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On the warping of Be star discs

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Abstract. The theory of radiatively-induced warps in accretion discs is applied to the discs of Be stars. It is found that these discs may develop warps in their inner regions, although once the warp amplitude is large enough then the interaction between the disc and fast radiatively-driven wind will determine its evolution. The warping is shown to be more important for later than earlier B stars. Although the interaction of the fast-wind with the disc will limit the amplitude of the warp, it cannot drive the warp radially outwards, and so the radial evolution of the warp depends on the dominant advective process within the disc.

Typical timescales associated with growing modes are shown to be short, of the order of days-weeks, although these are not likely to be the timescales inferred from observations of line-profile variations which are much longer, of the order of years.

Key words: stars: emission-line, Be – stars: rotation – circumstellar matter

1. Introduction

Be stars are accepted to have a dense, slowly expanding disc in their equatorial planes and a fast radiatively-driven wind over their polar regions (see Slettebak 1988 for a review). The emission line profiles generated in the disc have been monitored for many stars and are found to vary (e.g. Dachs 1987). Long-term variations in the ratio of the violet to red components of a line, V/R, are attributed to $m=1$ density waves in a *Keplerian* disc (Okazaki 1991, Papaloizou et al. 1992). Hanucshik (1996) also concludes that the disc is rotationally supported. Using the assumption that the disc is rotating at Keplerian velocities around the central star and is present throughout most of the star's life, Porter (1998) has recently showed that the radial velocity at the inner edge of the disc must be small ($v_r \lesssim 0.01 \mathrm{km\,s^{-1}}$). The conclusion of these studies is that the disc material orbits many times before it travels a significant radial distance - similar conditions to the material in an accretion disc.

Pringle (1996), Malony, Bagelman & Pringle (1996) and Armitage & Pringle (1997) have recently examined the possibility that a warp may be induced in an accretion disc due to the incident radiation field from the central object. They find that if the accretion disc is optically thick at the wavelength at which the disc emission is maximal then the disc may become unstable.

Here it is recognised that the physical situation examined by Pringle and co-workers and that of the Be star disc are similar: the Be star disc is being illuminated from both sides by the central B star; the Be star disc material orbits many times before moving a signifiant radial distance, and observational as well as theoretical studies have shown that at some radius the Be star discs are optically thick (e.g. Chokshi & Cohen 1987, Kastner & Mazzali 1989, Marlborough et al. 1997). Recently, two Be stars have been examined and have shown observational charactertistics of a warped disc (Hummel, 1998). In this paper it is attempted to apply instability criteria for radiatively-driven warping to the discs thought to exist around Be stars.

In Sect. 2 the disc structure is examined and the optically thick regions are identified. The criterion for the radiatively-driven warp instability to arise in the disc is then examined in Sect. 3. The evolution of a warp interacting with fast radiatively-driven wind is considered in Sect. 4. This is discussed in Sect. 5, and conclusions given in Sect. 6.

2. Disc structure

The discs around Be stars have often been modelled with the "disc model" due to Waters (1986). This assumes that the disc density follows a power law in radius and, from mass conservation within the disc, that the radial velocity also has a power law form. Typically, the disc has a constant opening angle θ. Although this model is emperical, is is very successful in reproducing many of the observations of Be star discs. (e.g. Waters 1986, Waters, Coté & Lamers 1987)

However, theoretical models of disc formation and evolution have been less successful. Models have been developed based on magnetism (Poe & Friend 1986), changing wind line-driving parameters (Chen, Marlborough & Waters 1992), wind compression (Bjorkman & Cassinelli 1993, Porter 1997), pulsation (Willson 1986) and viscous decretion (Lee, Saio & Osaki 1991). None of these models may simultaneously account for many of the observed features of discs. Porter's (1998) small upper limit on the radial velocity of the disc at the star-disc boundary appears to favour the viscous decretion disc model, but does not rule out other theories.

Whatever the mechanism which produces the disc, its properties must be very similar to the disc model of Waters (1986); with the new limit on the radial velocity (Porter 1998), the mass-loss rate in the disc is similar to that in the fast wind over the polar regions (also see Okazaki 1997). It is noted that as long as the viscosity is suitably defined then the disc model may be described using identical mathematics as accretion discs (e.g. Pringle 1981) - indeed for the viscous decretion discs (Lee et al. 1991) a viscosity is introduced from the outset. The main difference between the accretion discs and the decretion discs here (viscous or not) is essentially the boundary conditions - here the material drifts outwards; $v_r > 0$ and angular momentum is supplied *from* the star, possibly by pulsations (see Osaki 1986).

