

On the gravitational wave noise from unresolved extragalactic binaries

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Abstract. We calculate stochastic gravitational wave background produced by extragalactic merging binary white dwarfs at the LISA frequencies $10^{-3} - 10^{-2}$ Hz with account of a strong evolution of global star formation rate in the Universe recently established observationally. We show that for the observed global star formation history and modern cosmological models the extragalactic background is an order of magnitude smaller than the mean Galactic value. An early star formation burst at high redshifts can bring it at a higher level but still a few times lower than the mean Galactic one.

Key words: gravitational waves – stars: binaries: close

1. Introduction

Gravitational waves (GW) from unresolved binary stars in the Galaxy forms a confusion limit in the frequency domain of LISA laser space interferometer (Bender et al. 1996) $10^{-4} - 10^{-1}$ Hz that effectively adds to the detector's noise. Different aspects of this background have been studied earlier by Rosi and Zimmerman (1976), Lipunov & Postnov (1987), Lipunov, Postnov & Prokhorov (1987), Hils, Bender & Webbink (1990), Bender & Hils (1997), Giampieri & Polnarev (1997), Postnov & Prokhorov (1998).

The importance of the knowledge of binary confusion limit within the LISA frequency band is dictated by many factors – for example, it limits the angular resolution of the detector (Cutler 1997) and restricts the possibilities of observing possible relic GW backgrounds (see, e.g., discussion in Grishchuk 1997). Since LISA with its three arms is expected to operate as essentially single interferometer (Schutz 1997), it can only see noise sources above its intrinsic noise h_{rms} . In our previous paper (Postnov & Prokhorov 1998) we focused on the stochastic GW noise from galactic binaries and showed that, depending on the (unknown) galactic binary white dwarf merger rate, this noise becomes lower than the expected LISA rms noise at frequencies $f \sim 3 \times 10^{-2}$ Hz.¹ We concluded that to be de-

tectable by LISA, relic GW backgrounds should be as high as $\Omega_{GW} h_{100}^2 > 10^{-8}$ at 10^{-2} Hz.

Clearly, the galactic binary GW noise, tracing the distribution of stars in the Galaxy, should be highly anisotropic (of order of one magnitude higher in the direction of the Galactic center; see calculations by Lipunov et al. 1995) and thus modulated with LISA turning in its orbit. This distinctive feature of the galactic stochastic background can in fact be used to measure it by space-born interferometers (Giampieri & Polnarev 1997). This is not the case for a GW-background produced by extragalactic binaries, which becomes important at frequencies where galactic binaries do not form continuous noise ($f > 3 \times 10^{-3}$ Hz for 1-year integration).

A crude estimate shows that the isotropic extragalactic background is expected to be an order of magnitude smaller than the average GW noise from galactic binaries (see Lipunov et al. (1987, 1995), Hils et al. (1990)). All estimates of this background made so far, however, have not taken into account the fact of a strong global star formation evolution recently revealed by different astronomical observations (see the results of Canada-France-Hawaii survey of far galaxies with $z \lesssim 1$ (Lilly et al. 1996), the analysis of galaxies with Lyman jump at $z \gtrsim 2$ (Madau et al. 1996), and the analysis of the HST deep field survey of galaxies with $1 < z < 2$ (Connolly et al. 1997)). According to these studies, the global star formation rate strongly increases with redshift, peaking at $z \sim 2$ (see Fig. 1 below reproduced from Connolly et al. (1997), with points from Madau et al. (1996) corrected for extinction as in Boyle & Terlevich (1997)).

In a Euclidean space, the remote sources would contribute more and more to the energy density with distance r , as $\propto r$, leading to the photometric paradox; in a FRW expanding Universe this is not the case. However, the effects of proper density evolution of galaxies (star formation) with redshift, together with the proper evolution of the sources inside galaxies, could compensate for the cosmological energy density dilution. So a priori it is not at all clear if the remote extragalactic binaries can

¹ This limiting frequency is derived assuming integration time T and frequency f such that $fT = 1$; for a fixed integration time (say, 1 year) the frequency resolution is $\Delta f \approx 1/T \sim 10^{-7}$ Hz. Correspondingly,

the limiting frequency for the galactic white dwarf binary confusion limit would decrease as $(fT)^{-3/8}$, typically by 50 times at 1 mHz for 1 year integration time.

be neglected in studying extragalactic binary GW background. This is the motivation to our calculations.

