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Photocentre displacement of minor planets: analysis of Hipparcos astrometry*

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Abstract. We present a study of the phase effect (or photocentre offset) on the astrometry of minor planets. Analysis of the high precision Hipparcos observations of the largest asteroids shows the validity of the model. Minor planets Ceres, Pallas, Vesta and Iris show a brightness distribution close to the lunar scattering properties, while Juno and Hebe are found to be more uniformly bright objects. For general purposes, the observed astrometric positions of asteroids could be corrected for the systematic photocentre offset by assuming in a first approximation a spherical body with a brightness distribution following the law of Buratti & Veverka (1983).

Key words: Planets and satellites: Ceres, Pallas, Vesta – Minor planets, asteroids – ephemerides – astrometry

1. Introduction

Asteroids are of particular interest for dynamical studies of the solar system and its long-term evolution. Sufficiently accurate astrometric positions are, in the particular cases of a very close encounter, of considerable value for the determination of a perturbing body's mass (e.g. Hilton 1997; Viateau & Rapaport 1997). These small bodies can also be considered as test particles gravitating in the weak-field of the solar system, hence they are good candidates for tests of general relativity (e.g. Sitarski 1992; Shahid-Saless & Yeomans 1994). They are also important for the linking of the dynamical and geometrical quasi-inertial reference systems. Very accurate astrometric measurements of these small bodies in the reference frame of the extragalactic objects that could be achieved by interferometric means, e.g. DIVA (Bastian et al. 1996), GAIA (Lindegren & Perryman 1996), LIGHT (Yoshizawa et al. 1997), SIM (Unwin et al. 1996), or from the Earth over sufficiently long periods, would tied the two reference frames with an unprecedented precision and provide a test of Mach's principle to the micro-arcsecond level of accuracy. Nevertheless, neglecting the photoccentre offset, i.e. the offset between the centre of light and the centre of figure, would induce a considerable bias in such a link (Söderhjelm & Lindegren 1982).

The Hipparcos satellite launched in 1989 was the first instrument dedicated to providing astrometric measurements from space. Since a few solar system objects were included in the observing programme (mainly asteroids), very accurate astrometric positions related – via the Hipparcos catalogue – to the system of the International Celestial Reference System (ICRS) were obtained (Hestroffer et al. 1998). Compared to equivalent observations obtained with ground-based meridian circles, the Hipparcos astrometric measurements are, with a mean standard error of 10–15 mas (milliarcsecond), at least 10 times more accurate than the $0.''15 - 0.''25$ obtained at the La Palma or Bordeaux observatories since 1984. On the other hand, ground-based observations with CCD chips yield, or will yield in the near future, small-field astrometry of similar accuracy. Hence the phase effect, which was usually neglected up to now for asteroids, will have to be accounted for in the future.

It was well known, during the construction of the Hipparcos Solar System Objects Astrometric Catalogue, that the observed position corresponds to the position of the object's photocentre (Hestroffer & Mignard 1997a). The photocentre shows an offset from the body's centre of figure in the direction toward the sun, which depends mainly on solar phase angle, the apparent size of the object, its shape and its surface properties. However this offset could not be determined with sufficient accuracy during the construction of the Catalogue to be properly accounted for. Hence the published observations still refer to the photocentre, while any ephemerides refer to the centre of mass. As a result of the very accurate Hipparcos positions, the photocentre offset on the largest asteroids becomes for the first time observable (Hestroffer et al. 1995). We present in this paper the determination of the limb-darkening parameterization for the Hipparcos minor planets, and the effects on the improvement of the ephemerides for the largest asteroids observed during the mission.

