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Self oscillations of a forced inhomogeneous hydromagnetic cavity

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Abstract. We study the nonlinear development of MHD waves in a one dimensional cavity stratified by gravity and embedded in background, homogeneous, vertical magnetic field which produces an anisotropic viscosity. The plasma is compressible and its temperature is homogeneous over most of the cavity, rising to large values in a narrow 'transition zone' near its upper boundary.

Disturbances at the cavity's boundaries are assumed to be 'quasi resonant' with the fundamental mode of the discrete Alfvén spectrum. The linear behaviour of the cavity is given by a detailed computation of the Alfvén and sound eigenmodes and eigenfrequencies. The non linear behaviour of the fundamental Alfvén mode is studied by Galerkin and multiple scale analyses which reduce the full MHD equations to a two dimensional, driven, dissipative dynamical system.

Self oscillations of the Alfvén wave amplitude set in via a Hopf bifurcation as the background magnetic field is varied past a critical value ($B_{\rm cr} \simeq 1.5$ G). In solar chromospheric conditions the period of these oscillations lies in the hour range.

Key words: Sun: oscillations – MHD – waves

1. Introduction

Waves in plasma fluid systems in the solar chromosphere which are open to external disturbances are becoming a lively area of investigation which is attracting both theoretical, numerical and observational efforts in a broad set of conditions, ranging from pure hydrodynamic models (Fleck & Schmitz 1991, Schmitz & Fleck 1995), to linear (Chouduri et al. 1993a, Chouduri et al. 1993b, Hasan 1997, Hasan & Kneer 1986) and nonlinear (Hasan & Kneer 1990, Leibacher et al. 1982) hydromagnetic models, to fine scale observations of both acoustic and magnetoacoustic oscillations (Deubner & Fleck 1989, Fleck & Deubner 1989).

While there is a substantial agreement on the non homogeneous nature of the chromospheric plasma as introduced by gravity and by the underlying network structure, the nature, strength and rôle of the magnetic fields (Ploner & Solanki 1997, Remling et al. 1996, Sivaraman & Livingston 1982, Solanki 1989, Solanki et al. 1991) and of the disturbances driving the oscillations (Lee 1993, Moon & Yun 1992, Tarbell et al. 1991) still need considerable investigation.

Here we study the nonlinear behaviour of oscillations in the upper chromosphere (above the temperature minimum) focussing attention on the effects of gravitational stratification, transition region temperature rise and background magnetic field. Since it would be prohibitive to carry a general linear theory (Roberts 1988) over into the nonlinear regime, owing to the continuous spectra and to the singular eigenmodes introduced by the magnetic field, we consider a simple geometrical setting named *hydromagnetic cavity* (Nocera & Priest 1991).

The magnetic field in such a cavity is assumed to be homogeneous and vertical (an approximation applicable to the upper chromosphere, where the 'fanning out' of the magnetic lines of force decreases). Harmonic oscillations at the boundaries feed shear, linearly polarized Alfvén waves into the cavity. These propagate in the vertical direction and, to second order, drive slow magnetoacoustic waves with which they interact at third order, according to the basic equations of Sect. 2. We assume viscosity to be the main dissipation mechanism and take into account its anisotropic nature as introduced by the magnetic field. For the atmosphere model of Vernazza et al. (1973), this latter property holds true over $2/3$ of extent of the cavity, an approximation we must accept due to the lack a comprehensive theory of transport coefficients in weakly magnetized plasmas. We choose not to include the effects of heat conduction and radiative losses which are however important in the lower layers of the chromosphere.

The linear part of the work (Sects. 3-4) is calibrated so that the fundamental magnetoacoustic mode has a period ranging from 3 to 5 minutes. Considerable care is devoted to the computation of the Alfvén and magnetoacoustic eigenfrequencies and eigenmodes. The nonlinear part of the work (Sect. 5) features a regular perturbation analysis of a non trivial Galerkin base which, in quasi resonant conditions, reduce the full set of MHD equations to a two dimensional, driven, dissipative dynamical system, much in the same way as for the homogeneous plasma case (Nocera & Priest 1991).

The main result of the work is that, at variance with the homogeneous case, a Hopf bifurcation exists of the reduced dynamical system as the background magnetic field varies past

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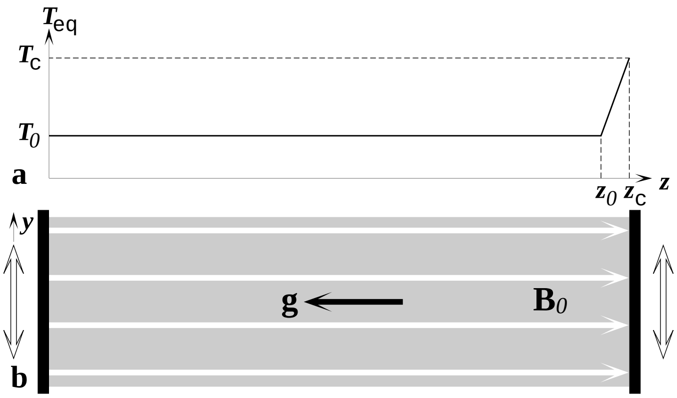

Fig. 1. A hydromagnetic cavity. **a** Equilibrium temperature profile. **b** The homogeneous gravity $\mathbf{g}$, equilibrium magnetic field $\mathbf{B}_0$ and the boundaries' shear motion (vertical arrows).

the critical value $B_{\rm cr} \simeq 1.5$ G (Sect. 5). This entrains 'self-oscillations' of the Alfvén wave's amplitude with a period of a few hours, depending on the frequency of the disturbances at the cavity's boundaries (Sect. 6).

2. Basic equations

We describe the solar chromosphere-transition region by a two deck model plasma embedded in homogeneous magnetic $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ and gravity $\mathbf{g} = -g\hat{\mathbf{z}}$ fields and forming a *hydromagnetic cavity* (Nocera and Priest 1991) bounded by two oscillating surfaces (Fig. 1). We choose viscosity as the relevant dissipative effect in the plasma. The following MHD equations govern the inhomogeneous plasma experiencing adiabatic processes (Priest 1982)

$$
\begin{aligned}
&\rho[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}] = -\nabla P + \frac{1}{4\pi}[\nabla\wedge\mathbf{B}]\wedge\mathbf{B} + \rho\mathbf{g} + \sigma,\\
&\frac{\partial \mathbf{B}}{\partial t} = \nabla\wedge[\mathbf{v}\wedge\mathbf{B}],\\
&\nabla\cdot\mathbf{B} = 0,\\
&\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0,\\
&\frac{\partial P}{\partial t} + \mathbf{v}\cdot\nabla P + \gamma P\,\nabla\cdot\mathbf{v} = 0.
\end{aligned}
\tag{2.1}
$$

Here $\mathbf{v}$ is the flow velocity, $\mathbf{B}$ is the magnetic field, P is the gas pressure, ρ is the mass density and γ the ratio of specific heats in the plasma. The momentum transfer due to ion-ion collisions in a fully ionized plasma is given by (Braginskii 1965):

$$
\begin{aligned}
&\sigma_\alpha = 3\frac{\partial}{\partial x_\beta}[\eta B_{\alpha\beta}B_{\gamma\lambda}\frac{\partial v_\gamma}{\partial x_\lambda}],\ B_{\mu\nu} = \frac{B_\mu B_\nu}{B^2} - \frac{1}{3}\delta_{\mu\nu},\\
&\eta \simeq 2\,10^{-15}\rho_i[\mathrm{g\ cm^{-3}}]T_i[\mathrm{K}]^{5/2}/(\Lambda Z^4)\ \mathrm{g\ cm^{-1}s^{-1}},
\end{aligned}
\tag{2.2}
$$

ρ_i, T_i, Z, Λ being the mass density, temperature, charge and collision Coulomb logarithm of the ions respectively.