2. GW noise from sources with changing frequency

We start with a short description of how a GW background is formed by a population of frequency-unresolved sources with changing frequency. The amplitude of the stochastic GW background is conventionally measured in terms of the energy density per logarithmic frequency interval related to the critical density to close the Universe

$$\Omega_{GW} = \frac{dE_{GW}}{d \ln f d^3 x \rho_{cr}}, \quad (1)$$

where $\rho_{cr} = 3H_0^2/8\pi \approx 1.9 \times 10^{-29} h_{100}^2 \text{ g cm}^{-3}$, $h_{100} = H_0/100 \text{ [km/s/Mpc]}$ is the present value of the Hubble constant; we shall use geometrical units $G = c = 1$ throughout the paper. As in the case of deterministic GW signals, one may equally use the characteristic dimensionless amplitude of metric variations h_c , which determines the signal-to-noise ratio on the detector $(S/N) = h_c/h_{rms}$. This amplitude is connected with Ω_{GW} through $h_c(f) = (1/2\pi)(H_0/f)\Omega_{GW}^{1/2}$.

To calculate the GW noise produced by some frequency-unresolved sources, we need know the number of sources per logarithmic frequency interval. At first glance, this would require knowing the precise formation and evolution of sources. However, when only GW carries away the angular momentum from the emitting source (as is the case for coalescing binary white dwarf forming the high-frequency part of the galactic binary GW background), the problem becomes very simple and physically clear.

Energy loss causes the change in the GW frequency $\omega = 2\pi f$ of emitting objects. In the case of a binary star with masses of the components M_1 and M_2 in a circular orbit of radius a the negative orbital energy, $E_{orb} = -M_1 M_2 / 2a \sim \mathcal{M} f_{orb}^{2/3}$ (here $\mathcal{M} = (M_1 + M_2)^{2/5} (M_1 M_2)^{3/5}$ is the so-called ‘‘chirp mass’’ of the system) is being lost, and the orbital frequency (hence, the GW frequency, which is twice the orbital one) increases.

For a good approximation, the conditions of star formation and evolution of astrophysical objects in our Galaxy may be viewed as stationary. This is true at least for last 5 billion years. Let the

formation rate of GW sources be \mathcal{R} . Inside the LISA frequency range, $10^{-4} - 10^{-1} \text{ Hz}$, only coalescing binary white dwarfs (WD) and binary neutron stars contribute. Even if binary neutron stars coalesce at a rate of 1/10000 yr in the Galaxy, their total number at any moment of time still should be much smaller than the number of WD binaries, and we restrict ourselves to considering only binary WD. As we showed in our previous paper (Postnov & Prokhorov 1998), the galactic rate of binary WD mergers may be restricted by the statistics of SN type Ia explosions, which are much more numerous than the observed close binary WDs. However, as we show below, the ratio of the extragalactic to galactic GW binary noise is almost independent of the exact value of \mathcal{R} . In real galaxies \mathcal{R} may be the function of

time, which will be accounted for by the star formation history SFR(t).

At some moment of time the number of sources per unit logarithmic frequency interval is

$$dN(t)/d \ln f \equiv N(f, t) = \mathcal{R}(t)(f/\dot{f}). \quad (2)$$

so the total energy emitted in GW per second per unit logarithmic frequency interval at frequency f by all such sources in the galaxy is

$$\begin{aligned} dE_{GW}/(dt d \ln f) &\equiv L(f)_{GW} = \tilde{L}(f)_{GW} N(f, t) \\ &= \tilde{L}(f)_{GW} \mathcal{R}(t) \times (f/\dot{f}), \end{aligned} \quad (3)$$

where $\tilde{L}(f)_{GW}$ is the GW luminosity (erg/s) of the typical source at frequency f , $\tilde{L}(f)_{GW} \propto f^{10/3}$ for binary systems.

