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LETTER

Letter to the Editor

Two dimensional cooling simulations of rotating neutron stars

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Abstract. The effect of rotation on the cooling of neutron stars is investigated. The thermal evolution equations are solved in two dimensions with full account of general relativistic effects. It is found that rotation is particularly important in the early epoch when the neutron star's interior is not yet isothermal. The polar surface temperature is up to 63% higher than the equatorial temperature. This temperature difference might be observable if the thermal radiation of a young, rapidly rotating neutron star is detected. In the intermediate epoch ($10^2 \lesssim t \lesssim 10^5$ yr), when the interior becomes isothermal, the polar temperature is still up to 31% higher than the equatorial temperature. Afterwards photon surface radiation dominates the cooling, and the surface becomes isothermal on a timescale of $\sim 10^7$ yr. Furthermore, the transition between the early and the intermediate epochs is delayed by several hundred years. An additional effect of rotation is the reduction of the neutrino luminosity due to the reduction of the central density with respect to the non-rotating case.

Key words: stars: neutron – stars: evolution – stars: rotation – relativity – X-rays: stars

1. Introduction

In the past three decades the cooling history of neutron stars was investigated by several authors (e.g. Tsuruta 1966, Richardson et al. 1982, Van Riper 1991, Schaab et al. 1996, Page 1997). Recent numerical simulations account for non-isothermal interior, as well as for general relativistic effects. Nevertheless, as far as we know, all investigations assumed spherical symmetry of geometry and temperature distribution.

As it was pointed out by Miralles et al. (1993), the effect of rapid rotation on the cooling of neutron stars can be as important as general relativistic effects, whereas the effect of slow rotation should be negligible. Although the assumption of slow rotation holds for most of the known pulsars, there exist a couple of millisecond pulsars (Lyne 1996), for which rotation should yield a rather different cooling behaviour. These millisecond pulsars are generally located in binary systems. It may however be expected that young, isolated millisecond pulsars can be detected in the near future, too. A candidate might be the supernova remnant of SN1987A. Although the observed neutrino burst lasting for ten seconds indicates that a neutron star was formed in the supernova, there is no evidence for the continued existence of it (s. Chevalier 1997 for a recent review). However, the neutron star can still be hidden by the surrounding matter, and the continued observations might reveal a rapidly rotating neutron star.

The aim of this letter is to study the effect of non-spherical geometry on the cooling of neutron stars. As far as we know, this is the first investigation of rotational effects beyond the isothermal core ansatz (Miralles et al. 1993) and also the first completely two dimensional simulation of neutron star cooling. The letter is organized as follows: We first derive the general relativistic equations of thermal evolution and describe the numerical method in Sect. 2. In Sect. 3, we apply the two dimensional cooling code to static and rotating neutron star models based on a relativistic equation of state including hyperonic degrees of freedom. Finally, we summarize our conclusions and discuss further improvements and applications of the current work in Sect. 4.

2. Equations and numerical method

Already a few seconds after the formation of a neutron star in a supernova, its interior settles down into catalysed, degenerate matter. The subsequent cooling involves only thermal processes and does not change the space time geometry. However, the structure of a neutron star depends on the rotational velocity, which generally decreases as the star loose angular momentum, e.g. due to emission of magnetic dipole radiation. Since the time scale for reaching hydrostatic equilibrium is much smaller than the time scale for the variation of angular velocity (s. Zeldovich & Novikov 1971, p. 239), one can treat the evolution of a neutron star in quasi stationary approximation. Though the partial transformation of rotational energy into thermal energy may considerably change the cooling behaviour of a

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neutron star (see, e.g., Van Riper 1991), and also the variation of space time geometry might have an effect on it, we study here, as a first step, the simplest case of constant angular velocity.

The stationary, axisymmetric, and asymptotic flat metric in quasi isotropic coordinates reads

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu}\mathrm{d}t^2 + \mathrm{e}^{2\phi}(\mathrm{d}\varphi - N^\varphi \mathrm{d}t)^2 + \mathrm{e}^{2\omega}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2), \tag{1}$$

where the metric coefficients $g_{\mu\nu} = g_{\mu\nu}(r, \theta)$ are functions of r and θ only. The metric coefficients are determined by the Einstein equation ($c = G = 1$) $\mathbf{G} = 8\pi\mathbf{T}$, and the energy-momentum conservation $\nabla \cdot \mathbf{T} = 0$. The obtained elliptic differential equations (Bonazzola et al. 1993) are solved via a finite difference scheme (Schaab 1998) once, before the cooling simulation starts.