We now assume that the physical quantities depend only on the cartesian coordinate z and reduce Eq. (2.1) to

$$
\begin{aligned}
&\rho\left[\frac{\partial v_x}{\partial t} + v_z\frac{\partial v_x}{\partial z}\right] = \frac{1}{4\pi}B_0\frac{\partial B_x}{\partial z} + \sigma_x,\\
&\rho\left[\frac{\partial v_y}{\partial t} + v_z\frac{\partial v_y}{\partial z}\right] = \frac{1}{4\pi}B_0\frac{\partial B_y}{\partial z} + \sigma_y,\\
&\rho\left[\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial z}\right] = -\frac{\partial}{\partial z}[P + \frac{1}{8\pi}(B_x^2 + B_y^2)] - \rho g + \sigma_z,\\
&\frac{\partial B_x}{\partial t} = B_0\frac{\partial v_x}{\partial z} - \frac{\partial}{\partial z}(v_z B_x),\\
&\frac{\partial B_y}{\partial t} = B_0\frac{\partial v_y}{\partial z} - \frac{\partial}{\partial z}(v_z B_y),\\
&\frac{\partial \rho}{\partial t} + v_z\frac{\partial \rho}{\partial z} = -\rho\frac{\partial v_z}{\partial z},\\
&\frac{\partial P}{\partial t} + v_z\frac{\partial P}{\partial z} = -\gamma P\frac{\partial v_z}{\partial z},
\end{aligned}
\tag{2.3}
$$

A *static* equilibrium solution of Eq. (2.3) is now chosen so that the height-dependent temperature $T = T_{\rm eq}$ equals the chromospheric value T_0 in region I ($0 < z < z_0$) and rises linearly to its coronal value $T_{\rm c}$ in the transition region II ($z_0 < z < z_{\rm c}$) as in Fig. 1:

$$
\begin{aligned}
&T_{\rm eq}(z) = \begin{cases} T_0 & \text{region I}\\ T_0 + (T_{\rm c} - T_0)\frac{z - z_0}{z_{\rm c} - z_0} & \text{region II}\end{cases},\\
&\rho_{\rm eq}(z) = \begin{cases} \mathrm{e}^{-\alpha z} & \text{region I}\\ T_{\rm eq}^{-\delta-1}(z) & \text{region II}\end{cases},\\
&P_{\rm eq}(z) = \rho_{\rm eq}(z)T_{\rm eq}(z),
\end{aligned}
\tag{2.4}
$$

where

$$
\alpha = \frac{z_{\rm c}}{H_T},\ H_T = \frac{k_{\rm B}T_0}{mg},\ z_{\rm p} = \frac{z_0}{z_{\rm c}},\ \delta = \frac{\alpha(1 - z_{\rm p})}{(T_{\rm c}/T_0 - 1)}. \tag{2.5}
$$

Let the equilibrium (2.4) be perturbed by shear, *linearly*, y-polarized Alfvén oscillations with a small magnetic Mach number $\varepsilon^{1/2} \ll 1$ which are driven by the *one-dimensional*, y-directed motion of the boundaries of the cavity (Fig. 1). Such motions provide a displacement of the plasma with speed v_y. According to Faraday's law, the resulting electric field $E_x = -v_y B_0/c$ induces a magnetic field B_y of order $\varepsilon^{1/2}$. This produces a variation of the magnetic pressure $B_y^2/(8\pi)$ which, in turn, drives motions in the z-direction with velocity v_z of order ε. According to Faraday's law again, the resulting electric field $E_x = v_z B_y/c$ induces a correction to the value of the magnetic field B_y of order $\varepsilon^{3/2}$. Last, the Lorentz force $B_0(\partial B_y/\partial z)/(4\pi)$ will accelerate the plasma in the y-direction at a rate of order $\varepsilon^{3/2}$. By inspection of Eq. (2.3) it is seen that, to leading order, this scheme produces corrections only to v_y and B_y which have the same linear, y-directed polarization as in the original Alfvén oscillation and thus leaves the remaining components of the magnetic field and velocity *unchanged*. It can be shown (Nocera et al. 1986) that this applies also when the viscous effects are considered through the stress tensor in Eq. (2.2). In the following we assume that these components identically vanish and propose the following ordering of the physical quantities in the plasma:

$$
\begin{aligned}
&\mathbf{v} = v_{\rm A_0}(0, \varepsilon^{1/2}v_{\rm T}, \varepsilon w_{\rm T}),\ \mathbf{B} = B_0(0, \varepsilon^{1/2}h_{\rm T}, 1),\\
&\rho = \rho_{\rm eq}(1 + \varepsilon d_{\rm T}),\ P = P_{\rm eq}(1 + \varepsilon p_{\rm T}).
\end{aligned}
\tag{2.6}
$$

The meaning of the subscript 'T' (for 'Total') will appear in Sect. 3. We now introduce the time $\tau_{\rm A} = z_{\rm c}/v_{\rm A_0}$, the Alfvén speed