For an isotropic background we find

$$\Omega_{GW}(f)\rho_{cr} = L(f)_{GW}/(4\pi \langle r \rangle^2) \quad (4)$$

where $\langle r \rangle$ is the inverse-square average distance to the typical source. Strictly speaking, this distance (as well as the binary chirp mass \mathcal{M}) may be a function of frequency since the binaries characterized by different \mathcal{M} may be differently distributed in the Galaxy. Since the real distribution of binaries of different mass in the Galaxy is poorly known, we can take, as an estimate, the mean photometric distance for a spheroidal distribution in the form $dN \propto \exp[-r/r_0] \exp[-(z/z_0)^2]$ (r is the radial distance to the Galactic center and z the height above the galactic plane) with $r_0 = 5 \text{ kpc}$ and $z_0 = 4.2 \text{ kpc}$ with $\langle r \rangle \approx 7.89 \text{ kpc}$.

For binary stars the energy reservoir radiated in GW is their orbital binding energy, which is a power law of the orbital frequency: $E \propto f^{2/3}$. Hence the frequency change (f/\dot{f}) may be found from the equation $dE/dt = (2/3)E(\dot{f}/f)$. By energy conservation law $dE/dt = (dE/dt)_{GW} + (dE/dt)_{\dots}$ where (...) means other possible loss/gain of energy. Finally, we obtain

$$(f/\dot{f}) = (2/3)E/((dE/dt)_{GW} + (dE/dt)_{\dots}) \quad (5)$$

and

$$L(f)_{GW} = (2/3)\mathcal{R}\tilde{E} \frac{1}{1 + \frac{(dE/dt)_{\dots}}{(dE/dt)_{GW}}} \quad (6)$$

Remarkably, if GW is the dominant source of energy removal, the resulting GW stochastic background depends only on the source formation rate:

$$\Omega_{GW}(f)\rho_{cr} = (2/3)\mathcal{R}\tilde{E}/(4\pi \langle r \rangle^2) \quad (7)$$

For the frequency range under consideration the assumption that only GW emission drives the orbital evolution of a close binary WD is a perfect approximation. The same relates to the assumption tacitly made that the orbit of all such binaries is circular. As we are interested in the blue end of this frequency range, we need not to consider much more complicated evolution of cataclysmic binaries whose orbital periods are always higher than a few tens of minutes.

3. GW noise from unresolved extragalactic binary stars at LISA frequencies

Let the detector's frequency be f . Then sources in galaxies within the comoving volume element $dV(z)$ at redshift z produce an isotropic GW background with the energy density near the detector (at $z = 0$)

$$\rho_{cr} d\Omega_{GW}(f) = \left(\frac{L(f')}{4\pi D(z)^2} \right) n(z) dV(z), \quad (8)$$

where $L(f)$ is the GW luminosity of one galaxy at the comoving frequency $f' = f(1+z)$, $D(z)$ is the photometric distance, $n(z) = n_G F(z)(1+z)^3$ is the comoving density of galaxies at redshift z with factor $F(z)$ describing the galactic density evolution, and n_G is the present day density of galaxies. In the case of no proper evolution $F(z) = 1$.

Transiting in a standard way from integration over proper volume to integration over cosmic time (e.g. Weinberg 1972) we arrive at

$$\rho_{cr} \Omega_{GW}(z^*) = n_G \int_0^{z^*} F(z) \left(\frac{L(f(1+z))}{1+z} \right) \frac{dt}{dz} dz, \quad (9)$$

where z^* marks the beginning of the star formation in the Universe. Noticing that $L(f') \propto E_{orb}(f(1+z))$ scales with redshift as $(1+z)^{2/3}$, using Eq. (9), and assuming that binary WD merging rate \mathcal{R} per proper volume changes with time only due to galactic density evolution $F(z)$, we finally obtain the ratio of extragalactic to galactic GW energy density produced by coalescing binary WD:

$$\frac{\Omega_{GW}(z^*)}{\Omega_{GW}} = 4\pi \langle r^2 \rangle n_G \int_0^{z^*} \frac{F(z)}{(1+z)^{-1/3}} \frac{dt}{dz} dz \quad (10)$$

The present-day galactic density can be rewritten in terms of Ω_b , the baryon fraction in stars relative to the critical density, assuming the stellar mass of the typical galaxy to be $M_G = 10^{11} M_\odot$:

$$n_G \approx 0.013 (\text{Mpc}^{-3}) \left(\frac{\Omega_b}{0.005} \right) h_{100}^2. \quad (11)$$

Here Ω_b is normalized to the observed present-day baryon density of luminous matter ($\Omega_b = 0.003$ inside bulges of spiral and in elliptical galaxies, $\Omega_b = 0.0015$ in disks of spiral galaxies; see Fukugita et al 1996, 1997 for further details). For $h_{100} = 0.7$ the numerical coefficient in (11) is in a good agreement with the measured local galactic density 0.0048Mpc^{-3} (Loveday et al. 1992).