2.1. Optical depth of the disc

The model of Waters (1986) is used to represent disc structure. It is assumed to have a constant opening angle θ, and has a density profile of $\rho = 10^{-11}\rho_0(r/R_*)^{-n}$g cm^{-3}, where ρ_0 is the density at the inner edge measured in 10^{-11}g cm^{-3}. The disc temperature is assumed to be $T_d = 0.5T_{\rm eff}$. It is also assumed to be isothermal, although this approximation may be easily relaxed (see Waters 1986 for a discussion on non-isothermality).

From this the optical depth of the disc may be calculated at a frequency ν at which νF_ν at the disc temperature T_d is maximum (where F_ν is the Planck function). Using cylindrical geometry, the optical depth of the disc τ_ν at a radius R is calculated from the prescription in Waters (1986). Using his notation

$$\left.\begin{aligned}
\tau_\nu(R) &= X_\lambda\, X_{*d}\, C\, R^{-2n+1} \\
X_\lambda &= \lambda^2 \frac{\left(1-{\rm e}^{-h\nu/kT_d}\right)}{\left(\frac{h\nu}{kT_d}\right)}\,(g_{ff}+g_{fb}) \\
X_{*d} &= 4.923\times 10^{13}\,\frac{\bar{z^2}}{T_d^{3/2}\mu^2}\gamma\rho_0^2\left(\frac{R_*}{R_\odot}\right) \\
C &= \int_0^\theta \cos^{2n-2}y\; dy
\end{aligned}\right\} \qquad (1)$$

where λ is the wavelength in cm, g_{ff} and g_{fb} are the Gaunt factors for free-free and free-bound emission respectively, $\bar{z^2}$ is the mean squared atomic charge, γ is the ratio of the number of electrons to ions, and μ is the mean atomic mass. The opening angle of the disc is θ and the rest of the symbols take their usual meanings.

Using a selection of model stars spanning the main-sequence B star range various quantities for the model can be calculated (Table 1). The wavelength λ_m at which νF_ν is maximal is displayed in column 7 of Table 1. To calculate the optical depth, it has been assumed that the base density in the discs for all the stars is identical: $\rho_0 = 1$. There seems to be little correlation between the spectral type of the star, and disc tracers such as equivelent width of Hα (see e.g. van Kerkwijk et al. 1995), and so the assumption of a common density normalization for all the stars is unlikely to be seriously in error. The calculation has also assumed that $n = 2.5$, and $\mu = 1.6^{-1}$.

Although the disc is optically thick in line transitions out to several tens of stellar radii (e.g. Chokshi & Cohen 1987, Kastner & Mazzali 1989), Eq. 1 does not yield $\tau > 1$ for λ_m. Therefore the wavelength at which the optical depth is greater than unity $\lambda_{\tau=1}$ at the inner edge of the disc is calculated, as is the fraction of emission at that wavelength to the maximal emission νF_ν. These quantities are presented in columns 8 and 9 of Table 1. The fraction of the disc emission at $\lambda_{\tau=1}$ is small for the earliest B star, rising to 60% for the latest spectral type considered. The effect of this partial optical thickness of the disc is likely to be on the magnitude of the warp, should the disc become unstable (see below). The discs around the earliest (latest) B stars will generate the smallest (largest) amplitude warp. Clearly, if the disc is to become unstable to radiation-induced warping, it must be unstable to these perturbations very close to the star.

3. Disc warping

The ciriterion that radiation-driven warping has been derived by Pringle (1996) and Malony, Bagelman & Pringle (1996). The typical timescale for the radiation torque t_Γ and the viscous timescale for damping warps t_{ν_2} are:

$$t_{\nu_2} = \frac{2R^2}{\nu_2}, \quad t_\Gamma = 12\pi G^{1/2} c \left(\frac{\Sigma M_*^{1/2} R^{3/2}}{L_*}\right) \qquad (2)$$

where ν_2 is the (R,z) component of the disc viscosity (see e.g. Pringle 1992) M_* and L_* are the mass and luminosity of the star, G and c are the gravitational constant and speed of light respectively, and Σ is the disc surface density. For instability, Pringle (1996) estimated that $t_{\nu_2} > 2\pi t_\Gamma$, a result which has been found consistent with numerical simulations (Pringle 1997, Armitage & Pringle 1997).