$v_{A_0} = B_0/[4\pi\rho_0]^{1/2}$ and we normalize the coordinate z to z_c, time to τ_A, and the physical quantities to their value at $z = 0$ (labelled by a '0'), using for simplicity the same symbols for the non dimensional quantities: $z \to zz_c$, $\rho \to \rho_0\rho$, $v_A \to v_{A_0}v_A$, $T \to T_0T$, $P_{eq} = [k_B\rho_0T_0/m]\rho T_{eq} \to P_0P$, $\eta \to \eta_0\eta$, $v_s = [\gamma k_B T_0/m]^{1/2}T_{eq}^{1/2} \to v_{s_0}v_s$, k_B being Boltzmann's constant and m the mean atomic mass. Up to $O(\varepsilon^{3/2})$, Eqs. (2.1) read

$$\begin{aligned}
&\frac{\partial h_T}{\partial t} = \frac{\partial v_T}{\partial z} - \varepsilon\frac{\partial}{\partial z}(w_T h_T),\\
&\frac{\partial v_T}{\partial t} = v_A^2\frac{\partial h_T}{\partial z} - \varepsilon\{v_A^2 d_T\frac{\partial h_T}{\partial z}\\
&+w_T\frac{\partial v_T}{\partial z} - \frac{1}{Re}\frac{\partial}{\partial z}[\eta h_T(2\frac{\partial w_T}{\partial z} + 3h_T\frac{\partial v_T}{\partial z})]\},\\
&\frac{\partial d_T}{\partial t} = -\frac{\partial w_T}{\partial z} - \frac{1}{\rho}\frac{\partial \rho}{\partial z}w_T,\\
&\frac{\partial p_T}{\partial t} = -\gamma\frac{\partial w_T}{\partial z} + \frac{\alpha w_T}{v_s^2},\\
&\frac{\partial w_T}{\partial t} = -\beta_g(d_T - p_T) - \frac{\beta}{\gamma}v_s^2\frac{\partial p_T}{\partial z} + \frac{4\eta}{3Re\rho}\frac{\partial^2 w_T}{\partial z^2}\\
&-\frac{1}{2}\frac{\partial}{\partial z}[(1 - \frac{4\eta}{Re}\frac{\partial}{\partial t})h_T^2]. \qquad (2.7)
\end{aligned}$$

In Eqs. (2.7) and in the following, $\rho \equiv \rho_{eq}, \eta \equiv \eta(\rho_{eq}, T_{eq})$ and

$$\beta = v_{s_0}^2/v_{A_0}^2,\ \beta_g = gz_c/v_{A_0}^2,\ Re = \rho_0 v_{A_0} z_c/\eta_0. \qquad (2.8)$$

Combining the first two equations in Eqs. (2.7) we arrive to the nonlinear wave equation:

$$\begin{aligned}
&\frac{\partial^2 h_T}{\partial t^2} - \frac{\partial}{\partial z}[v_A^2\frac{\partial h_T}{\partial z}] = -\varepsilon\frac{\partial}{\partial z}\{\frac{\partial}{\partial t}(w_T h_T) + v_A^2 d_T\frac{\partial h_T}{\partial z}\\
&+w_T\frac{\partial v_T}{\partial z} - \frac{1}{Re}\frac{\partial}{\partial z}[\eta h_T(2\frac{\partial w_T}{\partial z} + 3h_T\frac{\partial v_T}{\partial z})]\} \equiv \varepsilon\frac{\partial \mathcal{T}}{\partial z}. \qquad (2.9)
\end{aligned}$$

Let $h_B(z,t)$ be a function obeying the linear part and the boundary conditions of Eq. (2.9)

$$\frac{\partial^2 h_B}{\partial t^2} = \frac{\partial}{\partial z}[v_A^2\frac{\partial h_B}{\partial z}],\ h_B(0) = h_T(0),\ h_B(1) = h_T(1) \qquad (2.10)$$

and let $h(z,t)$ obey the remaining part of Eq. (2.9) and vanishing boundary conditions:

$$\frac{\partial^2 h}{\partial t^2} = \frac{\partial}{\partial z}[v_A^2\frac{\partial h}{\partial z}] + \varepsilon\frac{\partial \mathcal{T}}{\partial z},\ h(0) = h(1) = 0. \qquad (2.11)$$

It is plainly seen that

$$h_T = h_B + h \qquad (2.12)$$

is a solution of Eq. (2.9). In a similar way, introducing the notation

$$w_T = w_B + w, \qquad (2.13)$$

the last three equations in Eqs. (2.7) may be reduced to

$$\frac{\partial^2 w_B}{\partial t^2} - \beta v_s^2\frac{\partial^2 w_B}{\partial z^2} + \gamma\beta_g\frac{\partial w_B}{\partial z} - \frac{4\eta}{3Re\rho}\frac{\partial^3 w_B}{\partial z^2\partial t} = 0 \qquad (2.14)$$

and

$$\begin{aligned}
&\frac{\partial^2 w}{\partial t^2} - \beta v_s^2\frac{\partial^2 w}{\partial z^2} + \gamma\beta_g\frac{\partial w}{\partial z} - \frac{4\eta}{3Re\rho}\frac{\partial^3 w}{\partial z^2\partial t} =\\
&-\frac{1}{2\rho}\frac{\partial}{\partial z\partial t}[1 - \frac{2\eta}{Re}\frac{\partial}{\partial t}]h_T^2,\ w = 0 \text{ at } z = 0, 1\,. \qquad (2.15)
\end{aligned}$$

In this way the nonlinear boundary value problem (2.7) is split into three simpler problems which we tackle separately. The solution of Eq. (2.10) and the eigenmodes of the linear part of Eq. (2.11) will be found in Sect. 3. The solution of Eqs. (2.14) and (2.15) will be found in Sect. 4. Last, the solution of Eq. (2.11) will be found in Sect. 5.

3. Alfvén eigenmodes

In Sect. 5 the solution of Eq. (2.11) will be developed using the the eigenfunctions f of its linear part ($\mathcal{T} = 0$)

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial z}[v_A^2\frac{\partial f}{\partial z}],\ f(0) = f(1) = 0, \qquad (3.1)$$

whose Fourier transform

$$\hat{f}(z,\omega) = \int_{-\infty}^{\infty} e^{i\omega t} f(z,t)dt \qquad (3.2)$$

solves the boundary value problem

$$[v_A^2\hat{f}']' + \omega^2\hat{f} = 0,\ \hat{f}(0,\omega) = \hat{f}(1,\omega) = 0, \qquad (3.3)$$

'$\prime$' denoting $\mathrm{d}/\mathrm{d}z$. Calculating the Alfvén speed from the equilibrium density (2.4), using Bessel's functions J_1, Y_1, J_ν and $J_{-\nu}$, imposing normalization, boundary conditions and the continuity of f and f' at $z = z_p$ and dropping the '$\hat{}$', gives the set of eigenfunctions f_n

$$\begin{aligned}
&f_n = A_n\begin{cases} R_n\sqrt{\rho}[J_1(\vartheta_n^0) + K_n^0 Y_1(\vartheta_n^0)] \equiv f_n^0 & \text{region I}\\ T^{-\frac{\delta}{2}}[J_\nu(\vartheta_n^c) + K_n^c J_{-\nu}(\vartheta_n^c)] \equiv f_n^c & \text{region II}\end{cases},\\
&\beta_n^0 = \frac{2\omega_n}{\alpha},\ \beta_n^c = \beta_n^0\nu\rho_p,\ \vartheta_n^0 = \beta_n^0\sqrt{\rho},\ \vartheta_n^c = \beta_n^c T^{-\frac{1-\delta}{2}},\\
&\rho_p = \rho(z_p),\ R_n = \frac{f_n^c(z_p)}{f_n^0(z_p)},\ \int_0^1 |f_n|^2(z)\mathrm{d}z = 1,\\
&K_n^0 = -J_1(\beta_n^0)/Y_1(\beta_n^0),\ K_n^c = -[J_\nu(\vartheta_n^c)/J_{-\nu}(\vartheta_n^c)]_{z=1},\\
&\nu = \frac{\delta}{1-\delta}, \qquad (3.4)
\end{aligned}$$

where the constants A_n are chosen so that the eigenfunctions f_n are nomalized to 1. Note that, if ν is an integer, the Bessel functions Y_ν must be used since $J_{-\nu}$ is no longer independent of J_ν. However, in the applications to the solar atmosphere, $T_c/T_0 \gg 1$; thus we have $\delta \ll 1$ in Eq. (2.5) and so $0 < \nu \ll 1$. Since the f_n's solve the self-adjoint boundary value problem (3.3) with homogeneous boundary conditions, they form a complete orthonormal set on the interval $z \in (0, 1)$.