In the case of uniform rotation, $\Omega = \mathrm{const.}$, considered here, the equations for thermal evolution are (Miralles et al. 1993, Schaab 1998)

$$\partial_r \tilde{h}_1 + \frac{1}{r}\partial_\theta \tilde{h}_2 = -r\mathrm{e}^{\phi+2\omega}\left(\frac{1}{\Gamma}\mathrm{e}^{2\nu}\epsilon + \Gamma C_\mathrm{V}\partial_t \tilde{T}\right) \tag{2}$$

$$\partial_r \tilde{T} = -\frac{1}{r\kappa}\mathrm{e}^{-\nu-\phi}\tilde{h}_1 \tag{3}$$

$$\frac{1}{r}\partial_\theta \tilde{T} = -\frac{1}{r\kappa}\mathrm{e}^{-\nu-\phi}\tilde{h}_2, \tag{4}$$

where

$$\Gamma = \left(1 - \mathrm{e}^{2(\phi-\nu)}\left(\Omega - N^\varphi\right)^2\right)^{-\frac{1}{2}} \tag{5}$$

$$\tilde{h}_i = \frac{1}{\Gamma} r \mathrm{e}^{2\nu+\phi+\omega} h_i \tag{6}$$

$$\tilde{T} = \frac{1}{\Gamma}\mathrm{e}^{\nu} T. \tag{7}$$

$\mathbf{h}$ denote the heat flux 3-vector in the comoving frame, C_V the heat capacity, ϵ the neutrino emissivity, and κ the heat conductivity. The partial radial and angular differentials are abbreviated by ∂_r and ∂_θ, respectively. Thermal equilibrium is described by $\tilde{T} = \mathrm{const.}$.

At the surface of the neutron star the heat flux h_1 and h_2 is determined by the normal heat flux h_N

$$h_1(r = R) = h_\mathrm{N}^1 = \left(1 + \frac{1}{R^2}\left(\frac{\mathrm{d}R}{\mathrm{d}\theta}\right)^2\right)^{-\frac{1}{2}} h_\mathrm{N} \tag{8}$$

$$h_2(r = R) = h_\mathrm{N}^2 = -\left(1 + \frac{1}{R^2}\left(\frac{\mathrm{d}R}{\mathrm{d}\theta}\right)^2\right)^{-\frac{1}{2}} \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}\theta} h_\mathrm{N}, \tag{9}$$

where $R(\theta)$ is the r-coordinate of the surface. h_N is taken from a non-magnetic photosphere model which describes the temperature gradient in the region between $e = 10^{10}\,\mathrm{g\,cm^{-3}}$ and the star's surface (e.g. Gudmundson et al. 1983). In these models $h_\mathrm{N}(\theta)$ depends on the temperature at the density $e = 10^{10}\,\mathrm{g\,cm^{-3}}$ and on the surface gravity

$$g_\mathrm{s} = \Gamma\mathrm{e}^{-\nu-\omega}\left(\left(\partial_r \frac{1}{\Gamma}\mathrm{e}^{\nu} + \Gamma\mathrm{e}^{-\nu+2\phi}(\Omega - N^\varphi)\partial_r\Omega\right)^2 + \frac{1}{r^2}\left(\partial_\theta \frac{1}{\Gamma}\mathrm{e}^{\nu} + \Gamma\mathrm{e}^{-\nu+2\phi}(\Omega - N^\varphi)\partial_\theta\Omega\right)^2\right)^{1/2}\Bigg|_\mathrm{surface}. \tag{10}$$

The parabolic differential equations obtained after inserting Eqs. (3) and (4) into Eq. (2) are solved via an implicit finite difference scheme by using a alternating direction implicit method. This yields a non-linear equation system which can be solved iteratively. The obtained linear equation systems have tridiagonal coefficient matrices which can be inverted rather fast. The correctness of the two dimensional code was checked by comparing the outcome of it with simple, analytically solvable models and with the results of the one dimensional code described by Schaab et al. (1996).