To make further progress on this problem for *decretion* type discs, however, the angular momentum transport equation must be examined again. For a steady disc, a constant $\mathcal{C}$ is produced when the angular momentum equation is integrated (Pringle's 1981 Eq. 3.8). For decretion discs, this constant is the rate of angular momentum input at the inner boundary of the disc. If the central star rotates with an angular velocity of a fraction f_* of the Keplerian value, then the constant $\mathcal{C}$ should become $\mathcal{C} = (1-f_*)\dot{M}R^2\Omega$. This leads to a modification of Pringle's (1981) Eq. 3.9:

$$\nu_1\Sigma = \frac{\dot{M}}{3\pi}\left[1-(1-f_*)\left(\frac{R_*}{R}\right)^{1/2}\right], \qquad (3)$$

where ν_1 is the (R,ϕ) component of viscosity.

With this and the instability criterion Eq. 2 then becomes

$$\left(\frac{R_*}{R}\right)^{1/2} - (1-f_*)\left(\frac{R_*}{R}\right) - \frac{LR_*^{1/2}}{4\pi G^{1/2} c\eta M^{1/2}\dot{M}} < 0. \qquad (4)$$

Table 1. Stellar parameters for the stars considered from Schmidt-Kaler (1982). The mass loss rates (in $\mathrm{M_\odot yr^{-1}}$) come from the relation by Garmany et al. (1981). λ_m is the wavelength at which νF_ν at the temperature of the disc ($T_d = 0.5T_{\rm eff}$) is maximal. The wavelength at which the disc has an optical of unity at the inner edge is $\lambda_{\tau=1}$. The fraction of disc emission at $\lambda_{\tau=1}$ to that at λ_m is listed in the ninth column. The tenth column gives the growth timescale t_Γ for the warping instability at $r = R_*$ from Eq. 7. The final column gives the bending (from Eq. 12) at which the wind-disc interaction becomes significant.

Spectral type	M_* ($M_\odot$)	R_* ($R_\odot$)	logL_*	$T_{\rm eff}$ (10^4K)	log$\dot{M}_w$	λ_m (μm)	$\lambda_{\tau=1}$ (μm)	emission fraction	t_Γ (days)	$\psi-\theta$ (degrees)
B0	17.5	7.4	4.81	33.9	-7.5	0.22	1.29	0.04	0.5	47
B1	13.0	6.4	4.31	27.5	-8.4	0.27	1.23	0.08	0.9	40
B2	9.8	5.6	3.85	22.4	-9.2	0.33	1.19	0.15	1.7	35
B3	7.6	4.8	3.43	19.0	-9.8	0.39	1.18	0.22	2.6	30
B4	6.4	4.2	3.14	17.3	-10.4	0.42	1.20	0.26	3.3	26
B5	5.5	3.8	2.89	15.7	-10.8	0.47	1.21	0.31	4.3	24
B6	4.8	3.5	2.67	14.4	-11.2	0.51	1.21	0.37	5.4	22
B7	4.2	3.2	2.45	13.3	-11.6	0.55	1.22	0.43	6.7	20
B8	3.4	3.0	2.28	11.2	-11.8	0.67	1.18	0.60	8.0	18

This is essentially different from the expression of Armitage & Pringle (1997) (their Eq. 3) as the full expression of our Eq. 3 has been used instead of the approximation in the case of large R/R_*. As the disc is (partially) optically thick only in the inner parts then the full expression is required.

Setting Eq. 4 equal to zero then defines the critical radius R_c separating regions of instability and stability. With the replacement $r = R_c/R_*$, this becomes

$$\left.\begin{array}{l} r^{1/2} - (1-f_*) - gr = 0 \\ g = \dfrac{3.7\times10^{-2}}{\eta\dot{M}_{-10}}\left(\dfrac{L_*}{L_\odot}\right)\left(\dfrac{M_*}{M_\odot}\right)^{-1/2}\left(\dfrac{R_*}{R_\odot}\right)^{1/2} \end{array}\right\} \quad (5)$$

where $\eta = \nu_2/\nu_1$, and the mass loss rate $\dot{M}_{-10}$ is measured in $10^{-10}\mathrm{M_\odot yr^{-1}}$.