If $\alpha \gg 1$, the eigenvalue ω_n in Eq. (3.4) can be approximated by the WKB value

$$\omega_n \simeq 2\pi\frac{\alpha}{4}(n + \frac{1}{4}),\ \alpha \gg 1,\ \ n = 1, 2\ldots\,. \qquad (3.5)$$

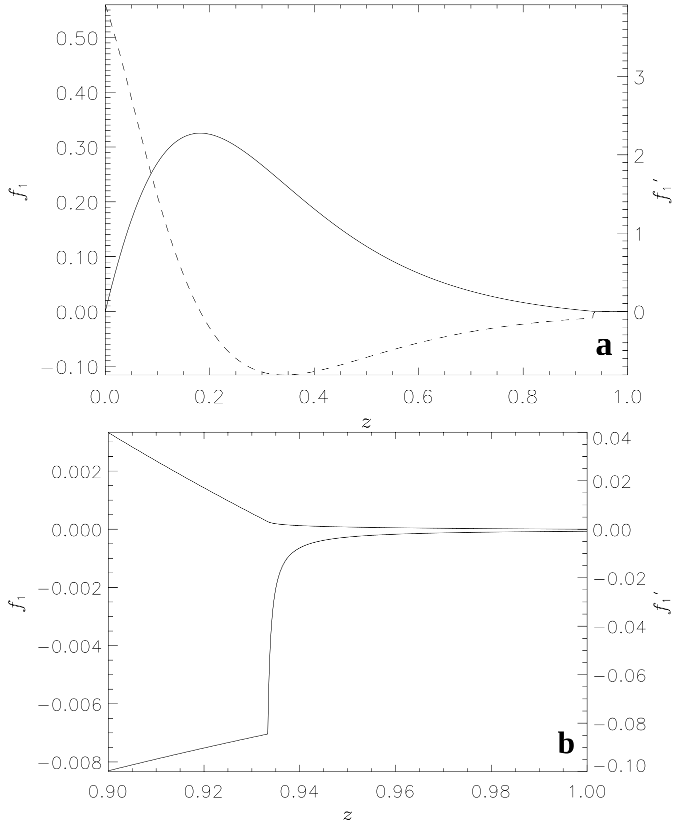

Fig. 2. **a** The Alfvén eigenmode (solid curve, left scale: f_1; dashed curve, right scale: f_1') vs. height. **b** Detail. Parameters as given in Table 2; $B_0 = 1.55$ G.

Table 1. Eigenfrequencies

B_0 (G)	$\omega_1/(2\pi\tau_{\rm A})$ WKB (10^{-3}Hz)	$\omega_1/(2\pi\tau_{\rm A})$ num. (10^{-3}Hz)	sound period (minutes)
1.0	1.8474	1.8512	4.5015
1.1	2.0321	2.0364	4.0923
1.2	2.2169	2.2215	3.7513
1.3	2.4016	2.4066	3.4627
1.4	2.5864	2.5917	3.2154
1.5	2.7711	2.7768	3.0010
1.6	2.9559	2.9620	2.8134

However, satisfactory matching of the linear eigenmodes at the base of the transition zone $z = z_{\rm p}$ requires that the values of the eigenfrequencies ω_n be known with an accuracy higher than the one provided for $\alpha \gg 1$ in Eq. (3.5). To achieve this, matching at $z = z_{\rm p}$ is cast as a root-finding problem; this is solved for ω_n by the Dekker-Brent method (Press et al. 1992) to produce the eigenfunctions n Eq. (3.4). Column 3 in Table 1 shows the frequency of the first mode compared with its WKB extimate (3.5) (column 2) for several values of the equilibrium magnetic field (column 1). The mode f_1 is shown in Fig. 2. The parameters used here and in the rest of the work (Table 2) are chosen so that the magnetic pressure fluctuations (at frequency $2\omega_1$) drive magnetoacoustic oscillations, according to Eq. (2.15), with a period of 3 to 5 minutes (Table 1, column 4). To calculate the

Table 2. Parameters used in the computations

T_0(K)	$T_{\rm c}$(K)	ρ_0 (g cm^{-3})	$m/m_{\rm p}$
10^4	10^6	2.4 10^{-12}	1.0
z_0 (cm)	$z_{\rm c}$(cm)	H_T (cm)	α
1.4 10^8	1.5 10^8	3.0 10^7	5.0

scale length H_T in Eq. (2.5) we adopted the solar surface gravity $g \simeq 2.74\,10^4\ {\rm cm\,s^{-2}}$.

We now revert to Eq. (2.10) and prescribe the following boundary conditions

$$h_{\rm B} = 2\Re[Z_1 {\rm e}^{-{\rm i}\omega_{\rm B}t}] \text{ at } z = 0, 1,\ \omega_{\rm B} = \omega_1 + \varepsilon\Delta\omega. \tag{3.6}$$

In this near-resonance condition, Eq. (2.10) approximately reduces to Eq. (3.1) and thus the solution (3.4) with $n = 1$ can be used to solve Eq. (2.10):

$$\begin{aligned} h_{\rm B} &= 2\Re[Z_1(f_1 + \mathcal{O}(\varepsilon)){\rm e}^{-{\rm i}\omega_{\rm B}t}]\,, \\ v_{\rm B} &= -2v_{\rm A}^2\Im[Z_1(f_1' + \mathcal{O}(\varepsilon)){\rm e}^{-{\rm i}\omega_{\rm B}t}/\omega_{\rm B}]\,. \end{aligned} \tag{3.7}$$

The velocity $v_{\rm B}$ has been calculated using the second equation in Eq. (2.7) to leading order ($\partial v_{\rm B}/\partial t = v_{\rm A}^2\partial h_{\rm B}/\partial z$) and describes the mechanical motion at the boundaries.

