

## Letter to the Editor

# GRB afterglows: from ultra-relativistic to non-relativistic phase

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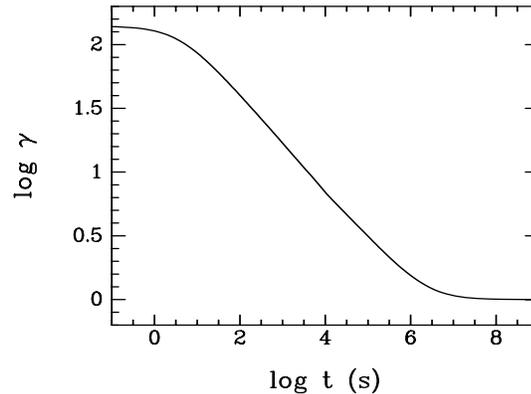
**Abstract.** Postburst evolution of adiabatic fireballs that produce  $\gamma$ -ray bursts is studied. Emphasis has been put on the transition from the highly relativistic phase to the non-relativistic phase, which, according to our calculation, should happen much earlier than previously expected. The theoretical light curves decline a little sharper at the non-relativistic stage than at early times, but still can fit the observations well. However, disagreements are obvious when  $t \geq 100$  d, implying a large initial energy  $E_0$ , a low interstellar density  $n$ , or the possible existence of a persistent energy source at the center of the fireball.

**Key words:** gamma rays: bursts – shock waves – stars: neutron – ISM: jets and outflows – relativity

## 1. Introduction

Since their discovery about thirty years ago (Klebesadel et al. 1973),  $\gamma$ -ray bursts (GRBs) have made one of the biggest mysteries in astrophysics (Fishman & Meegan 1995). The Italian-Dutch BeppoSAX satellite opened up a new era in the field in early 1997. By the end of May 1998, afterglows have been observed in X-rays from about a dozen events, in optical wavelengths in several cases (GRB 970228, 970508, 971214, 980326, 980329, 980519, and possibly GRB 980425), and even in radio from GRB 970508, 980329, and GRB 980519. Very recently, GRB 971214 was reported to have released an enormous energy of  $3 \times 10^{53}$  erg in  $\gamma$ -rays alone (Kulkarni et al. 1998; Wijers 1998); and GRB 980425 seems to be associated with a supernova (Galama et al. 1998b), these facts have aroused researcher's more fever and may provide crucial clues to our understanding of GRBs.

The fireball model has become the most popular and successful model of GRBs (Mészáros et al. 1994; Fenimore et al. 1996, and references therein). After producing the main GRB, the fireball will continue to expand as a thin shell into the interstellar medium (ISM), generating an ultra-relativistic shock. Afterglows at longer wavelengths are produced by the shocked ISM (Mészáros & Rees 1997; Waxman 1997a,b; Tavani 1997; Sari 1997; Wijers et al. 1997; Huang et al. 1998; Dai



**Fig. 1.** Evolution of the fireball's Lorentz factor

& Lu 1998a,b). It has been derived that for adiabatic expansion,  $R \propto t^{1/4}$ ,  $\gamma \propto t^{-3/8}$ , where  $R$  is the shock radius measured in the burster's static frame,  $\gamma$  is the Lorentz factor of the shocked ISM measured by the observer and  $t$  is the observed time. These scaling laws are valid only at the ultra-relativistic stage.

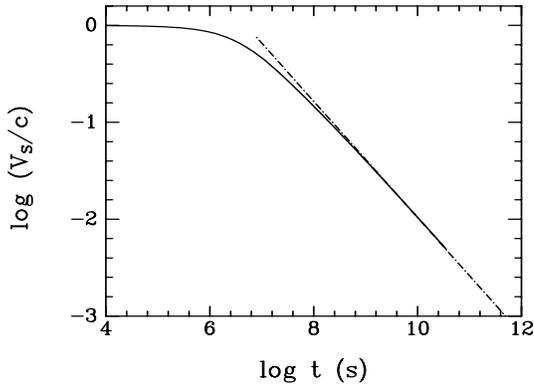
The purpose of this *Letter* is to study numerically the full evolution of adiabatic fireballs from the ultra-relativistic phase to the non-relativistic phase. It is found that radiation during the mildly relativistic phase ( $2 \leq \gamma \leq 5$ ) and the non-relativistic phase ( $\gamma \leq 2$ ), which was obviously neglected in previous studies, is of great importance.

