Astron. Astrophys. 336, L77–L80 (1998)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Theoretical vs. semi-empirical relative ages of globular clusters

L. Pulone[1,2], **M. Salaris**[3,4], **A. Weiss**[3], **and R. Buonanno**[1]

[1] Osservatorio Astronomico di Roma, Via dell'Osservatorio 2, I-00040 Monte Porzio Catone (Roma), Italy
[2] European Southern Observatory, Karl-Schwarzschild-Strasse 2, D-85748 Garching, Germany
[3] Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85748 Garching, Germany
[4] Astrophysics Research Institute, Liverpool John Moores University, Byrom Street, Liverpool L3 3AF, UK

Received 14 May 1998 / Accepted 7 July 1998

Abstract. Theoretical relative ages of galactic globular clusters have recently been challenged by a semi-empirical relation. It was used to point out that tested sets of isochrones were unable to reproduce the relation and yield internally inconsistent relative ages. We find that differential cluster ages derived with the isochrones by Salaris & Weiss (1998) are reliable and internally consistent. We also show that this consistency depends on using the lower absolute ages determined by SW98, which therefore receive more empirical support. Moreover, we discuss the effect of the clusters absolute age on the evaluation of their differential ages, and its connection with the question of their age dispersion.

Key words: stars: evolution – population II – galaxy: evolution – globular clusters: general

1. Two opponent views of relative cluster ages

The determination of galactic globular cluster (GC) ages has recently come into a state of flux, first due to updated stellar physics and lately due to the HIPPARCOS results. Theoretical isochrones were challenged by Buonanno et al. (1998, hereafter BCP), who have presented a self-consistent method for the determination of relative cluster ages, which as much as possible makes use of observational properties and tries to minimize the input from theoretical isochrones. They argue that three tested sets of isochrones fail to reproduce the empirical relation between $\triangle(B-V)_{\rm TO}^{\rm RGB}$ (the (B-V) difference between the TO and the base of the RGB) and [Fe/H] at a given age. Furthermore, relative ages based on brightness or colour differences are inconsistent in all cases. In this *Letter* we will confront the isochrones by Salaris & Weiss (1997, 1998; hereafter SW97 & SW98) with the observational results and show that this set passes the test to a great extent.

SW97 have determined GC ages by means of the traditional theoretical approach of comparing theoretical isochrones with an observed CMD. The differences with respect to related works are (i) that they use the very latest (canonical) input physics, (ii) that also α-element enhancement in the GC composition is taken into account (e.g. in the opacity tables) and (iii) that they determined *absolute* ages only for the very few clusters where both CMD morphology and available data guarantee an accurate dating. The brightness difference between Turn-Off (TO) and Zero-Age Horizontal Branch (ZAHB), $\triangle V_{\rm TO}^{\rm HB}$, was used as the absolute age indicator. The imposed constraints result in a rather small sample of 7 clusters of all metallicities (including disk GC in SW98) suitable for the determination of absolute ages (see SW97 and SW98 for details). All other ages in their sample of 31 GC have been determined differentially with respect to these template clusters by using the (theoretical) dependence of $\triangle(B-V)_{\rm TO}^{\rm RGB}$ on age. For minimizing the errors arising from uncertainties in the adopted colour transformations and the mixing length calibration, SW97 and SW98 evaluated the relative ages only within metallicity groups. Whether the relative ages would also be valid across metallicity group boundaries, SW97 checked in only one case. This and other checks (e.g. availability of two clusters with an absolute age determination in the same metallicity group, comparison of the template clusters distances obtained from the theoretical ZAHB models with main-sequence fitting distances using HIPPARCOS subdwarfs) resulted in the confirmation of the internal consistency of the SW97 and SW98 ages and of the reliability of their theoretical isochrones.