This equation yields Armitage & Pringle's result (their Eq. 5) if the $(1-f_*)$ term is absent. Eq. 5 is a quadratic equation for $r^{1/2}$, and so it may produce zero, one or two solutions depending on the value of g. The solution is

$$r = \left(\frac{1 \pm \sqrt{1-4g(1-f_*)}}{2g}\right)^2. \quad (6)$$

If only one solution is produced, then it is the inner boundary beyond which warping instabilities may occur; if two solutions are produced (both with $r \geq 1$), then the region between them is *stable* to the warping instability. If $4g(1-f_*) > 1$ then no real solutions are obtained. This occurs when Eq. 4 is obeyed for all values of r, i.e. the whole disc may be unstable to warping - in this case the warping occurs everywhere the disc is optically thick.

To calculate the function g the mass loss rate in the disc and η needs to be supplied. This is problematical for both quantities. As the viscosity may not be isotropic (see Pringle 1992), η may not necessarily be unity. Also, although the disc density may be derived via observations of Be star disc continuum emission in the IR (see e.g. Waters 1986) reasonably accurately, the radial velocity in the disc, and hence the mass-loss rate is unknown. Decretion disc models produce low radial velocities (see e.g. Okazaki 1997), and from angular momentum considerations the radial velocity at the stellar surface must be $v_r \lesssim 0.01\mathrm{km\,s^{-1}}$ (Porter 1998, but see Porter's discussion). Low velocities such as these imply mass-loss rates in Be star discs similar to those for the radiation-driven wind over the poles (Okazaki 1997). However, if the disc *is* a viscous decretion disc, then there is no *a priori* reason that its mass loss rate and the fast wind's mass loss rate should be the same as two different mechanisms are acting.

Clearly, there is some uncertainty here. To keep the calculation general, an "ignorance" parameter Δ is introduced such that $\eta\dot{M}_{-10}$ is replaced by $\Delta\eta\dot{M}_w$, where $\dot{M}_w$ is the mass-loss rate of the fast wind over the pole (taken to be that from Garmany et al.'s 1981 emperical relation; column 6 Table 1). With this introduction, the function $4g(1-f_*)$ is calculated for the sample of Be stars and is shown in Fig. 1.

Fig. 1 shows that unless Δ is large, the disc is unstable for all radii where it is optically thick. If the mass loss rate in the disc is large, or the viscosity is very anisotropic, then Eq. 6 does have two real solutions, and a region of the disc will be stable. It is found that the inner solution, where it exists, is located within the star ($r < 1$), and therefore only the second solution is meaningful. This outer solution then provides a minimum radius at which the disc becomes unstable. As an example calculation, the outer solutions are calculated, where they exist and $r > 1$ for values of $\Delta = 100, 1000$ and are displayed in Fig. 2.

3.1. Growth and precession rates

How fast do the unstable modes grow when they are excited? The growth timescale from Eq. 2 t_Γ in years is

$$t_\Gamma = 1.4\left(\frac{R_*}{R_\odot}\right)^{\frac{5}{2}}\left(\frac{M_*}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{L_*}{L_\odot}\right)^{-1}\rho_0\tan\theta\,\left(\frac{R}{R_*}\right)^{\frac{5}{2}-n}. \quad (7)$$

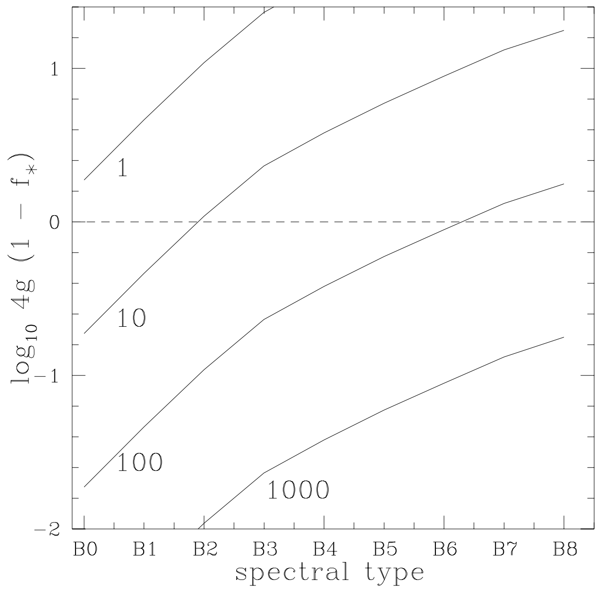

Fig. 1. The function $4g(1-f_*)$ defined in the text. The four lines are labelled with the different values of Δ they correspond to. The dotted line is the limit above which no real solutions are obtained for Eq. 6.