## 2. Previous studies

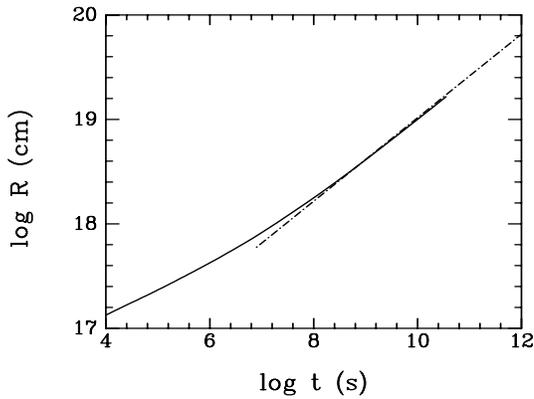
In case of adiabatic expansion, the shocked ISM's Lorentz factor evolves based on (Sari 1997; Waxman 1997a,b; Tavani 1997):

$$\gamma \approx (200 - 400) E_{51}^{1/8} n_0^{-1/8} t_s^{-3/8}, \quad (1)$$

where  $E_0 = E_{51} \times 10^{51}$  erg is the original fireball energy, of which about a half is believed to have been released as  $\gamma$ -rays during the GRB phase,  $n = n_0 \text{ cm}^{-3}$  is the number density of the unshocked ISM, and  $t_s$  is  $t$  in units of second.  $R(t)$  can be derived from  $\gamma^2 R^3 \approx E_0 / (4\pi n m_p c^2)$ , where  $m_p$  is the proton mass and  $c$  the velocity of light. Flux density at observing frequency  $\nu$  then declines as  $S_\nu \propto t^{3(1-p)/4}$ , where  $p$  is the index characterizing the power-law distribution of the



**Fig. 2.** Evolution of the shock's velocity. Full line is our result and dash-dotted line is analytic solution for non-relativistic shock wave (Eq. (9))



**Fig. 3.** The fireball's radius vs. time. Full line is our result and dash-dotted line is analytic solution (Eq. (8))

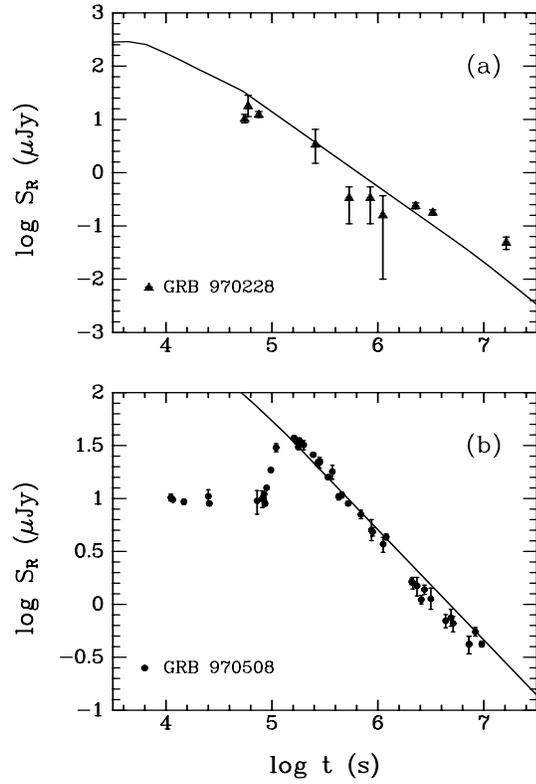
shocked ISM electrons,  $dn'_e/d\gamma_e \propto \gamma_e^{-p}$ . These expressions are valid only when  $\gamma \gg 1$ . In general, X-ray and optical afterglows were observed to follow power-law decays, and such a fireball/blastwave model agrees with observations quite well.