We now recall briefly the BCP method for determining homogeneous relative ages for GC. The first step is to define a sample of coeval clusters in a wide range of metallicities. This requires age determinations, which at least must be able to define correctly what is "coeval". To this scope, BCP used a variant of the vertical method used by SW97, employing a point on the main-sequence 0.05 mag redder than the TO. The sensitivity of this new absolute age indicator ($\triangle V^{0.05}$) and its reliability have been investigated in great detail in BCP. In total, a sample of 11 clusters covering $[{\rm Fe/H}] = -2.2 \cdots -0.5$ was defined, whose absolute ages agreed within ± 1 Gyr. While the absolute age of this group depended on the set of isochrones used (D'Antona

Send offprint requests to: L. Pulone
Correspondence to: L.Pulone

et al. 1997; VandenBerg 1996, private communication to BCP; Straniero & Chieffi 1996, p.c. to BCP), membership within the group was basically invariant. From this, the empirical relation of $\triangle(B-V)_{\rm TO}^{\rm RGB}$ vs. [Fe/H] (at the common age of the group of clusters) is obtained (Fig. 11 of BCP), which in turn can be compared to the predicted one. Here, BCP noted that none of the adopted isochrone sets agreed with the empirical relation. It was also demonstrated that the discrepancy is independent of the colour-transformations employed and thus the inconsistency could arise from the theoretical models themselves.

In the second step 6 clusters of definitely lower age than that of the calibrating group have been used to determine the change in $\triangle(B-V)_{\rm TO}^{\rm RGB}$ with age at given [Fe/H]. The ages of the young clusters relative to the calibrating ones were obtained by means of the $\triangle V^{0.05}$ method. Again, theoretical models have to be used for this step; only theoretical predictions for the stellar brightness, but not for colours, are used. This finally yielded an empirical expression relating age differences with two observational quantities, namely $\triangle(B-V)_{\rm TO}^{\rm RGB}$ and metallicity (Eq. 3 and Fig. 16 of BCP). This relationship is then applied for deriving relative ages for GC whose ZAHB level cannot be properly defined (blue or scarcely populated HB). For all other clusters in the BCP sample the average of the $\triangle V^{0.05}$ and the $\triangle(B-V)_{\rm TO}^{\rm RGB}$ relative ages is used.

Considering that the BCP relationship depends itself on theoretical isochrones which were demonstrated to be inconsistent in at least one respect, it is worthwhile to question the reliability of the BCP-relation, which can be valid only if the theoretical isochrones do not suffer from systematic errors in the definition of the coeval clusters. Second, since SW97 and SW98 claim the internal consistency of their results, both in terms of absolute and relative ages, one could ask whether this result depends on the smaller sample of clusters selected by SW97 and SW98. In the following we will therefore re-derive the BCP relationship using the SW98 isochrones where theoretical input is necessary.

2. Data set and coeval sample

The reference GC data set is that of BCP with the addition of NGC 6171 (a metal-rich halo GC included in SW97) and the exclusion of the three disk GC 47 Tuc, M 71 and NGC 6352. As extensively discussed in Alonso et al. (1997), $\triangle(B-V)_{\rm TO}^{\rm RGB}$ and $\triangle V_{\rm TO}^{\rm HB}$ yield significantly different ages for these clusters, if the typical GC helium content (0.24) is assumed. The problem can be resolved with a nearly solar value (0.27; SW98) for the disk clusters, but this would add an unwanted additional degree of freedom for the semi-empirical relationship investigated here. Moreover, given the strong dependence of the horizontal-branch luminosity on the He abundance, the inclusion of these clusters would affect the present comparison with the models because it would affect the selection of the sample of coeval clusters. The reader should be well aware of this difference between BCP and the present paper, because at the metal-rich end of the relationship $\triangle(B-V)_{\rm TO}^{\rm RGB} vs$[Fe/H] BCP found the largest discrepancies between observations and models.