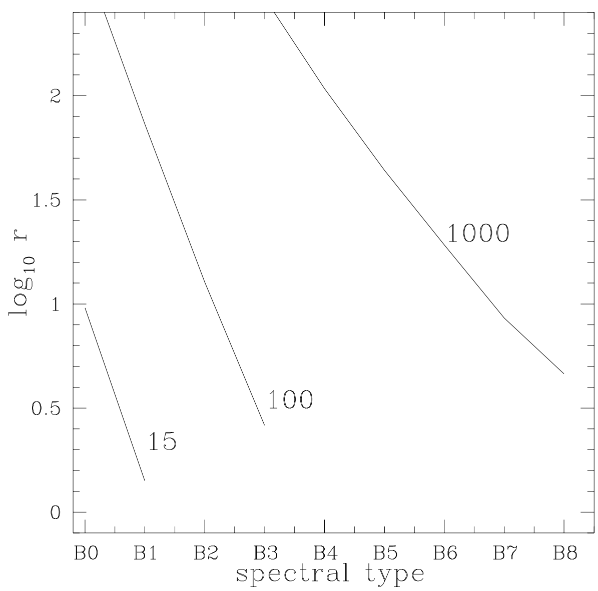

Fig. 2. The outer solution to Eq. 5, for the labelled values of Δ. There are no solutions which have $r>1$ for $\Delta \lesssim 12$ for the stellar parameters in Table 1.

The precession timescale of the warp is $T \approx 4\pi t_\Gamma$ which, when $R/R_* \approx 1$, $\rho_0 \approx 1$ and tan$\theta \approx 0.1$ becomes

$$T \approx 1.7 \left(\frac{R_*}{R_\odot}\right)^{\frac{5}{2}} \left(\frac{M_*}{M_\odot}\right)^{\frac{1}{2}} \left(\frac{L_*}{L_\odot}\right)^{-1} \text{ yr.} \tag{8}$$

The growth timescales are displayed in column 10 of Table 1. It should be pointed out that these timescales are short, typically $\mathcal{O}$(days). Consequently, precesion timescales are $\mathcal{O}$(months). The observational implications of these timescales is returned to later in the discussion.

4. Disc-fast wind interaction

Once the disc starts to warp, it will interact with the fast, radiatively driven wind from the star (which has radial velocities of $\sim 10^3 \mathrm{km\,s^{-1}}$, see e.g. Snow 1981). A bending mode will present a non-negligible area to the fast wind and the fast wind will interact via its ram-pressure. In order to attempt to describe the evolution of the warp in the optically-thin regions of the disc where the radiatively-driven warping mechanism does not operate, some typical timescales are now calculated.

4.1. Latitudinal timescale and magnitude of the warp

The fast wind will interact with the warped disc and attempt to deflect the warp toward the plane of neutral stability - the equatorial plane. This interaction is complex, however, as it depends critically on the amplitude, (or local inclination) of the warp, and is therefore a function of the warp itself. The timescale for this deflection is denoted as the wind-disc timescale t_{wd} and is now estimated. The local disc opening angle is ψ, (for bends ψ is a monotonically increasing function of R), and the opening angle of a stationary disc is θ (see Fig. 1, Porter (1997). The velocity of the wind perpendicular to the disc surface is $v_\perp = v_w \sin(\psi-\theta)$, where v_w is the velocity of the fast wind (which is assumed not to have a meridional component - although see Bjorkamn & Cassinelli 1993, and Owocki, Cramner & Gayley 1996) The ram pressure directed toward the equatorial plane is then $\rho_w v_\perp^2$ where ρ_w is the density of the wind. This pressure acts on an area of $\Phi R\, \delta R/\cos\psi$, where Φ is the angle over which the warp exists (e.g. for a simple inclined disc, $\Phi = \pi$).