However, we notice that afterglows from GRB 970228 and GRB 970508 have followed simple power-law decays for as long as 190 days and 80 days respectively, while in Eq. (1), even  $t = 30$  d will lead to  $\gamma \sim 1$ . We stress that the overall evolution of the postburst fireball can not be regarded as a simple one-phase process. One should be careful in applying those scaling laws at later times. In fact, it is clear from Eq. (1) that the expansion will become mildly relativistic ( $2 \leq \gamma \leq 5$ ) when  $t \geq 10$  d, and will cease to be relativistic about 30 days later at the latest.

### 3. Our model

We now propose a refined model to describe the full evolution of the postburst fireballs. As usual, we suppose that the main GRB occurs at,

$$R_0 = \left( \frac{3E_0}{4\pi n m_p c^2 \eta^2} \right)^{1/3} = 10^{16} E_{51}^{1/3} n_0^{-1/3} \eta_{300}^{-2/3} \text{ cm}, \quad (2)$$



**Fig. 4a and b.** Optical afterglows in R band: **a** GRB 970228, **b** GRB 970508. Full lines are our theoretical results. Observed fluxes are from Pedersen et al. (1998), Galama et al. (1998a), Garcia et al. (1998)

where  $\eta = 300\eta_{300} = E_0/(M_0 c^2)$ ,  $M_0$  is the mass of the contaminating baryons. In the subsequent expansion, jump conditions for the shock can be described as (Blandford & McKee 1976):

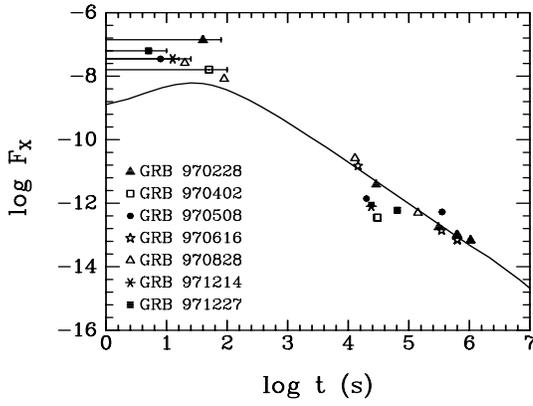
$$n' = \frac{\hat{\gamma}\gamma + 1}{\hat{\gamma} - 1} n, \quad (3)$$

$$e' = \frac{\hat{\gamma}\gamma + 1}{\hat{\gamma} - 1} (\gamma - 1) n m_p c^2, \quad (4)$$

$$\Gamma^2 = \frac{(\gamma + 1)[\hat{\gamma}(\gamma - 1) + 1]^2}{\hat{\gamma}(2 - \hat{\gamma})(\gamma - 1) + 2}, \quad (5)$$

where  $n'$  and  $e'$  are the electron number density and energy density of the shocked ISM respectively in the frame co-moving with the shell,  $\Gamma$  is the Lorentz factor of the shock, and  $\hat{\gamma}$  is the adiabatic index of the ISM, for which we have derived an approximation,  $\hat{\gamma} \approx (4\gamma + 1)/(3\gamma)$ , consistent with the requirement that  $\hat{\gamma} \approx 4/3$  for an extremely relativistic blastwave and  $\hat{\gamma} \approx 5/3$  for the non-relativistic Sedov shock. For  $\gamma \gg 1$ , Eqs. (3) – (5) reduce to  $n' = 4\gamma n$ ,  $e' = 4\gamma^2 n m_p c^2$  and  $\Gamma = \sqrt{2}\gamma$ , which are just the starting point of previous studies. Here we expect that our equations are valid for describing relativistic shocks as well as non-relativistic blastwaves.