2. Photocentre offset

Assuming that the observed object is spherical and the scattering of light on the surface follows the reciprocity principle, the photocentre offset is along the bisector of the directions to-

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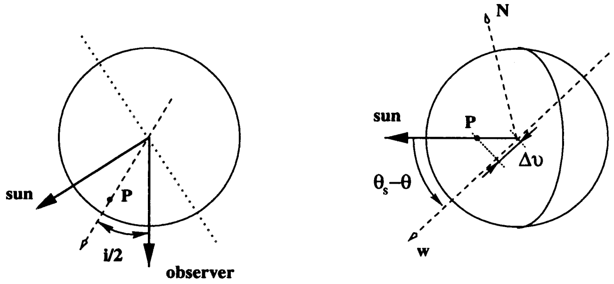

Fig. 1. Geometry of the photocentre offset. Left: for a spherical object scattering light in accordance with the reciprocity principle, the photocentre P is located on the phase angle bisector. Right: in the tangent plane, the position angle of the sun and the Hipparcos scanning direction $\mathbf{w}$ are $\theta_{\rm s}$ and θ respectively. Projected in the tangent plane and in the Hipparcos scanning direction, the photocentre offset is Δv

ward the observer and the sun (Lindegren 1987). It is given in differential coordinates (Standish et al. 1992) by:

$$\begin{pmatrix} \Delta\alpha\cos\delta \\ \Delta\delta \end{pmatrix} = \begin{pmatrix} \sin\theta_{\rm s} \\ \cos\theta_{\rm s} \end{pmatrix} C(i)\, \sin(i/2)\, \phi/2 \tag{1}$$

where ϕ is the apparent diameter, i is the solar phase angle and $\theta_{\rm s}$ is the position angle in the tangent plane of the sub-solar point (see Fig. 1). For a Hipparcos observation, this offset reduces to a single correction over a great circle (ESA 1997) whose position angle is θ, so that we have:

$$\Delta v = \cos(\theta_{\rm s} - \theta)\, C(i)\, \sin(i/2)\, \phi/2 \tag{2}$$

The function $C(i)$ depends on the actual brightness distribution over the visible surface, and it can be given with sufficient accuracy by the first terms of its series expansion. Moreover, given the astrometric noise of the Hipparcos measurement and the size of the phase effect, it is not possible to determine the different parameters of the elaborated light-scattering models of Lumme & Bowell (1981) or Hapke (1981). On the other hand, we will see that applying the more simple law of Buratti & Veverka (1983) is sufficient to correct the astrometric data for the photocentre displacement.

For a Lambertian sphere we have (Lindegren 1977):

$$C(i) = \frac{3\pi}{4}\, \frac{\cos^3(i/2)}{\sin i + (\pi - i)\cos i}$$

and the expansion:

$$C(i) = \frac{3}{4} + \frac{3}{32}\, i^2 - \frac{1}{4\pi}\, i^3 + o\,(i^4) \tag{3}$$

When considering a theoretical uniform brightness $I(\mu, \mu_o) =$ constant, we have (Hestroffer et al. 1995):

$$C(i) = \frac{8}{3\pi}\, \sin(i/2) = \frac{4}{3\pi}\, i + o\,(i^3) \tag{4}$$

For a normalised brightness distribution following the empirical law of Minnaert $I(\mu, \mu_o) = \mu_o^k\, \mu^{k-1}$, we find when $k > 1/2$:

$$\begin{aligned} C(i) \;=\; & \frac{2k+1}{2k+2} \\ & + \frac{(k+1/2)}{8\,(k+1)\,\Gamma(k)\,\Gamma(k+1/2)} \times \\ & \Big\{ 2\,\Gamma(k)\,\Gamma(k+3/2) + \\ & \Big(\Gamma(k-1/2) + 2\,\Gamma(k+1/2)\Big)\, \Gamma(k+1) \Big\}\; i^2 \\ & + O\,(i^3) \quad ; \quad (k > 1/2) \end{aligned} \tag{5}$$

where the parameter $k = k(i)$ also depends on the phase angle. Using the Lommel-Seeliger law $I(\mu, \mu_o) = \mu_o/(\mu + \mu_o)$ which is known to better exhibit the lunar scattering properties than the law of Minnaert, we have (Lindegren 1977):

$$C(i) = \frac{2}{3\pi}\, \frac{\sin i + (\pi - i)\cos i}{\cos(i/2) - \sin^2(i/2)\log\cot(i/4)} \tag{6}$$

and the approximation:

$$C(i) = \frac{2}{3} - \left(\frac{1}{4} + \frac{\log i - \log 4}{6} \right) i^2 + o\,(i^3) \tag{7}$$