The acceleration g_{wd} toward the equatorial plane provided by this interaction is equal to the force due to the ram-pressure (resolved into the z direction), divided by the mass dm on which it acts $dm = \Phi R\, \delta R \rho_d H$, where ρ_d is the disc density and $H = R\,\tan\theta$ is the scale height of the disc. Hence, the acceleration is:

$$g_{wd} = \left(\frac{\rho_w v_w^2}{\rho_d}\right) \frac{\sin^2(\psi-\theta)}{R\tan\theta}. \tag{9}$$

The timescale for "flattening out" warps by this wind-disc interaction depends on how far the disc has moved from the equatorial plane $\delta H = R(\tan\psi - \tan\theta)$. Assuming that both θ and ψ are small, then the timescale is $t_{wd}^2 \sim R(\psi-\theta)/g_{wd}$, or

$$t_{wd} = R \left(\frac{\rho_d}{\rho_w v_w^2}\right)^{\frac{1}{2}} \left(\frac{\theta}{\psi-\theta}\right)^{\frac{1}{2}}. \tag{10}$$

Setting $\rho_w = \dot{M}_w / 4\pi R^2 v_w$, and again using $\rho_d = 10^{-11}\rho_0 (R/R_*)^{-n}$, this timescale (in seconds) becomes

$$t_{wd} = 6.8 \times 10^4 \left(\frac{R}{R_*}\right)^{2-\frac{n}{2}} \left(\frac{R_*}{R_\odot}\right)^2 \frac{\rho_0^{\frac{1}{2}}}{v_8^{\frac{1}{2}} \dot{M}_w^{\frac{1}{2}}} \left(\frac{\theta}{\psi-\theta}\right)^{\frac{1}{2}} \tag{11}$$

where v_8 is the wind velocity in 10^8cm s^{-1}($10^3 \mathrm{km\,s^{-1}}$) and the wind mass-loss rate $\dot{M}_w$ is measured in $10^{-10} \mathrm{M_\odot yr^{-1}}$.

This expression may now be used to estimate the magnitude of bending modes which may grow via the effect of radiation before they are seriously peturbed by the wind-disc interaction. This is done by calculating the bending $(\psi - \theta)$ at which the wind-disc timescale t_{wd} is equal to the radiatively-driven warping timescale t_Γ. This yields

$$\theta(\psi - \theta) = 1.6 \times 10^{-6} \left(\frac{M_*}{M_\odot}\right)^{-1} \left(\frac{R_*}{R_\odot}\right)^{-1} \left(\frac{L_*}{L_\odot}\right)^{2} \times \frac{1}{\rho_0 v_8 \dot{M}_w} \left(\frac{R}{R_*}\right)^{n-1}. \quad (12)$$

Assuming $v_8 \approx 1$, and values of $\dot{M}_w$ from Garmany et al.'s (1981) relation, the values of $\psi - \theta$ are listed in column 9 of Table 1 at the inner edges of the disc ($R/R_* \approx 1$), for $\theta = 0.1$rad - it is clear that large warps can develop before they are seriously affected by the wind-disc interaction. In fact for the early B stars the approximation that ψ is small is severely strained.

4.2. Is the warp driven to larger radii

Clearly, the fast wind will attempt to force the warp to larger radii. The effectiveness of this depends on the ratio of the radial momentum flux in the wind to the gravitational attraction of the disc. If this ratio is greater (less) than unity then the warp will (will not) be driven outwards.

The wind's momentum flux is simply the product of mass-loss rate and the wind velocity, weighted by the ratio of the solid angle of interaction to 4πsteradians. The area of interaction is $\Phi R \delta R \sin(\phi - \theta)/(\cos\theta \cos\psi)$, and so the wind's momentum flux $\mathcal{F}$ is

$$\mathcal{F}_m = \dot{M}_w v_w \left(\frac{\delta R \sin(\psi - \theta)}{2R \cos\theta \cos\psi}\right). \quad (13)$$

The gravitational interaction of the star and the mass over which the wind's momentum flux acts is

$$\mathcal{F}_g = \frac{GM_*}{R^2} dm = \frac{\Phi G M_* \rho_d H \delta R}{R}. \quad (14)$$

Scaling the wind's mass-loss rate and velocity to $10^{-10} \mathrm{M_\odot yr^{-1}}$, and $10^8 \mathrm{cm\ s^{-1}}$ respectively, and using the power law expression for the disc, the ratio of the two terms above is

$$\frac{\mathcal{F}_m}{\mathcal{F}_g} = 5.4 \times 10^{-4} \dot{M}_w v_8 \left(\frac{M_*}{M_\odot}\right)^{-1} \left(\frac{R_*}{R_\odot}\right)^{-1} \left(\frac{R}{R_*}\right)^{n} \times \left(\frac{\sin(\psi - \theta)}{\sin\theta \cos\psi}\right). \quad (15)$$

In the last section it was found that ψ will be tens of degrees when the wind-disc interaction takes placeand so the trigonometric term will numerically be ∼10s. Therefore, it is clear that the fast wind *cannot* drive the warps to large radii.