The kinetic energy of the shocked ISM in the fireball is  $E_k = \sigma \beta^2 \Gamma^2 (4/3) \pi R^3 n m_p c^2$  (Blandford & McKee 1976), where  $\beta = (1 - 1/\Gamma^2)^{1/2}$ , and  $\sigma$  is a coefficient:  $\sigma \rightarrow 0.35$



**Fig. 5.** X-ray afterglows (0.1–10 keV, in unit of  $\text{erg cm}^{-2} \text{s}^{-1}$ ). Full line is our theoretical result. For the observed fluxes, please see Dai et al. (1998)

when  $\beta \rightarrow 1$  and  $\sigma \rightarrow 0.73$  when  $\beta \rightarrow 0$ . We will use an approximate equation for  $\sigma$ :  $\sigma = 0.73 - 0.38\beta$ . As usual, the expansion is assumed to be adiabatic, during which energy is conserved, so we have,

$$\frac{4}{3}\pi\sigma\beta^2\Gamma^2 R^3 n m_p c^2 = E_k = \frac{E_0}{2}. \quad (6)$$

In order to study the evolution of  $\gamma$ , we should add the following differential equation (Huang et al. 1998),

$$\frac{dR}{dt} = \frac{\sqrt{\Gamma^2 - 1}}{\Gamma} \frac{\gamma}{\gamma - \sqrt{\gamma^2 - 1}} c, \quad (7)$$

Eqs. (2)–(7) present a perfect description of the shock. Given initial values of  $E_0$  and  $M_0$ ,  $R(t)$  and  $\gamma(t)$  can be evaluated numerically. But under the assumption that  $\gamma \gg 1$ , we can derive a simple analytic solution,  $R^4 \approx R_0^4 + 8kt$ ,  $\gamma \approx (cR^3/k)^{-1/2}$ , where  $k = E_0/(4\pi n m_p c)$ . Additionally if  $R \gg R_0$ , that is  $t \gg \tau$ , where  $\tau$  refers to the duration of the main GRB, then  $R \approx 1.06 \times 10^{16} (E_{51}/n_0)^{1/4} t^{1/4}$  cm,  $\gamma \approx 273 (E_{51}/n_0)^{1/8} t^{-3/8}$ . These expressions are consistent with previous studies.

#### 4. Numerical results

We have evaluated the propagation of the blastwave numerically, taking  $E_0 = 10^{51}$  erg,  $n = 1 \text{ cm}^{-3}$ , and  $M_0 = 2 \times 10^{-6} M_\odot$ . Fig. 1 is the evolution of  $\gamma$ , which follows the power-law expression of Eq. (1) quite well when  $10^1 \text{ s} \leq t \leq 10^6 \text{ s}$ . However, it is clear that  $\gamma$  ceases to be much larger than 1 when  $t \geq 10^6 \text{ s}$ , so that Eq. (1) is no longer applicable. At least, it is problematic to assume that the shock wave was still ultra-relativistic, therefore one should be cautious in applying those scaling laws such as  $R \propto t^{1/4}$ ,  $\gamma \propto t^{-3/8}$ , and  $S_\nu \propto t^{3(1-p)/4}$ . This fact has been completely ignored in previous studies.

Fig. 2 illustrates the evolution of the shock wave's velocity ( $V_s$ ).  $V_s$  differs markedly from  $c$  after  $\sim 10^6 \text{ s}$ , and the evolution enters the non-relativistic phase, for which a simple analytic solution is available (Lang 1980):

$$R = 1.15(E_k t^2 / \rho)^{1/5}, \quad (8)$$

$$V_s = 3R/(10t), \quad (9)$$

where  $\rho = n m_p$ . The dash-dotted line in Fig. 2 is plotted according to Eq. (9). Our result is in good agreement with it. Fig. 3 shows our evolution of radius, also plotted are results from Eq. (8). We see from Figs. 3 and 4 that our refined model is applicable for both relativistic and non-relativistic expansion.

To compare with observations, we need to calculate synchrotron radiation from the shocked ISM. As usual, electrons in the shocked ISM are assumed to follow a power-law distribution, with an index of  $p$ ; the magnetic field energy density in the co-moving frame is supposed to be a fraction  $\xi_B^2$  of the energy density,  $B'^2/8\pi = \xi_B^2 e'$ ; and the electrons are supposed to carry a fraction  $\xi_e$  of the energy,  $\gamma_{e,\min} m_e c^2 = \xi_e \gamma m_p c^2$ . The luminosity distance to the GRB source is designated as  $D_L$ . We have plotted in Figs. 4 and 5 the calculated R band flux densities ( $S_R$ ) and 0.1–10 keV fluxes ( $F_X$ ) respectively, and compared them with observed afterglows.