The analytical expression introduced by Buratti & Veverka (1983) reproduces in a simplified manner the more refined models of Lumme & Bowell (1981) and Hapke (1981). In this case the brightness distribution is given by:

$$I(\mu, \mu_o, i) = A\, \frac{\mu_o}{\mu + \mu_o}\, f(i) + (1 - A)\, \mu_o \tag{8}$$

and we obtain by numerical integration:

$$C(i) = \frac{3 - A}{4 - A} + 13/8\, \frac{(3 - A)\,(4/13 - A)}{(4 - A)^2}\, i + o\,(i^2) \tag{9}$$

Following Buratti & Veverka (1983) the zero-phase geometric albedo p, the parameter A and Lumme's multiple scattering parameter $Q_{\rm M}$ are related by:

$$\begin{aligned} f(0) \;&=\; \frac{6\,p}{1 + \frac{3\,(2 - 3\,p)}{2\,(2/Q_{\rm M} - 1)}} \\ A \;&=\; \frac{4 - 6\,p}{4 - f(0)} \end{aligned}$$

This yields $A \simeq 1$ for the low-albedo asteroid (1) Ceres ($p = 0.08$), and $A \simeq 0.9$ for (4) Vesta ($p = 0.3$). Assuming the mean value $A = 0.95$ we have the approximation for the photocentre displacement of a given asteroid $C(i) = 0.670 + 0.045\, i + o\,(i^2)$, where i is in radians.

Comparison of the first terms obtained in Eq. (5) and Eq. (9) yields the relation $A \sim 2\,(1 - k)$. For the Hipparcos observations occurring at larger solar phase angle, the limb darkening is stronger (French & Veverka 1983). Taking the results obtained from the analysis of the Hipparcos modulated photometry of minor planet (1) Ceres (Hestroffer & Mignard 1997b; Hestroffer & Mignard 1997c), we assume $A = 0.8$ ($k = 0.6$). Thus the following values were adopted:

$$C(i) \;=\; 0.670 + 0.045\, i + o\,(i^2) \quad ; \quad (i \leq 0.2) \tag{10}$$

$$C(i) \;=\; 0.686 + 0.037\, i + o\,(i^2) \quad ; \quad (0.2 < i) \tag{11}$$

However, the difference in the photocentre offset between Eq. (10) and Eq. (11) is less than one milliarcsecond for a typical main-belt asteroid. Compared to the much more refined light scattering models of Lumme & Bowell (1981) or Hapke (1981), the Minnaert law yields a value of the phase-effect correction accurate to ∼5%. Hence an expression of the form $C(i) = (2k+1)/(2k+2)+0.04\,i$ could usefully be used to determine a limb-darkening parameterization from astrometric measurements over the range $0 \leq i \leq 0.5$ rad.

3. Hipparcos observations

The observations of a solar system object by Hipparcos always occurred around the quadratures, i.e. when the solar phase is maximal. At the largest phase angle ($\sim 30^\circ$ for a ground-based observation or a telescope orbiting around the earth), the photocentre offset can reach ∼20% of the observed object's apparent radius. For the great majority of the Hipparcos asteroids it does not exceed 10 mas (see Table 1). Only for the largest bodies can the phase effect be of the same order as the astrometric measurements noise (roughly 10–15 mas). A correction for the photocentre offset was applied for instance during the analysis of the observations of the planetary satellites (Morrison et al. 1997).

Here we study the effect on the improvement of the ephemerides; since the phase effect is proportional to the apparent diameter, we restrict our study to the largest bodies. We will both validate the model established in the previous section, and derive the best value for the limb-darkening parameterization from a least-squares fit. The Hipparcos astrometric data for the asteroids result from two data reduction consortia (FAST and NDAC). Here, no distinction is made between the FAST and NDAC data although they do not strictly speaking correspond to the same point on the surface of the object; in this paper we consider that both positions are independent.

Although it is of little consequence here, a transformation between the FK5 and the ICRS reference frames was applied for the calculation of the ephemeris. Neglecting such a transformation would yield different solutions for the correction to the osculating elements (mainly for the perihelion and node arguments and the inclination). This transformation is given by considering only the time-dependent rotation between the two systems which components are of the order of 20 mas at epoch J1991.25 (Mignard et al. 1997).