Since the f_n's form a complete set, $h(z,t)$ can be expanded in a series of f_n:

$$h(z,t) = 2\Re\sum_1^\infty b_n(t) f_n(z). \tag{3.8}$$

Using Eq. (2.12) and (3.7) we write $h_{\rm T}$ in the form of an expansion

$$h_{\rm T}(z,t) = 2\Re\sum_1^\infty H_n(t){\rm e}^{-{\rm i}\omega_n t} f_n(z) + \mathcal{O}(\varepsilon). \tag{3.9}$$

The coefficients of these two expansion are clearly related:

$$b_1 = (H_1 - Z_1){\rm e}^{-{\rm i}\omega_1 t},\ b_n = H_n {\rm e}^{-{\rm i}\omega_n t},\ n > 1. \tag{3.10}$$

4. Magnetoacoustic eigenmodes

Eq. (2.15) describes the evolution of a sound wave driven by the fluctuations of the magnetic pressure and it is most naturally solved by finding the Green's function

$$G(z,\zeta) = \frac{v_1(z)v_2(\zeta)\mathcal{H}(\zeta - z) + v_1(\zeta)v_2(z)\mathcal{H}(z-\zeta)}{[-v_2(z)\partial v_1(z)/\partial z]_{z=0}} \tag{4.1}$$

of the operator acting on the Furier transform of $w(z,t)$

$$\hat{w}(z,\omega) = \int_{-\infty}^{\infty} {\rm e}^{{\rm i}\omega t} w(z,t) dt, \tag{4.2}$$

where the fundamental solutions $v_{1,2}$ obey the Fourier-transformed free part of Eq. (2.15) ($h_{\rm T} = 0$):

$$\begin{aligned} &\alpha[\theta(z)v_{1,2}']' - \omega^2\theta'(z)v_{1,2} = 0,\ v_1(0) = v_2(1) = 0, \\ &\theta(z) = {\rm e}^{-\alpha\int_0^z [v_{\rm s}^2(\zeta) - a/\rho(\zeta)]^{-1}{\rm d}\zeta},\ \ a = \frac{4{\rm i}\omega}{3\beta Re}. \end{aligned} \tag{4.3}$$

Calculating $v_{\rm s}$ from $T_{\rm eq}$ (Eq. 2.4), in region I we have

$$v^0_{1,2} = MF(\alpha_+, \alpha_-, 1, \rho/a) + N\Phi(\alpha_+, \alpha_-, 1, \rho/a),$$
$$\alpha_\pm = \frac{1 \pm k}{2},\ k = [1 - (\frac{\omega}{\omega_g})^2]^{1/2},\ \omega_g = (\frac{z_{\rm c}}{v_{\rm A_0}})\frac{\gamma g}{2v_{\rm s_0}} \quad (4.4)$$

where F is the hypergeometric function, M and N are arbitrary constants and ω_g is the normalized cutoff frequency for sound wawes calculated at the base of the atmosphere. The value 1 of the third argument of F stems from the otherwise convenient density model $\rho \sim {\rm e}^{-\alpha z}$ of region I and introduces a 'pathology' in the second solution of Eq. (4.3). The familiar series expansion for $\Phi(a, b, c, z)$ holds for $|z| < 1$ and it is inapplicable in our case. A more general expressions is (Nikiforov and Uvarov 1988):

$$\Phi(a, b, 1, z) = -\lim_{c\to 0}\frac{\partial}{\partial c}\{F(a, b, 1-c, z) +$$
$$+z^c F(a+c, b+c, 1+c, z)/\Gamma(1+c)\}. \quad (4.5)$$

To solve Eq. (4.3) in region II, we assume that $1 - z_{\rm p} \ll 1$ (a situation which well represents the narrow solar transition region) and develop $v_{1,2}$ in a Taylor series:

$$v^{\rm c}_{1,2} = a_0 + a_1(z-z_{\rm p}) + a_2(z-z_{\rm p})^2,\ a_2 = -\frac{a_0\omega^2 + \alpha a_1}{2 - 2a/\rho_{\rm p}}. \quad (4.6)$$

Matching of solutions (4.4) and (4.6)s at $z = z_{\rm p}$ gives v_1

$$v_1 = \begin{cases} \Phi(1/a)F(\rho/a) - F(1/a)\Phi(\rho/a) & \text{region I} \\ a_0 + a_1(z-z_{\rm p}) + a_2(z-z_{\rm p})^2 & \text{region II} \end{cases},$$
$$a_1 = [\Phi(1/a)\partial F(\rho/a)/\partial z - F(1/a)\partial\Phi(\rho/a)/\partial z]_{z=z_{\rm p}},$$
$$a_0 = \Phi(1/a)F(\rho_{\rm p}/a) - F(1/a)\Phi(\rho_{\rm p}/a), \quad (4.7)$$

which vanishes at $z = 0$ and v_2 which vanishes at $z = 1$:

$$v_2 = \begin{cases} F(\rho/a)\mathcal{N}\Phi - \Phi(\rho/a)\mathcal{N}F & \text{region I} \\ 1 + a_1(z-z_{\rm p}) + a_2(z-z_{\rm p})^2 & \text{region II} \end{cases},$$
$$\mathcal{N}f = \frac{[\partial f(\rho/a)/\partial z - a_1 f(\rho/a)]_{z=z_{\rm p}}}{[F(\rho/a)\partial\Phi(\rho/a)/\partial z - \Phi(\rho/a)\partial F(\rho/a)/\partial z]_{z=z_{\rm p}}},$$
$$a_1 = -\frac{2\beta(1-a/\rho_{\rm p}) - 4\omega^2(1-z_{\rm p})^2}{(1-z_{\rm p})[2\beta(1-a/\rho_{\rm p}) - \alpha\beta(1-z_{\rm p})]}. \quad (4.8)$$

Using the Green's function (4.1) and setting $\hat{h^2_{\rm T}}(z,\omega) = \int_{-\infty}^{\infty}{\rm e}^{{\rm i}\omega t}h^2_{\rm T}(z,t)dt$, we have

$$\hat{w} = \frac{-{\rm i}}{4\sqrt{\beta}}\frac{\omega}{\omega_g}\int_0^1 G(z,\zeta)\theta'\{[(1+2{\rm i}\omega\eta/Re)\hat{h^2_{\rm T}}]'/\rho\}{\rm d}\zeta. \quad (4.9)$$