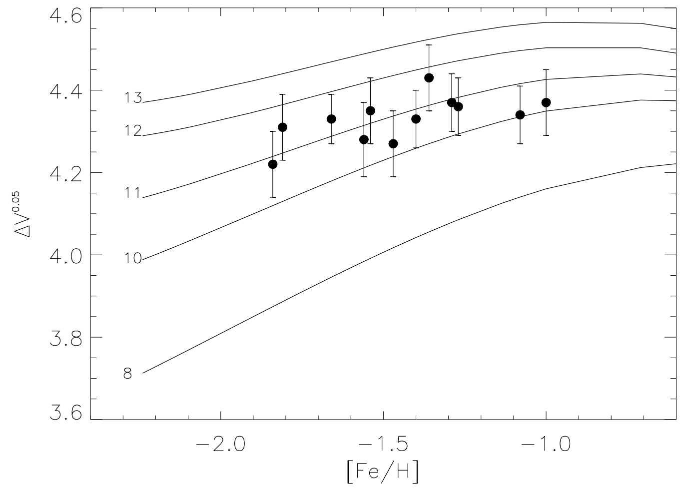

Fig. 1. $\triangle V^{0.05}$ for the 12 coeval clusters compared to the theoretical values by SW98 for several ages (in Gyr)

Adopting this data set we have selected a sample of coeval clusters, on the base of the observed $\triangle V^{0.05}$ values and their errors, and the absolute ages as determined from the SW98 isochrones. The largest possible sample of coeval clusters is different from BCP and it is shown in Fig. 1 (see also data in Table 1); individual ages range from 10 to 12 Gyr and overlap within the errors, providing an average age of 10.9 Gyr. The sample spans the metallicity range -2.0$<[Fe/H]<$-1.0, typical of the bulk of halo GC, and, contrarily to BCP, it does not include any of the most metal poor clusters with [Fe/H]<-2.0. This is in agreement with the results by SW97 and SW98 who found that the most metal poor GC are on average older than the other GC.

The result that the coeval clusters are not the same as in BCP is at odds with their conclusion that the membership to the coeval group does not depend on the isochrones considered. The main reason for this occurrence is the lower absolute age found for the individual GC with the SW98 isochrones. As previously described, the average age of the coeval GC is 10.9 Gyr, while BCP considered an average age of 15 Gyr. As it is evident from Figs. 7 and 8 in BCP, the shape of the theoretical relation between $\triangle V^{0.05}$ and [Fe/H] depends on the absolute age of the isochrones. For example, at ages around 12 Gyr, the derivative $\delta\triangle V^{0.05}/\delta({\rm age})$ at low metallicities is quite different from the case of 14 or 16 Gyr, independently of the set of colour transformations and bolometric corrections used.

If we would have shifted by $\approx$ +0.2 mag the $\triangle V^{0.05}$ values of the isochrones used in BCP (see their Fig. 8) in such a way that the average age of the coeval clusters is around 11 Gyr, we would have excluded the most metal poor ones from the coeval sample, thus recovering the result obtained with the SW98 isochrones.

Another effect of the lower absolute age of the GC is that now we find included in the coeval sample two clusters, Pal 5 and Arp 2, considered as young by BCP.

Table 1. Data for the coeval GC sample

Cluster	$\triangle V^{0.05}$	$\triangle(B-V)_{\rm TO}^{\rm RGB}$	[Fe/H]
NGC 362	4.36±0.07	0.263±0.01	−1.27
NGC 1261	4.37±0.07	0.261±0.01	−1.29
NGC 3201	4.28±0.09	0.243±0.01	−1.56
NGC 5272 (M 3)	4.33±0.06	0.249±0.01	−1.66
NGC 5904 (M 5)	4.33±0.07	0.250±0.01	−1.40
NGC 6101	4.31±0.08	0.239±0.01	−1.81
NGC 6121 (M 4)	4.43±0.08	0.248±0.01	−1.36
NGC 6171	4.37±0.08	0.260±0.01	−1.00
NGC 6362	4.34±0.07	0.264±0.01	−1.08
NGC 6584	4.35±0.08	0.248±0.01	−1.54
Pal 5	4.27±0.08	0.266±0.01	−1.47
Arp 2	4.22±0.08	0.248±0.01	−1.84