In Fig. 4a, we take  $p = 2.5$ ,  $\xi_B^2 = 0.01$ ,  $\xi_e = 1$ , and  $D_L = 3$  Gpc. Other parameters such as  $E_0$ ,  $n$ ,  $M_0$  are the same as in Fig. 1. Generally the theoretical light curve fits GRB 970228 well. However, a problem appears at later times ( $t \geq 10^7 \text{ s}$ ) when the blastwave becomes non-relativistic. Our model predicts a sharper decline, consistent with the conclusion that has been drawn by Wijers et al. (1997), but the observed flux decays obviously slower. An overall power-law decline lasting for more than  $10^7 \text{ s}$  would require a much larger  $E_0$  or a much smaller  $n$  to ensure that the blastwave is uniformly ultra-relativistic. However, due to the limited observational data points, it is arbitrary to make any further conclusions.

For the optical afterglow from GRB 970508, a set of physical parameters have recently been carefully derived as:  $E_0 = 3.7 \times 10^{52}$  erg,  $n = 0.035 \text{ cm}^{-3}$ ,  $\xi_e = 0.13$ ,  $\xi_B^2 = 0.068$ ,  $p = 2.2$ ,  $D_L \approx 4$  Gpc (Wijers & Galama 1998; Galama et al. 1998c,d). In Fig. 4b, we have taken these values, with  $M_0 = 7.4 \times 10^{-5} M_\odot$ . Since  $E_0$  is large and  $n$  is small, the period during which the ultra-relativistic approximation is valid is substantially extended. In fact,  $\gamma$  keeps to be larger than 1.5 even when  $t > 1.2 \times 10^7 \text{ s}$ , and the theoretical light curve are well presented by a single straight line, which fits observations satisfactorily. We note that if  $p = 2.3$  were assumed, the observational data could be reproduced even better. The afterglow from GRB 970508 seemed to decay more slowly after about 80 days, implying the presence of a constant component (Pedersen et al. 1998; Galama et al. 1998a; Garcia et al. 1998), maybe its host galaxy. The observed optical flux peaked about two days later, possibly associated with an X-ray outburst (Piro et al. 1998). This feature could not be explained by a simple blastwave model. Our theoretical light curve peaks several hours later at optical wavelengths and several tens of seconds later in X-rays.

The theoretical X-ray light curve is plotted and compared with observations in Fig. 5, where parameters are evaluated the same as in Fig. 4a. X-ray counterparts usually fade away in less than two weeks when the expansion is still relativistic, so observed light curves are well represented by power-law relations.

## 5. Discussion

The simple fireball model predicts a power-law decay for GRB afterglows, in rough agreement with observations. However, the fact that the blastwave might enter the non-relativistic phase when the afterglows are still detectable has been widely ignored. Here we stress that the blastwave generated by a typical fireball will cease to be ultra-relativistic in  $\sim 10^6$  s. One should be cautious in applying those simple scaling laws such as  $\gamma \propto t^{-3/8}$ ,  $R \propto t^{1/4}$ , and  $S_\nu \propto t^{3(1-p)/4}$  (for adiabatic expansion) at such late times as  $t \geq 10^6$  s.

We have derived refined equations to describe the full evolution of postburst fireballs. Our model is consistent with previous studies in both the ultra-relativistic phase and the non-relativistic phase. The predicted afterglows decay slightly sharper at the non-relativistic stage. The observed overall power-law decay lasting for several months is usually considered as a strong proof to the popular fireball model. Here we point out that it has in fact raised a problem, since it requires a large  $E_0$  and/or a low  $n$ . For example, recent detection of an enormous burst GRB 971214 with  $E_0 \sim 3 \times 10^{53}$  erg might ameliorate the ultra-relativistic assumption to some extent (Kulkarni et al. 1998; Wijers 1998). The solution could also be that there is a persistent energy source at the center of the GRB source (Dai & Lu 1998b).

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