We now revert to Eq. (2.14). For $|1-\beta| \gg \varepsilon$, $w_{\rm B}$ oscillates at frequency which is off-resonant with any multiple of ω_n, $n = 1, 2, ...$, thus being immaterial for the dynamics of the Alfvén eigenmodes. In Eq. (4.9) it suffices to give $h_{\rm T}$ up to $\mathcal{O}(1)$, for $w_{\rm T}$ will later be used in Eq. (2.9), where it appears in terms which are already $\mathcal{O}(\varepsilon)$. Then, using Eqs. (3.9) (without the $\mathcal{O}(\varepsilon)$ corrections) and (4.9), the first few relevant terms of $w_{\rm T}$ are

$$w_{\rm T} = 2\Im[2\omega_1 H_1^2 g_1 {\rm e}^{-2{\rm i}\omega_1 t} + (\omega_1+\omega_2)H_1H_2g_2{\rm e}^{-{\rm i}(\omega_1+\omega_2)t}$$
$$+2\omega_2H_2^2g_3{\rm e}^{-2{\rm i}\omega_2 t} + (\omega_1-\omega_2)H_1H_2^*g_4{\rm e}^{-{\rm i}(\omega_1-\omega_2)t}]. \quad (4.10)$$

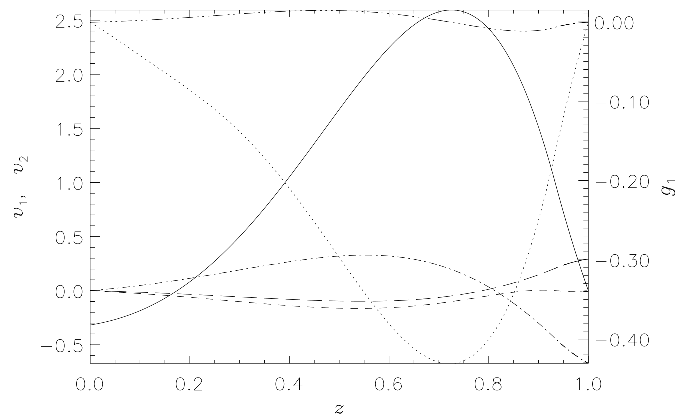

Fig. 3. Left scale: the two fundamental solutions (solid line: $\Re v_2$; short-dashed line: $\Im v_2$; long-dashed line: $\Re v_1$; dash-dotted line: $\Im v_1$) vs. height. Right scale: the magnetoacoustic mode (dotted line: $\Re g_1$; dashed-triple dotted line: $\Im g_1$). Parameters as given in Table 2; $B_0 = 1.55$ G, $Re_0 = 1000$.

Here we give only the expression for mode 1:

$$g_1(z) = \frac{[\mathcal{H}(z_{\rm p}-z)\varphi_1(z) + \mathcal{H}(z-z_{\rm p})\varphi_2(z)]}{2(\beta - 4{\rm i}\omega_1/(3Re))[v_2(z)\partial v_1(z)/\partial z]_{z=0}},$$
$$\varphi_1(z) = [U_1(z)v_2(z) + v_1(z)\delta U(z)] + v_1(z)\Lambda_2,$$
$$\varphi_2(z) = v_2(z)[U_1(z_{\rm p}) + \Lambda_1],\ \delta U(z) = U_2(z_{\rm p}) - U_2(z),$$
$$\Lambda_{1,2} = v_{1,2}(z_{\rm p})[\partial(1+2{\rm i}\omega_1\eta/Re)f_1^2/\partial z]_{z=z_{\rm p}^+}(1-z_{\rm p}),$$
$$U_{1,2}(z) = (1 + 2\frac{{\rm i}\omega_1}{Re})\int_0^z v_{1,2}(\zeta)[\partial f_1^2(\zeta)/\partial\zeta]{\rm d}\zeta. \quad (4.11)$$

The solutions $v_{1,2}$ (Fig. 3) are found by computing the hypergeometric function F (Press et al 1992) and the limit and the derivative in Eq. (4.5) by the Richardson deferred approach to the limit (Press el al 1992). Integrating in Eq. (4.11) by a 64 point Gauss quadrature formula (Press et al 1992) yields g_1 (Fig. 3). In Eq. (2.2) and in the following, the values $Z = 1$, $\gamma = 5/3$ and $\Lambda = 10$ are used.

5. The Hopf bifurcation

Substituting Eq. (3.8) in Eq. (2.11), using the orthonormality property of the functions f_m and projecting upon the mode f_n we obtain an ordinary differential equation:

$$\frac{{\rm d}^2 b_n}{{\rm d}t^2} + \omega_n^2 b_n = \varepsilon\int_0^1\frac{\partial\mathcal{T}}{\partial z}f_n(z){\rm d}z. \quad (5.1)$$

We solve this equation by regular perturbations (Kevorkian and Cole 1981) and set

$$b_n(t) = b_n^{(0)}(t_0, t_1) + \varepsilon b_n^{(1)}(t_0, t_1) + \ldots, \quad (5.2)$$

$t_0 = t$ being the 'fast' time and the 'slow time', $t_1 = \varepsilon t$, being defined so that $\partial/\partial t_1 = \mathcal{O}(\varepsilon)$. The zeroth and the first order approximations to Eq. (5.1) thus read

$$\frac{{\rm d}b_n^{(0)}}{{\rm d}t_0^2} + \omega_n^2 b_n^{(0)} = 0, \quad (5.3)$$

$$\frac{\mathrm{d}b_n^{(1)}}{\mathrm{d}t_0^2}+\omega_n^2 b_n^{(1)}=-2\frac{\partial^2 b_n^{(0)}}{\partial t_0\partial t_1}+\int_0^1\frac{\partial\mathcal{T}^{(0)}}{\partial z}f_n(z)\mathrm{d}z, \quad (5.4)$$

$\mathcal{T}^{(0)}=\mathcal{T}|_{\varepsilon=0}$ being the zeroth order approximation to $\mathcal{T}$. It is now convenient to write te solution of Eq. (5.3) as (see Eq. 3.10)

$$b_1^{(0)}=2\Re[(H_1(t_1)-Z_1)\mathrm{e}^{-\mathrm{i}\omega_1 t_0}],$$
$$b_n^{(0)}=2\Re[H_n(t_1)\mathrm{e}^{-\mathrm{i}\omega_n t_0}],\ n>1, \quad (5.5)$$

where Z_1 is the constant amplitude appearing in Eqs. (3.6) and (3.7). We note that a 'secular' term appears in $b_n^{(1)}$ if the right hand side of Eq. (5.4) has a term proportional to $\exp(-\mathrm{i}\omega_n t_0)$. We set this term to zero by requiring that

$$2\mathrm{i}\frac{\partial H_n}{\partial t_1}=-\frac{1}{2\pi}\int_0^{2\pi/\omega_n}\mathrm{e}^{\mathrm{i}\omega_n t_0}\mathrm{d}t_0\int_0^1 f_n(z)\frac{\partial\mathcal{T}^{(0)}}{\partial z}\mathrm{d}z. \quad (5.6)$$