3. Differential ages

The first step to evaluate GC relative ages following the technique by BCP is the calibration of a relation between $\triangle(B-V)_{\rm TO}^{\rm RGB}$ and metallicity for the subset of coeval clusters. A linear fit to the data (taking into account the $\triangle(B-V)_{\rm TO}^{\rm RGB}$ error given in Table 1 and a typical error of ±0.15 dex in metallicity) provides

$$\triangle(B-V)_{\rm TO}^{\rm RGB} = (0.028 \pm 0.013)[{\rm Fe/H}] + (0.294 \pm 0.020) \quad (1)$$

The next step is the evaluation of the ratio $\delta/\Delta t_9$, where δ is the difference between the observed $\triangle(B-V)_{\rm TO}^{\rm RGB}$ and the one *expected* on the basis of Eq. (1), and Δt_9 is the relative age (in Gyr) with respect to the coeval clusters, as determined from $\triangle V^{0.05}$. We determined this ratio for the four younger clusters NGC 1851, Pal 12, Rup 106 and Ter 7, (see BCP for a discussion), then averaged these four values (having checked that they are not significantly correlated with metallicity) and get:

$$\delta/\Delta t_9 = -0.0158 \pm 0.0057 \ {\rm mag/Gyr} \quad (2)$$

This quantity is smaller than that of BCP (-0.0093), implying a larger sensitivity of $\triangle(B-V)_{\rm TO}^{\rm RGB}$ to age and therefore smaller age differences for an observed $\triangle(B-V)_{\rm TO}^{\rm RGB}$. By combining Eq. (1) and Eq. (2) we obtain the final relation between $\triangle(B-V)_{\rm TO}^{\rm RGB}$, [Fe/H] and Δt_9:

$$\Delta t_9 = (-0.0158 \pm 0.0057)^{-1} \cdot [(\triangle(B-V)_{\rm TO}^{\rm RGB} - (0.028 \pm 0.013)[{\rm Fe/H}] - (0.294 \pm 0.020)] \quad (3a)$$

This equation allows the proper calculation of the error in Δt_9 by standard error propagation. Taking into account the errors in $\triangle(B-V)_{\rm TO}^{\rm RGB}$ and [Fe/H] as well, one gets errors of the order of ±2.0 Gyr in the derived values of Δt_9 (see the vertical error bars in Fig. 3). Without the errors Eq. (3a) can be simplified to

$$\Delta t_9 = -63.3\triangle(B-V)_{\rm TO}^{\rm RGB} + 1.8[{\rm Fe/H}] + 18.6 \quad (3b)$$

The coefficients depend of course on the absolute age of the coeval sample; for the sake of comparison we recall here that the corresponding values obtained by BCP were (-107.5, 4.3, 33.3).

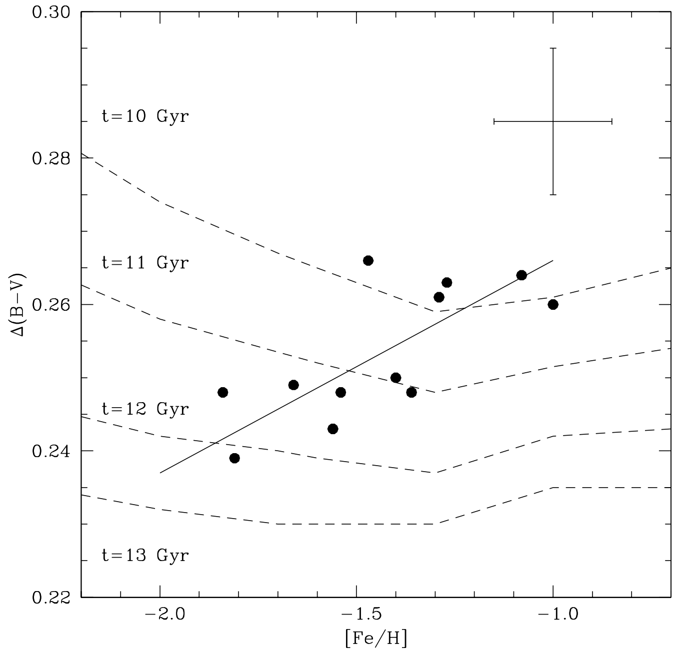

Fig. 2. Predictions of the SW98 isochrones (dashed lines) for $\triangle(B-V)_{\rm TO}^{\rm RGB}$ as a function of [Fe/H], compared with Eq. (1) (solid). Filled circles correspond to the observational points for the coeval clusters. A typical error bar is shown as well)