The mutual perturbations of minor planets can yield effects of the same order as the phase effect. In order to avoid other systematic effects as much as possible, the perturbations of (1) Ceres, (2) Pallas and (4) Vesta in addition to those of the nine major planets (whose positions and masses were taken from the DE200 solution) were accounted for in the ephemeris. The adopted masses for the perturbing minor planets were taken from Viateau & Rapaport (1997).

The O-Cs are given in Figs. 2 to 5 for Ceres, Pallas, Vesta and Iris. They lie generally in the range $\pm 0\farcs2$ and show essentially the errors due to the relatively poor ephemeris. The larger values observed for Pallas arise from the fact that its osculating elements were erroneously given in a hybrid (FK5, B1950) system (Viateau & Rapaport 1995). Next, we determine the correction to the osculating elements by a fit to the Hipparcos observations from two sets of observed minus calculated positions (O-C), where the second set is corrected for the photocentre offset as given by Eqs. (2) and (11). This yield two solutions for the new osculating elements and the corresponding post-fit residuals.

4. Results

Comparison between the residuals for the two solution sets in Figs. 2 to 5 clearly shows the improvement obtained when the calculated (or observed) places are corrected for the phase effect. In all cases the rms of the residuals drops to approximately 10 mas (see Table 2), in good agreement with the standard deviation of a single observation. Depending on the actual correlation between the limb-darkening parameter and the corrections to the osculating elements, neglecting the phase effect can yield a significant bias in the derivation of the orbit.

It is stressed that the astrometric measurements over the time span of a transit are one-dimensional. As a consequence of the particular scanning law of the Hipparcos satellite[1], the photocentre offset as seen on the sky regularly vanishes or projects to a small quantity on the scanning direction. In this particular case the coefficient $C(i)$ is indeterminate. Moreover the observations are not uniformly spread over the limited range in phase angle. Thus, although the Hipparcos astrometric measurements are sensitive to the phase effect, they are not particularly well adapted to derive a limb-darkening parameterization for a single object. First attempts to determine simultaneously the corrections to the osculating elements and a parameterization of the form $C(i) = a + b\,i$ from the observations were unsuccessful, because of a large correlation between the unknowns a and b. Hence a single parameter, averaged over the solar phase angle, $\langle C(i) \rangle$, was determined. The precision of this mean parameter ranges between 5% for Vesta and 22% for Iris (see Table 2). The residuals obtained with this single parameter fit, although not minimal, are still acceptable, and are not statistically different from those obtained by applying the correction given by Eqs. (2) and (11). For the other asteroids, the standard deviation on this determined averaged parameter is even larger, so that more data would be necessary to determine the limb-darkening parameterization with a better accuracy.

5. Discussion

The addition of the photocentre offset in the comparison between the observed and calculated positions yields smaller residuals. The best result is obtained for (4) Vesta where the rms of the residuals drops from 14.0 to 7.2 mas, the latter value being closer to the mean standard deviation of a single observation of this asteroid. Not surprisingly the improvement is betterer for the largest bodies; it is nevertheless still significant for the smaller planet (7) Iris, whose apparent diameter was ranged between $0\farcs09$ and $0\farcs25$, with a median value of $0\farcs12$.

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[1] The spin axis of the satellite precessed around the direction toward the sun with a constant angle of 43°.

Table 1. Maximum value of the photocentre offset for the Hipparcos observations; estimated from:
$|\Delta\upsilon_{\max}| \leq \frac{3}{4}\ \cos(47°)\ \sin(i_{\max}/2)\ \phi_{\max}/2$