The time-redshift relation in a FRW cosmological model with arbitrary matter (Ω_M) and vacuum (Ω_Λ) density is

$$dt = \frac{1}{H_0} \frac{dz}{(1+z) \sqrt{(1+z)^2 (1 + \Omega_M z) - z(2+z) \Omega_\Lambda}} \quad (12)$$

(notice that Ω_M includes both visual baryon density Ω_b and dark matter).

For a flat ($\Omega_M = 1$) FRW universe without cosmological term ($\Omega_\Lambda = 0$) $dt/dz = (1+z)^{-5/2}$ and without source evolution ($F(z) = 1$) the integrand in Eq. (10) turns into $(1+z)^{-17/6}$, so that²

$$\frac{\Omega_{GW}(z^*)}{\Omega_{GW}} \approx 0.03 \left(\frac{\Omega_b}{0.005} \right) h_{100} \left(\frac{\langle r \rangle}{10 \text{kpc}} \right)^2 [1 - (1+z^*)^{-11/6}]. \quad (13)$$

Clearly, the extragalactic background is small in comparison with the galactic one if no cosmological evolution of global star formation rate is included.

Now we wish to take into account the evolution of WD merging rate with time (redshift). Let $G(t)$ be the time dependence of the event rate of interest after a δ -function-like star formation burst. This function can be considered as a Green function for arbitrary law of star formation SFR(t). Then the comoving event rate at any time can be calculated as

$$\mathcal{R}_G(t) = \int_{t(z_*)}^t \text{SFR}(\tau) G(t - \tau) d\tau \quad (14)$$

(here $t(z_*)$ is the initial moment of star formation). The event rate per proper time observed from a layer dz at redshift z is

$$\mathcal{R}(z) = n(z) \mathcal{R}_G(z) dV(z). \quad (15)$$

Clearly, the evolution of the comoving galactic density $n(t(z))$ can be considered in the same way, but since we derive from observations the comoving luminosity density as a function of redshift, it is impossible to separate the star formation rate inside galaxies and the comoving galactic density evolution without making special model assumptions. We will assume that expressing the comoving density of galaxies through the luminous baryon density Ω_b (11) reflects real global SFR evolution without knowing how precisely has the comoving density of different types of galaxies changed with time. This remains valid until Ω_b is assumed constant. For example, Ω_b may change in models with non-zero cosmological constant at stages when its influence on the expansion of the Universe becomes dominant.

The observed star formation rate evolution SFR(z) was taken from Connolly et al. (1997). In fact, the absolute value of the star formation rate is deduced from the observed UV luminosity density, which requires knowledge of the initial mass function of stars and models of stellar evolution (Madau et al. 1997). For the relative star formation these requirements become weaker (at least in so far as we assume the same parameters of the star formation and evolution at any time). We reproduce this function (normalized such that $\text{SFR}(z=0) = 1$) in Fig. 1 with data from Madau et al. (1996) (open squares) being corrected for extinction as in Boyle and Terlevich (1997) (then the global SFR coincides with the evolution of the comoving density of

² Eq. (14) in paper (Postnov & Prokhorov (1998)) was calculated for $F(z) = (1+z)^3$, so it should be substituted by this equation for non-evolving sources.