3. Results

We consider a superfluid neutron star model basing on the relativistic Hartree-Fock equation of state labelled RHF8 in Huber et al. (1997), which accounts for hyperonic degrees of freedom. The global properties of uniformly rotating models with fixed gravitational mass $M = 1.5\,M_\odot$ and angular velocity $\Omega = 0$, 0.5, and $0.99\,\Omega_\mathrm{K}$ are summarized in Table 1. Ω_K denotes the maximum possible Kepler angular velocity, above which mass shedding sets in. All models allow for both the direct nucleon Urca and for the direct hyperon Urca processes (cf. Prakash et al. 1992). All direct Urca processes are suppressed by nucleon and lambda pairing below the respective critical temperature (cf. Schaab et al. 1998). The ingredients to the cooling simulations are similar to those discussed by Schaab et al. (1996) in detail and are published on the Web (http://www.physik.uni-muenchen.de/sektion/suessmann /astro/cool/schaab.0198/input.html).

Fig. 1 shows the evolution of the surface temperature as measured by a distant observer. It is possible to distinguish three epochs of evolution. In the first epoch $t \lesssim 100$ yr, large temperature gradients occur in the interior of the neutron star. Radial temperature gradients were already found in one dimensional simulations (see, for example, Richardson et al. 1982). In rotating neutron stars, azimuthal temperature gradients, which cause transverse heat flow, $h_2 \neq 0$, exist, too. The polar temperature is by 16% or 63% higher than the equatorial temperature for the models rotating with $\Omega = 0.5\,\Omega_\mathrm{K}$ and $0.99\,\Omega_\mathrm{K}$, respectively (see also Fig. 2). After about 100 yr the cooling wave reaches the surface and the interior becomes isothermal, $\tilde{T} = \mathrm{const.}$ The temperature deviation between pole and equator is now mainly caused by the smaller surface gravity g_s at the equator, since $T_\mathrm{s} \propto g_\mathrm{s}^{1/4}$ for fixed internal temperature. Whereas the temperature deviation is still high in the case of nearly Kepler rotation ($\sim 31\%$), it almost disappears for the model with

Table 1. Models of uniformly rotating neutron stars with fixed gravitational mass $M = 1.5 M_\odot$ and angular velocity $\Omega = 0$, 0.5, and $0.99\,\Omega_{\rm K}$

Model	$\Omega = 0$	$\Omega = 0.5\,\Omega_{\rm K}$	$\Omega = 0.99\,\Omega_{\rm K}$
M [$M_\odot$]	1.5	1.5	1.5
$M_{\rm B}$ [$M_\odot$]	1.694	1.693	1.672
$R_{\rm eq}^\infty$ [km]	12.81	13.25	17.03
Ω [s^{-1}]	0	2631	5208
$e_{\rm c}$ [10^{14} g cm^{-3}]	11.67	10.84	7.208
$n_{\rm c}$ [fm^{-3}]	0.6285	0.5888	0.4065
$g_{\rm s}^{\rm eq}$ [10^{14}cm^{2}s^{-1}]	1.648	1.459	0.546
$g_{\rm s}^{\rm p}$ [10^{14}cm^{2}s^{-1}]	1.648	1.630	1.396

Entries are the gravitational mass M, the baryonic mass $M_{\rm B}$, the circumferential radius as measured by a distant observer $R_{\rm eq}^\infty$, the angular velocity Ω, the central mass density $e_{\rm c}$, the central baryon density $n_{\rm c}$, and the surface gravity $g_{\rm s}$.