Planet	nobs	$i_{\min}$ [deg]	$i_{\max}$ [deg]	$\phi_{\min}$ [arcsec]	$\phi_{\max}$ [arcsec]	$\Delta\upsilon_{\max}$ [mas]
1	64	14.19	22.41	0.35	0.67	33[a]
2	62	13.53	27.37	0.19	0.41	25[a]
3	59	14.20	26.21	0.09	0.26	14[a]
4	57	17.96	26.06	0.23	0.40	23[a]
5	80	15.80	28.73	0.06	0.13	7
6	91	14.94	30.65	0.08	0.22	14[a]
7	66	16.58	31.67	0.09	0.25	17[a]
8	56	17.22	29.92	0.06	0.13	8
9	38	19.56	28.10	0.08	0.17	10
10	50	13.54	19.05	0.15	0.23	9
11	68	15.78	24.92	0.07	0.12	6
12	22	16.29	29.24	0.07	0.11	7
13	33	18.48	24.63	0.11	0.17	9
14	45	14.04	27.89	0.06	0.15	9
15	83	13.59	25.89	0.11	0.20	11[a]
16	48	14.16	22.97	0.10	0.19	9
18	98	15.23	27.31	0.06	0.14	8
19	30	16.39	28.47	0.12	0.18	11
20	60	17.96	28.59	0.07	0.13	8
22	61	15.90	22.54	0.05	0.08	4
23	65	13.72	28.37	0.05	0.09	5
27	35	17.05	29.42	0.05	0.11	7
28	33	19.85	24.07	0.07	0.09	4
29	74	17.22	24.94	0.10	0.18	10
30	48	16.44	28.29	0.06	0.11	6
31	14	17.53	24.00	0.13	0.19	10
37	32	14.23	22.83	0.06	0.10	4
39	108	13.62	23.56	0.06	0.11	5
40	102	18.14	26.98	0.05	0.11	6
42	50	20.67	32.30	0.06	0.13	9
44	52	20.51	28.34	0.04	0.07	4
51	14	19.98	26.37	0.11	0.12	7
63	12	19.53	26.52	0.07	0.09	5
88	36	17.59	23.75	0.11	0.17	9
115	33	21.32	28.35	0.04	0.09	5
129	40	18.21	24.91	0.06	0.08	4
192	32	18.89	25.90	0.07	0.10	5
196	14	16.59	19.24	0.07	0.08	3
216	21	17.14	25.94	0.07	0.10	5
230	35	16.68	25.26	0.06	0.09	5
324	73	17.46	32.60	0.11	0.36	25[a]
349	91	13.89	21.29	0.05	0.09	4
354	98	13.84	23.16	0.06	0.11	5
451	29	14.35	20.29	0.12	0.15	6
471	112	14.87	25.28	0.06	0.11	6[a]
511	62	14.64	22.05	0.13	0.21	10
532	38	14.22	24.74	0.09	0.17	9
704	81	14.05	22.04	0.14	0.21	10

[a] Discussed in this paper

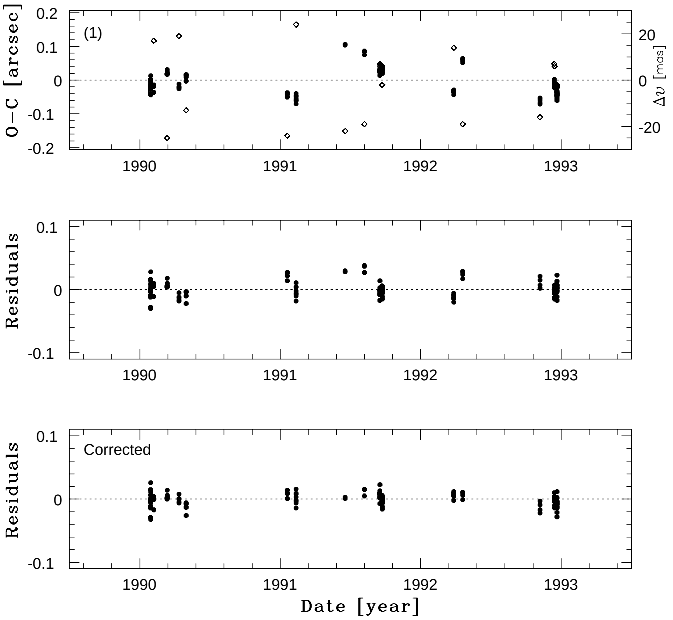

Fig. 2. Improvement of the residuals for (1) Ceres when the correction for the photocentre offset is applied. Upper panel: initial O-Cs where the ephemerides are calculated from the osculating elements for the year 1996 (Batrakov 1995). The photocentre offset $\Delta\upsilon$ calculated from Eqs. (2) and (11) isshown as open squares, values are given on the right-hand label. Middle panel: residuals obtained after compensation for the correction to the 6 osculating elements, without taking the phase effect into account. Lower panel: same as before, but the O-Cs were corrected for the phase effect

