that the duration of the signal for those objects is longer than for high mass stars.

The present results indicate that distances up to 2.7 kpc may be probed by the present planned sensitivity of VIRGO. Nevertheless, if most of neutron stars have masses around 1.4  $M_{\odot}$  then the distances to be probed are of the order of 1 kpc. Moreover, the detection probability and the frequency of the events depend on the badly known distribution of those sources in the Galaxy. It is possible that there may be a large number of SGRs to be discovered yet (Hurley et al. 1994a,b), since the present experiments are probably missing sources with either too long or too short mean time intervals between bursts.

SGRs should be considered as potential sources of GWs, and one of the goals of this paper is to call the attention to such a class of objects. It is worth mentioning that the detection of GWs in coincidence with  $\gamma$ -photons may represent an unambiguous possibility of identifying a given class of sources, a desiderata for all GW experiments.

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