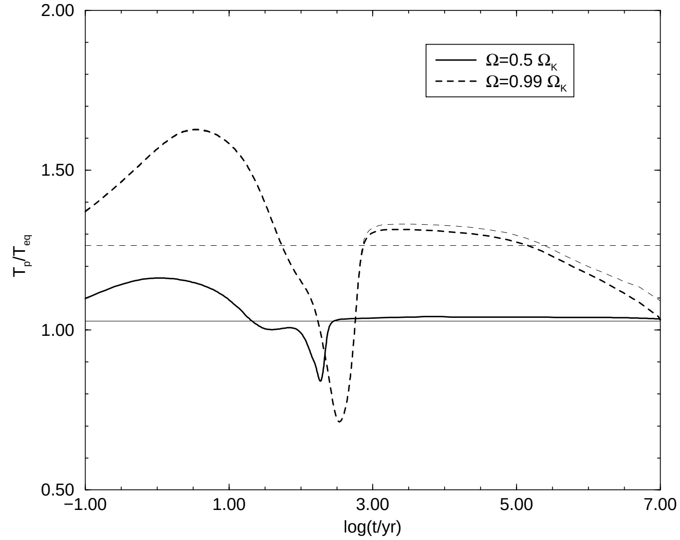

Fig. 2. Fraction $T_{\rm s}^{\rm p}/T_{\rm s}^{\rm eq}$ of polar and equatorial surface temperature as measured by a distant observer for the two rotating models. The corresponding relations $(g_{\rm s}^{\rm eq}/g_{\rm s}^{\rm p})^{1/4}$ of surface gravity are shown by the horizontal lines. The thin dashed curve is calculated under the assumption that $T_{\rm s} \propto g_{\rm s}^{1/4}$ holds even for small surface temperatures.

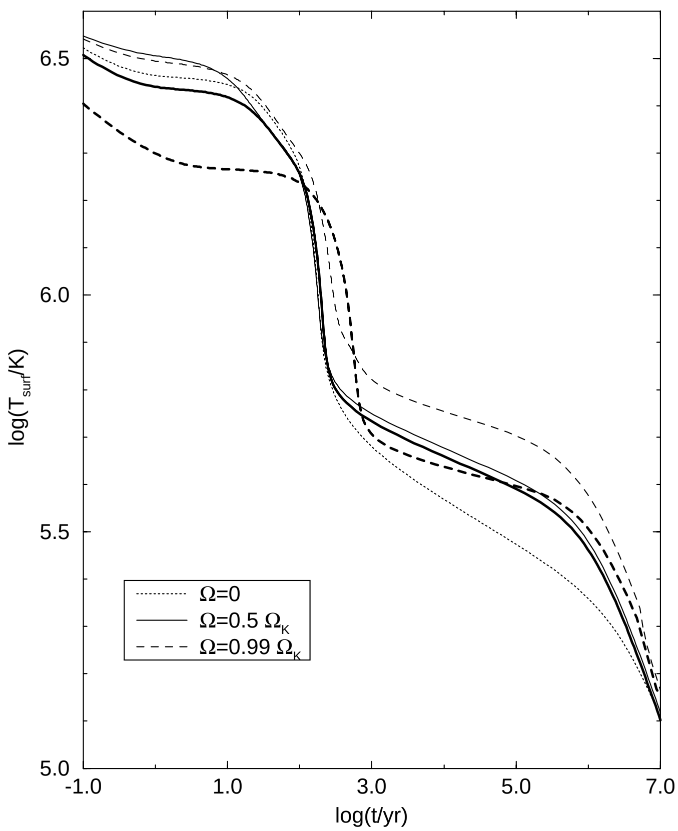

Fig. 1. Thermal evolution of rotating, superfluid neutron stars. The thick (thin) curves correspond to the equatorial (polar) surface temperature as measured by a distant observer. The considered models are given in Table 1.

$\Omega = 0.5\,\Omega_{\rm K}$ ($\sim 4\%$). After about 10^5 yr, the photon surface radiation dominates the cooling. The temperature distribution tends to a new equilibrium with ${\rm d}T_{\rm m}(\theta)/{\rm d}t = {\rm const.}$, on a timescale of $\sim 10^7$ yr. $T_{\rm m}(\theta)$ denotes the temperature at the inner boundary of the photosphere at $e = 10^{10}\,{\rm g\,cm^{-3}}$. Since the heat capacity is only weakly temperature dependent for $T \lesssim 10^9$ K in the outer crust, the surface temperature tends to an isothermal state, whereas the temperature in the outer crust varies with the surface gravity. This behaviour is accompanied by the breakdown of the scaling $T_{\rm s} \propto g_{\rm s}^{1/4}$ of the surface temperature with surface gravity for small surface temperatures. However, as one can see by comparing the thin dashed curve, which assumes $T_{\rm s} \propto g_{\rm s}^{1/4}$ over the whole temperature range, with the thick curve in Fig. 2, this is only a small effect and cannot explain the tendency of the surface temperature fraction to unity.