Since $\mathcal{T}^{(0)}$ in Eq. (5.6) is the zeroth order approximation to $\mathcal{T}$, we may approximate $\partial v_\mathrm{T}/\partial z$ in $\mathcal{T}$ by $\partial h_\mathrm{T}/\partial t$ since these terms differ by a quantity which is $\mathcal{O}(\varepsilon)$ (see the first equation in Eq. (2.7)). For the same reason, we will neglect quantities $\mathcal{O}(\varepsilon)$ in Eqs. (3.6) and (3.9). A further simplification arises if we consider the case in which $|H_1|\gg|H_n|$ for $n>1$ in Eq. (3.9), a situation studied in detail by Nocera and Priest (1991). Recent investigations (Nocera and Ruderman 1998) indeed confirm that this hypothesis (the 'pump wave approximation') holds remarkably well near the stationary states of Eqs. (2.7). These assumptions allow us to write Eqs. (3.9), (4.10), the density fluctuation d_T in Eq. (2.7) and the expression for $\mathcal{T}^{(0)}$ as follows

$$h_\mathrm{T}(z,t_0,t_1)=2\Re[H_1(t_1)\mathrm{e}^{-\mathrm{i}\omega_1 t_0}f_1(z)],$$
$$w_\mathrm{T}(z,t_0,t_1)=4\omega_1\Im[H_1^2(t_1)\mathrm{e}^{-2\mathrm{i}\omega_1 t_0}g_1(z)],$$
$$d_\mathrm{T}(z,t_0,t_1)=2\Im\{H_1^2(t_1)\mathrm{e}^{-2\mathrm{i}\omega_1 t_0}[\rho(z)g_1(z)]'/\rho(z)\},$$
$$\mathcal{T}^{(0)}(z,t_0,t_1)=-\{\frac{\partial}{\partial t}(w_\mathrm{T}h_\mathrm{T})+v_\mathrm{A}^2 d_\mathrm{T}\frac{\partial h_\mathrm{T}}{\partial z}$$
$$+w_\mathrm{T}\frac{\partial h_\mathrm{T}}{\partial t}-\frac{1}{Re}\frac{\partial}{\partial z}[\eta h_\mathrm{T}(2\frac{\partial w_\mathrm{T}}{\partial z}+3h_\mathrm{T}\frac{\partial h_\mathrm{T}}{\partial t})]\}. \quad (5.7)$$

Now the averaging operation in Eq. (5.6) can be carried out and, after some algebra, we rewrite Eq. (5.6) for mode $n=1$ as

$$\frac{\mathrm{d}x}{\mathrm{d}\tau}=-\Delta y+(\mu x+y)u^2,$$
$$\frac{\mathrm{d}y}{\mathrm{d}\tau}=\Delta x+(\mu y-x)u^2-\Delta, \quad (5.8)$$

where

$$x+\mathrm{i}y=H_1Z_1,\ u=|x+\mathrm{i}y|,\ \tau=|\Im\xi||Z_1|^2\varepsilon t,$$
$$\xi=\frac{1}{2\mathrm{i}\omega_1}\int_0^1\{\frac{v_\mathrm{A}^2}{\rho}[\rho g_1]'[f_1']^2+\frac{\mathrm{i}\omega_1\eta}{Re}(4g_1'+3f_1^2)f_1f_1''\}\mathrm{d}z,$$
$$\mu=\Re\xi/|\Im\xi|,\ \Delta=\Delta\omega/(|\Im\xi||Z_1|^2). \quad (5.9)$$

In Eq. (5.8) $\Re\xi$ acts a coefficient of nonlinear damping through the terms proportional do μ. To clarify its relation to viscosity, we note that, if $Re\gg 10^2$, after some algebra

$$\Re\xi\sim\xi_0/Re, \quad (5.10)$$

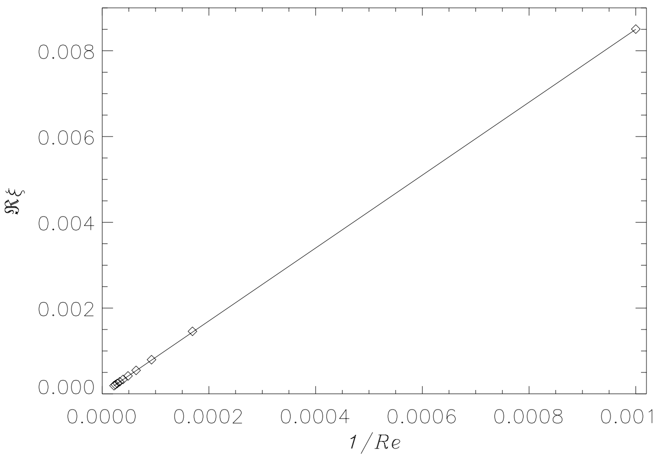

Fig. 4. The real part of the damping coefficient ξ vs. the normalized viscosity $1/Re$. The $\diamond$'s represent numerical data. The solid line represents the analytical estimate (5.10) with $\xi_0=8.5$. Parameters as given in Table 2; $B_0=1.5$ G.

ξ_0 being approximately independent of Re. This formula states that $\Re\xi$ is proportional to viscosity and agrees with the numerical value of ξ (Fig. 4) computed from Eq. (5.9) where the one (two) dimensional integration of terms involving f_1 (g_1) is performed by a 64 (64^2) point Gauss quadrature formula (Press et al. 1992).

The system (5.8) is identical with the one governing an homogeneous cavity (Nocera & Priest 1991, Eq. (4.1)). Note a misprint ($\Delta\omega\to-\Delta\omega$) in their Eq. (4.2a). The modulus $u_0(\mu,\Delta)=|x_0(\mu,\Delta)+\mathrm{i}y_0(\mu,\Delta)|$ of Eq. (5.8) is given implicitely by

$$\Delta=-u_0^4\pm u_0^3[1-\mu^2(u_0^2-1)]^{1/2}/(u_0^2-1). \quad (5.11)$$

By linearizing Eq. (5.8) we see that the small perturbations $(\delta x,\delta y)$ of these equilibria evolve as

$$\frac{\mathrm{d}\delta x}{\mathrm{d}\tau}=[\mu u_0^2+2x_0(\mu x_0+y_0)]\delta x$$
$$+[-\Delta+u_0^2+2y_0(\mu x_0+y_0)]\delta y,$$
$$\frac{\mathrm{d}\delta y}{\mathrm{d}\tau}=[\Delta-u_0^2+2x_0(\mu y_0-x_0)]\delta x$$
$$+[\mu u_0^2+2y_0(\mu y_0-x_0)]\delta y. \quad (5.12)$$

If $(\delta x,\delta y)\sim\exp(\lambda\tau)$, λ obeys

$$\Pi(\lambda)=\lambda^2-4\mu u_0^2\lambda+3(1+\mu^2)u_0^4-4\Delta u_0^2+\Delta^2=0. \quad (5.13)$$

Nocera & Priest (1991, Fig. 2) showed that the equilibria in Eq. (5.11) are multi-valued functions of Δ and that they come in triplets. Thus for each equilibrium branch in the triplet, λ obeys a second degree equation, which, in principle, gives six possible values for the growth rate for a given value of μ and Δ. Here we are interested in the Hopf bifurcations of the equilibria. These appear if a couple of imaginary solutions, $\pm\mathrm{i}|\lambda|$, of $\Pi(\lambda)=0$ exists (which implies $\mu=0$) and if $\partial\lambda/\partial\mu|_{\mu=0}\neq 0$ (Guckenheimer & Holmes 1983, Theorem 3.4.2). At variance with the homogeneous case (Nocera & Priest 1991), this can now happen, for in Eq. (5.10) ξ_0 changes sign as B_0 varies (Fig.