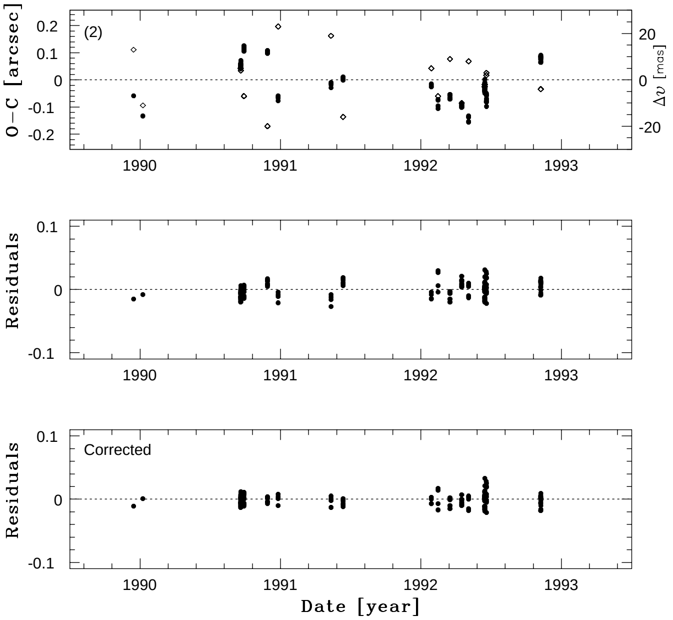

Fig. 3. Same as Fig. 2 for (2) Pallas

Adopting the same procedure as in the previous sections, the residuals obtained for (324) Bamberga are not improved although the photocentre correction is of the same order of magnitude as for the minor planets already considered. Moreover