Besides these effects on the angular dependency of the temperature, rotation has also a net effect on cooling in the intermediate epoch. This effect is caused by the reduction of the neutrino luminosity, which sensitively depends on the central density. A further effect of rotation on cooling of neutron stars is the lengthening of the thermal diffusion time by several hundred years. Additionally the drop of the surface temperature is smoother in the case of rotation (see also Miralles et al. 1993), since the cooling wave reaches the polar region earlier than the equator. During the cooling wave is reaching the different surface regions, the fraction $T_{\rm s}^{\rm p}/T_{\rm s}^{\rm eq}$ falls below unity (s. Fig. 2).

4. Conclusions and discussion

In this letter, we have studied the effect of non-spherical geometry on the cooling of neutron stars. Our general finding is that rapid rotation plays a significant role in the thermal evolution. This role is particularly important in the early epoch, when the star's interior is not yet isothermal. It is precisely this epoch, when even isolated pulsars might rotate very rapidly. The angular dependent radial and transverse heat flow cause azimuthal temperature gradients in the interior of the star and thus also at the surface. The polar surface temperature is by up to 63% higher than the equatorial temperature. The simulation of this non-isothermal epoch cannot be performed with isothermal core approaches or one dimensional codes.

In the intermediate epoch, heat conduction is unimportant, since the interior is nearly isothermal. Nevertheless two significant, though smaller, effects remain. The first effect is the reduction of the central density with respect to the static, non-rotating case, if one fixes the gravitational mass or the rest mass. Since the neutrino emissivity depends sensitively on the density, especially in the vicinity of threshold densities for fast neutrino emission processes, the neutrino luminosity of the star is reduced as well. The second effect is the angular dependency of the surface temperature caused by the angular dependent surface gravity. The polar surface temperature is still by up to 31% higher than the equatorial temperature. Both effects could also be studied with one dimensional codes or even within the isothermal core ansatz (see Miralles et al. 1993). Whereas Miralles et al. (1993) assumed that the interior is still isothermal in the late photon cooling epoch, we find that the surface temperature distribution itself tends to an isothermal equilibrium, which implies that the temperature distribution in the outer crust becomes angular dependent.

The transition between the early and the intermediate epochs turns out to be delayed by rotation. Because of the enlargement of the crust in the equatorial plane, the cooling wave formed in the inner region of the star needs several hundred years longer to reach the surface. Furthermore, the transition becomes smoother, since the radial diffusion time it not isotropic any more (see also Miralles et al. 1993).

The surface temperature, especially in the intermediate epoch, is rather sensitive to micro physical parameters (superfluid energy gap, neutrino emissivity, equation of state, etc.; see Schaab et al. 1996 for a recent review). The comparison of the net effect with observation might therefore be difficult. Nevertheless, the comparison of the observed soft X-ray spectra with the theoretical obtained spectra may reveal the angular dependency of the surface temperature due to rotational effects. The deduction of the spectrum from the surface temperature distribution is, unfortunately, not at all trivial in non-spherical geometry (Cunningham 1975). So far, only the thermal spectra of slowly rotating neutron stars ($\Omega = 70.37\mathrm{s}^{-1}$ for the Vela pulsar 0833-45) are known. It may however be expected that the thermal spectra of a young, isolated millisecond pulsar can be detected in the near future. For example, the period $P = 1.56$ ms of the fastest pulsar B1937+21 (Backer et al. 1982) known today corresponds to a ratio $\Omega/\Omega_\mathrm{K} = 0.77$, which lies between the respective ratios of our two rotating models.

Since we neglected some important effects, like differential rotation and the loss of angular momentum, our work can only be understood as a first step towards two dimensional simulation of cooling of rotating neutron stars. Of course, the two dimensional code presented here may also serve to investigate other non-spherically symmetric effects, such as the effect of strong magnetic fields on heat conductivity and neutrino emissivity. The inclusion of differential rotation and strong magnetic fields, as well as the mentioned deduction of the photon spectra will be addressed to future work.

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