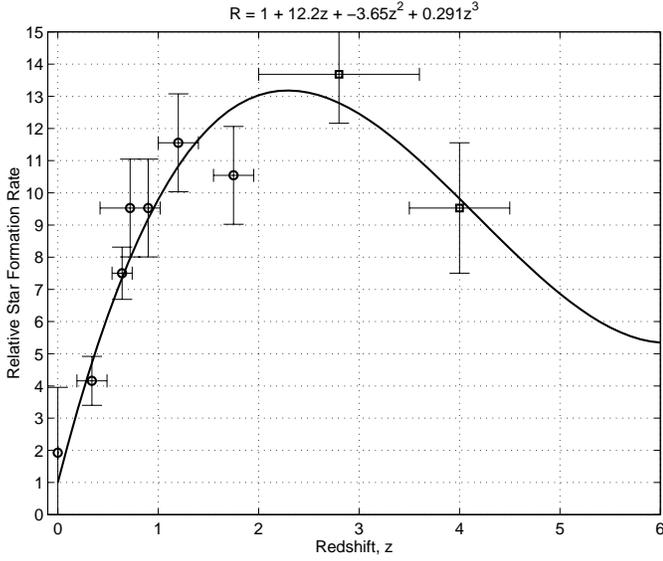


Fig. 1. The evolution of global star formation rate in the Universe from Connolly et al (1997) normalized such that the present star formation rate in solar masses per year per a $10^{11} M_{\odot}$ galaxy is unity. Open squares at large redshifts represent data from Madau et al. (1996) corrected for extinction as in Bovle and Terlevich (1997).

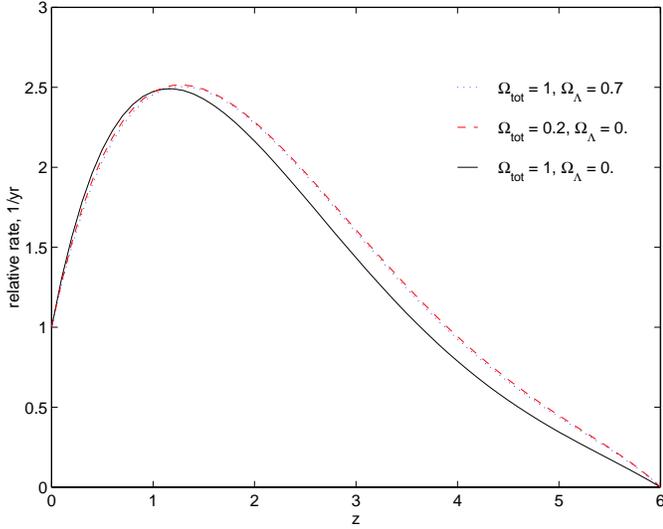


Fig. 2. The dependence of binary WD coalescence rate (normalized to the present Galactic rate) on redshift z for three representative cosmological models: a standard one ($\Omega_{tot} = 1, \Omega_{\Lambda} = 0$), a flat one with cosmological constant ($\Omega_{tot} = 1, \Omega_{\Lambda} = 0.7$), and an open one without cosmological constant ($\Omega_{tot} = 0.2, \Omega_{\Lambda} = 0$).

quasars). For numerical calculations this SFR can be approximated by the polynomial

$$\text{SFR}(z) \approx 1 + 12.2z - 3.65z^2 + 0.291z^3. \quad (16)$$

Fig. 2 shows the dependence of the binary white dwarf coalescence rate per a $10^{11} M_{\odot}$ galaxy on redshift calculated using Eq. (14) in units of the (precisely unknown) galactic rate \mathcal{R}_{wd} for different representative cosmological models: the standard (Λ CDM: $\Omega_{tot} = 1, \Omega_{\Lambda} = 0$), a flat with the cosmological

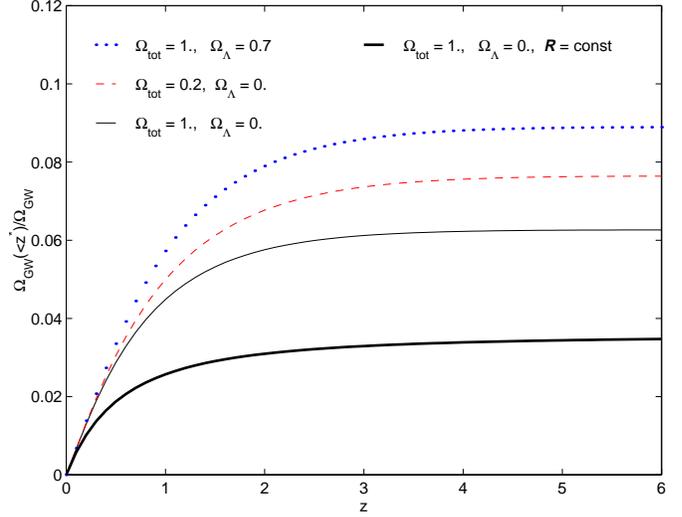


Fig. 3. The ratio of the energy density of the extragalactic stochastic GW background formed by merging binary WD $\Omega_{GW}(< z_*)$ to the mean Galactic level at frequency $f = 0.01$ Hz as a function of parameter z_* , the epoch of the initial star formation in the Universe. The global star formation rate history has the form (16). Cosmological models are the same as in Fig. 2.