5. Discussion

From the preceding sections it seems that Be star discs *do* posess the correct attributes to enable them to become unstable to radiatively-driven warping. If this is so, then they should be observable. In fact it may well be that this is indeed the case - Hummel (1998) has indeed interpreted line-profile variations in γCas (B0IVe) and 59 Cyg (B1.5Vnne) to be due to a warped inner disc. This adds a large degree of credibility to the above approach. Hummel *did* consider the radiatively-induced warping mechanism as an explanation for the observations, but rejected the mechanism as he found a prohibitively large mimimum radius for the instability to operate ($10^7 R_*$). The inconsistency between his result and that presented here lies in the application of accretion disc theory to *decretion* discs of Be stars. (This involved approximating the accretion efficiency $\epsilon = L/\dot{M}c^2$ by the ratio of the Schwarzschild radius to the stellar radius, leading to an error of $\sim 5 \times 10^3$. This value is squared to yield the inner radius for instability, which gives an overestimate of $\sim 2 \times 10^7$, bringing Hummel's value into line with that here.)

A feature of the development of the warps which is specific to Be stars is the action of the central star's radiatively driven wind. This introduces an extra (warp dependant) effective viscosity leading to a different evolution of bending modes.

5.1. Development of a warp

In the region of the disc where it is optically thick, radiatively induced warps grow on the timescale t_Γ. These warps are unmolested until they become large amplitude bends (defined from Eq. 12). At this point the wind-disc interaction will start to domiate over the induced warping, and the bending mode will oscillate about the equatorial plane under the action of the fast wind's ram pressure. If it is still within the optically thick part of the disc, then the combined effects of the induced warping, and the wind-disc interaction will limit the oscillation's amplitude. However, when the mode propagates out of the optically thick part of the disc, then the radiation-induced warping becomes inoperable, and the evolution of the mode is determined entirely by the wind-disc interaction. It has been found that the warp will not be driven to large radii under this interaction, and will only propagate outwards via the action of viscosity (for decretion discs) or advection. However, it is difficult to determine whether the warp will be amplified or whether it will decay due to the effects of the wind - further work on this point is necessary (Porter, in preparation).

5.2. Observational signatures

The timescales for the growth of radiatively-driven warps are shown in Table 1. Although modes may be excited over this short timescale, it is not necessarily the timescale over which observed disc tracers indicate the warp. Once a mode propagates outward beyond the optically thick part of the disc, then the timescale for its evolution and precession will be determined by the wind-disc timescale (Eq. 11). Consider a bending mode grows to its maximum amplitude where the wind-disc interaction becomes effective. Once it has propagated out in the disc and its amplitude has decreases, then the timescale the mode evolves at falls (see Eq. 11).

The observational properties of such a mode are not defined simply by the optically thick part of the disc defined in Sect. 2. Typically, the hydrogen lines form within several tens of stellar radii (e.g. Hanuschik et al. 1988) and it is these which are commonly used to infer properties about the disc. Consequently, the warps inferred from the observations are likely to have longer timescales than the growth timescales, and precession rates in Eqs. 7 and 8.

For example, consider a warp which had propagated to $10R_*$, and has $\psi - \theta = 0.1$. The precession timescale for the wind-disc interaction $T = 4\pi t_{wd}$, ranges from 170 days for a B0 star to 4200 days for a B8 star, considerably longer than the timescale over which the mode grows. Therefore it is likely that observationally derived precession timescales will range from several hundreds, to several thousand days. This is broadly consistent with Hummel (1998), who ascribes timescales of ~ 1000days for precession of the discs around γ Cas and 59 Cyg.

6. Conclusion

The theory of radiatively-induced warping of discs has been applied to Be stars and found to be important, especially for late B stars. The timescales associated with the growth of warps is short $\mathcal{O}$(days), although these may not be the relevant timescales over which the modes are examined observationally. It is noted that warping has been observed in Be star discs already (Hummel 1998), and that the observationally derived precession timescales fall in the range in which the simple argument predicts.

A scenario has been presented in which radiatively-driven warps are created in the central parts of the disc and propagate outwards under the influence of the interaction of the disc and fast stellar wind.

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