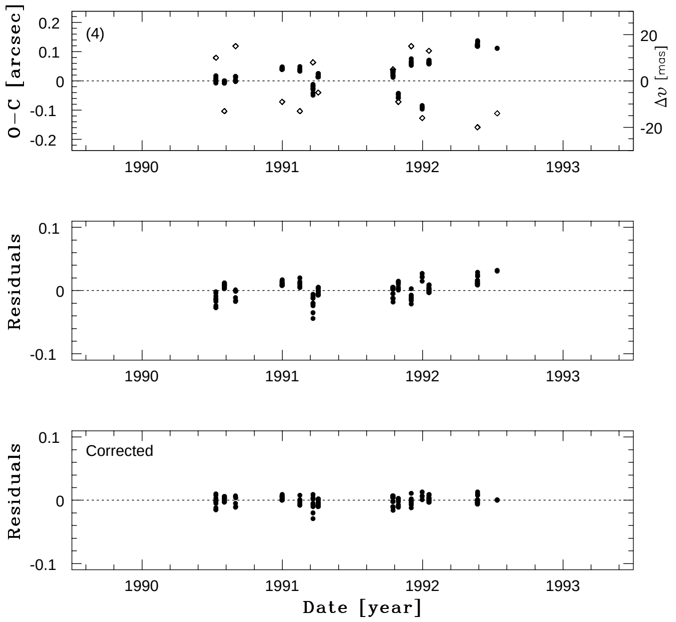

Fig. 4. Same as Fig. 2 for (4) Vesta

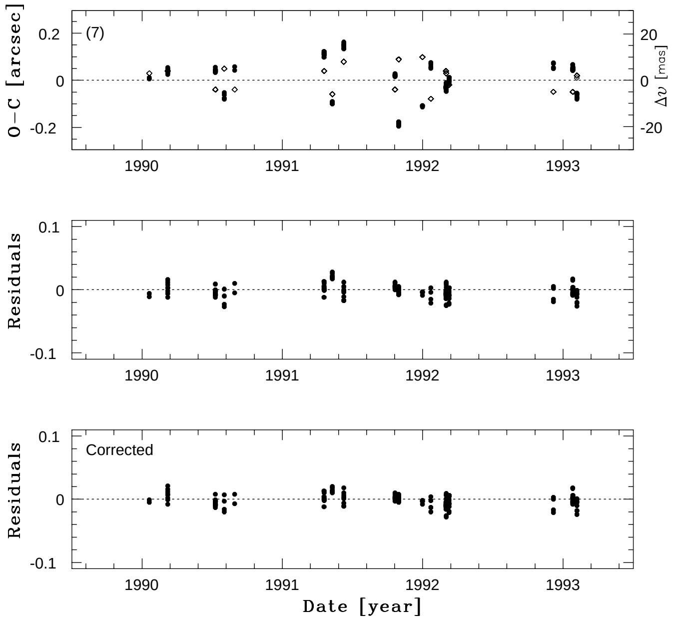

Fig. 5. Same as Fig. 2 for (7) Iris

Table 2. Improvement of the rms of the residuals. The diameter and maximum apparent diameter are listed in the first two rows. The rms of the residuals, once the data have been correctd for the phase offset as given by Eqs. (2) and (11) and the improvement with respect to the uncorrected data, are given in the third and fourth rows. The last rows give the median value of the solar phase angle, the best-fit averaged parameter $\langle C(i)\rangle$ (see text) and its formal error

Planet	Ceres	Pallas	Vesta	Iris
Diameter [km]	913	523	501	203
$\phi_{\max}$ [arcsec]	0.67	0.41	0.40	0.25
A priori values				
rms [mas]	9.6	9.7	7.2	9.7
ratio (%)	23	21	48	7
Least squares				
$\langle i\rangle$	17°4	18°2	21°4	22°9
$\langle C(i)\rangle$	0.63	0.62	0.75	0.68
$\pm 1\sigma$	0.06	0.06	0.04	0.15

some unexplained systematic effects still remain. It is hence more difficult to assess the validity of the applied photocentre offset correction. Unfortunately, determination of the averaged single parameter $\langle C(i)\rangle = 0.09 \pm 0.75$ is very poor because of a high correlation with the other unknowns. Thus more data are necessary to validate or determine the limb-darkening parameterization of this asteroid.

For the smaller asteroids (3) Juno ($A = 0.95$) and (6) Hebe ($A = 0.94$), there is a slight degradation of the distribution of the residuals (-4%) when we apply the a priori photocentre-offset correction. On the other hand, the residuals are improved when this correction is smaller and follows the expression given in Eq. (4). Since it is unlikely that the adopted diameters are too large by 75%, these asteroids show a more uniform brightness distribution. Because the phase effect is considerably smaller in this case, the residuals are only slightly improved (less than 1%). A test on the data of (15) Eunomia and (471) Papagena with both parameterized brightness distributions shows no significant difference, either on the dispersion of the residuals or on the correction to the osculating elements. Although a higher improvement of the residuals is obtained with a photocentre correction following Eq. (4), the correction for the photocentre offset in these cases is too small to distinguish the different models with the available data.

The effect of shape or albedo spots were not taken into account in this study. These may have significant influence on the photocentre displacement of the largest bodies. Hence the poor fit obtained for (324) Bamberga could still be the consequence of the changing body's along-scan-size during the mission. As seen in Sect. 4, the Hipparcos astrometry is not particularly well adapted to derive the scattering parameters of the elaborated models of Lumme & Bowell (1981) or Hapke (1981). The more simple law of Buratti & Veverka (1983) should provide a good approximation for the photocentre offset of the Hipparcos asteroids. However, our knowledged of the parameter A of this law, for asteroids observed at large phase angles, is rather poor. Model-predicted values would be of particular interest for further analysis of the Hipparcos data, and may explain the disagreement observed for (3) Juno and (6) Hebe.

6. Conclusion

A parameterization for the limb-darkening to apply to the observations of the Hipparcos asteroids is given. When adopting the simplified model of Buratti & Veverka (1983) for the averaged brightness distribution, a very good fit to the Hipparcos astrometric observations of the largest minor planets (1) Ceres, (2) Pallas, (4) Vesta and (7) Iris is obtained. On the other hand, better agreement to the Hipparcos data of (3) Juno and (6) Hebe is obtained when one assumes a more uniform brightness distribution. The assumption of spherical bodies and homogeneous brightness distribution, for these large objects, appears to be adequate. For the smallest asteroids the phase effect is small relative to the precision of the measurement. For general purposes, a parameterization following the Law of Buratti & Veverka, given in this paper by Eq. (10) and Eq. (11), should provide a good approximation to the photocentre correction.

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