constant (Λ CDM: $\Omega_{tot} = 1, \Omega_{\Lambda} = 0.7$, and an open one without the cosmological constant (Λ CDM: $\Omega_{tot} = 0.2, \Omega_{\Lambda} = 0$). The Green function $G(t)$ for binary WD coalescence rate was taken from our numerical calculations (Lipunov & Postnov 1988; Lipunov et al. 1995; Lipunov, Postnov & Prokhorov 1996) which to a good approximation is

$$G(t) = \frac{R_0}{1 + k \frac{t - t(z_*)}{t_H}}, \quad (17)$$

where t_H is the Hubble time, k is a numerical factor, R_0 is a constant calculated assuming that the present Galactic binary WD coalescence rate is the mean value for a constant star formation:

$$\mathcal{R} = \frac{1}{t_H - t(z_*)} \int_{t(z_*)}^{t_H} G(\tau) d\tau \quad (18)$$

It is seen from this figure that the choice of the cosmological model affects this function insignificantly.

After having obtained $\mathcal{R}_G(z)$, we can easily calculate, substituting $F(z) = \mathcal{R}_G(z)$ into Eq. (10), the ratio $\Omega_{GW}(< z_*) / \Omega_{GW}(z = 0)$ for different cosmological models. The results are presented in Fig. 3 for the same models as in the previous figure. For comparison, the thick solid line shows the value $\Omega_{GW}(< z_*) / \Omega_{GW}$ assuming a constant star formation rate in the Universe (Eq. [13]).

Our consideration of the stochastic GW background formation in the process of extragalactic binary WD merging remains valid until redshifts where "boundary effects" of frequency distribution of binary stars are unimportant. In the frequency range $f_{obs} = 10^{-3} - 10^{-2}$ Hz this is always the case because the

maximum proper frequency of the noise $f_{em} = f_{obs}(1+z) < 10f_{em} < 0.1$ Hz for $z_* < 6$.

4. Conclusions

Our results show the importance of taking into account effects of the global star formation rate evolution. It is seen that in some realistic cosmological models the stochastic background produced by extragalactic merging binary WD can be about 0.1 of the mean galactic value. For a 1-year LISA observation the mean (i.e. angle-averaged) Galactic background becomes "transparent" at $f \sim 3 \times 10^{-3}$ Hz, so at higher frequencies the search for other GW backgrounds becomes possible. The level of the extragalactic binary WD background (as well as any noise produced by astrophysical sources) is proportional to the fraction of baryonic matter Ω_b , which lies in the range $0.004 < \Omega_b h_{100}^2 < 0.02$ (Fukugita et al. 1997). Although most baryons are still in the form of ionized gas, we can substitute, as an upper limit, the value $\Omega_b h_{100}^2 = 0.02$ into Eq. (10), thus increasing the extragalactic GW background by four times. Such an extreme situation is feasible, for instance, if all baryons had passed a stellar stage during an early star formation burst at higher redshifts $z \gtrsim 6 - 10$, where spheroidal systems formed rapidly (the so-called "third population stars"; e.g. the model of Eggen, Lynden-Bell & Sandage 1962). The evolutionary history of the oldest ellipticals and low-surface brightness galaxy may also differ significantly from the global average star formation discussed above. The traces of the early star formation is difficult to obtain by direct studies of the UV luminosity density evolution because of a strong dust extinction. Perhaps, the very detection of an isotropic GW background, together with independent studies of far-IR background, would help revealing the true star formation history at high redshifts.

At present, the lack of observational data on the SFR behaviour at high redshift does not allow us to make more robust estimates. We conclude that unless the global star formation rate goes on increasing with a redshift at $z > 3$, the extragalactic GW background energy density $\Omega_{GW} \sim 10^{-9}$ is ten times smaller than the mean galactic value in the LISA frequency range $10^{-4} - 10^{-1}$ Hz.

The galactic binary GW noise will be modulated by the LISA orbital motion, whereas the extragalactic one will not. If the latter is comparable with galactic values at some galactic latitudes, it can impede detection by LISA of some interesting relic cosmological GW backgrounds (e.g. Grishchuk 1997), and specific statistical features of the relic GW should be used to separate them against the noise from astrophysical sources.

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