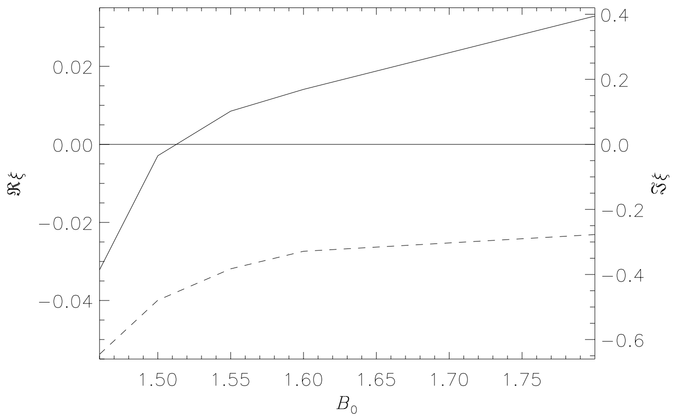

Fig. 5. The damping coefficient (solid curve, left scale: $\Re\xi$; dashed curve, right scale: $\Im\xi$) vs. the background magnetic field B_0. Parameters as given in Table 2; $Re = 1000$.

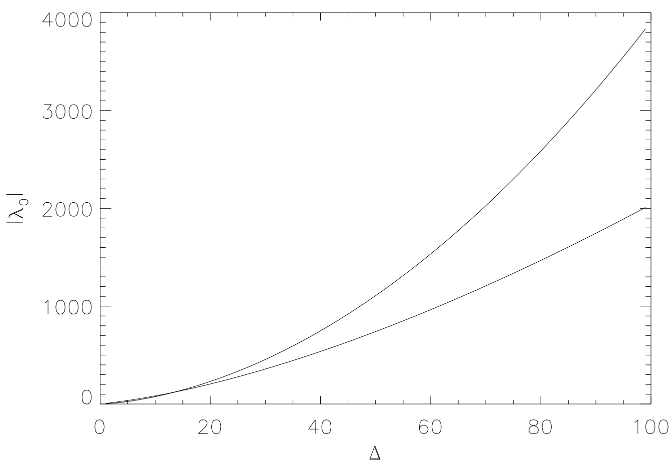

Fig. 6. The angular frequency $|\lambda_0|$ of the Hopf orbit versus the normalized frequency mismatch Δ. Two branches are possible which intersect at $\Delta = 27/2$.

5). The reasons for such change of sign will be discussed in Sect. 6. Thus $\Re\xi = 0$ at $B_{\rm cr} \simeq 1.5$ G. According to Table 1 this corresponds to a period of the magnetoacoustic mode of 3.0 minutes. When $\Re\xi \simeq 0$, we set $\lambda \simeq \pm{\rm i}|\lambda_0| + \mathcal{O}(\Re\xi)$ and combine Eqs. (5.11) and (5.13) to get

$$|\lambda_0|^6 - \Delta(6 + \frac{\Delta}{3})|\lambda_0|^4 - 4\Delta^3|\lambda_0|^2 + \Delta^4(27 + \frac{4}{3}\Delta) = 0,$$
$$\lambda \simeq \pm{\rm i}|\lambda_0| + 2\mu u_0^2 \Rightarrow \partial\lambda/\partial\mu = 2u_0^2 \neq 0. \qquad (5.14)$$

The discriminant of the cubic equation in Eq. (5.14) is $-(\Delta/3)^7(2\Delta - 27)^2(\Delta^2 + 117\Delta/4 + 648)$; since $|\lambda_0|$ has to be real and positive, only the two roots for $\Delta > 0$ are acceptable. These roots are shown in Fig. 6. We conclude that a family of orbits exists around the equilibrium points $u_0(\mu = 0)$, whose radius varies smoothly with μ and whose period is approximately

$$P = 2\pi/|\lambda_0|. \qquad (5.15)$$

Notice that $P \to 0$ as Δ increases. However, for the perturbation scheme to be consistent, $|\Delta| \ll \varepsilon^{-1/2}$.

6. Conclusions

In this work we studied the effect of a magnetic field on the nonlinear oscillations of a hydromagnetic cavity whose equilibrium state is stratified by gravity. The main assumptions adopted were the homogeneous nature of the field and a simple two-deck model for the equilibrium temperature which is meant to represent the solar upper chromosphere-transition region.

As a main result, it was shown that the variation of the strength of the magnetic field past a critical value ($B_{\rm cr} \simeq 1.5$ G) triggers 'self oscillations' (Hopf bifurcations) which appear as modulations of the amplitude of the Alfvén wave propagating in the cavity. This phenomenon relies on the dissipation parameter in our problem 'changing sign' as the magnetic field varies. At first sight, this fact may seem surprising: recent investigations reveal that it is due to 'negative energy' waves (Bologna and Nocera 1998), which appear in our problem, much in the same way as in hydrodynamic shear flows. It is known that these waves are amplified, rather than damped, by viscosity (Ostrovskii et al. 1986).

Beside this main result, several points are worth noticing. Our work features one of the rare attempts to apply a regular perturbation procedure to a non trivial Galerkin base which unfolds through several mathematical subtleties and numerical intricacies. Despite the considerable amount of parameters which characterize our physical problem, it is remarkable that the final result is a parameter independent relation between the frequency of the nonlinear modulation and the mismatch between the frequency of the first Alfvén eigenmode and the forcing acting at the boundary of the cavity. The simplicity of this algebraic relation (a cubic equation) should also be regarded as an encouraging result and could perhaps stimulate observations of possible modulations in the time series coming from non homogeneous hydromagnetic cavities.

Flux tubes in the solar chromosphere are excellent candidates for such observations, in view of the instrumental accuracy nowadays available. Using Eq. (5.15), Fig. 6 and the bound on the frequency mismatch ($|\Delta| \ll \varepsilon^{-1/2}$) discussed in Sect. 5 and considering that the modulations take place over the slow time $\tau = \varepsilon t$, a lower bound for the modulation period is $P_{\rm min} \geq 2\pi\tau_{\rm A}/(|\lambda_0|\varepsilon)$. For the parameters of Table 2 and $\varepsilon \approx 10^{-4}$, $\Delta = 60$ ($|\lambda_0| \simeq 1500$ for the upper branch of Fig. 6), we have $P_{\rm min} \simeq 6$ hours.

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