4. Discussion

In Sect. 1 we recalled that BCP found an inconsistency in the theoretical isochrones by comparing the empirical relation Eq. (1) with them. In Fig. 2 we show the same comparison and, in the considered range of metallicity, we did not find the same problem: the empirical relation crosses a 2-Gyr range in the isochrones, between 10 and 12 Gyr, which is identical to the variation of ages determined from $\triangle V^{0.05}$ (Fig. 1). It is also evident that the spread of the observational points and the errors associated to them (see Table 1) is large enough to make the 11 Gyr isochrone fully compatible with the empirical relation.

To further substantiate our claim that the SW98 isochrones – and therefore the semi-empirical relation derived in the previous section – are trustworthy, we show in Fig. 3 the comparison between the differential ages (with respect to t=10.9 Gyr) obtained with both methods for the 21 clusters in common (the coeval GC with the exception of NGC6121 and NGC6362, the 4 young ones and the blue HB clusters NGC288, NGC1904, NGC2298, NGC6254, NGC6397, NGC6752 and NGC7492) with $-2.0 < [{\rm Fe/H}] < -1.0$ on the Zinn & West (1984) metallicity scale, as adopted in the data set. Before doing so, we had to correct the ages found by SW98 for the different metallicity scale they have used, namely the Carretta & Gratton (1997) one. This new metallicity scale lowers the absolute ages by ≈0.8 Gyr when used instead of the Zinn & West (1984) one (SW98). Therefore, we have increased by this amount the ages given in

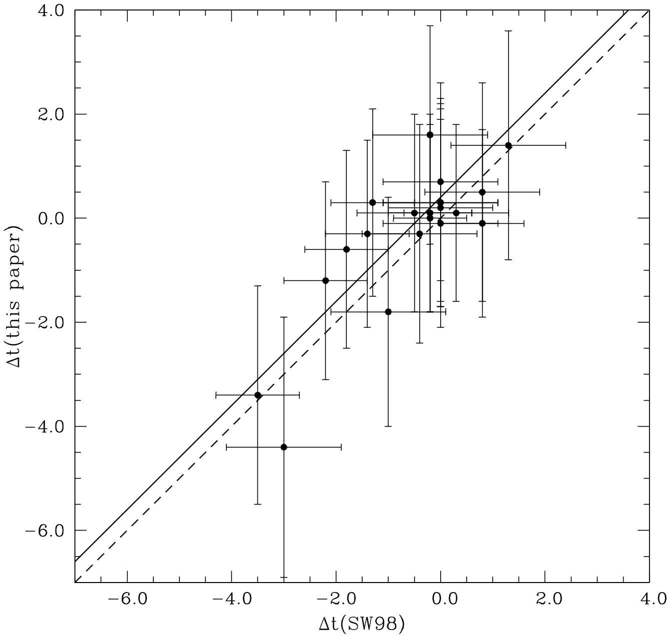

Fig. 3. Comparison of the GC differential ages (in Gyr; with respect to t=10.9) as obtained by SW98 and from Eq. (3a). The dashed line displays the 1:1 relation, the solid one the regression line of the points.

SW98, and we have computed their differential ages with respect to the zero-point of the semi-empirical relative ages, fixed at 10.9 Gyr.

The agreement between the two sets of differential ages is excellent: there is just a zero point shift of 0.4 ± 0.5 Gyr, well within the errors associated with the age determinations. This small zero point shift reflects basically the fact that for the coeval clusters in common (10 out of 12), SW98 find an average age (after the correction for the different metallicity scale) of 10.6 Gyr instead of 10.9 Gyr. When considering only the 7 blue HB clusters, one finds a zero point shift of 0.7 ± 0.8 Gyr, in good agreement with the results from the complete sample of 21 clusters.

The dependence of Eqs. (3a-b) on the absolute age of the GC affects also the derived age spread among the clusters. To quantify whether a real intrinsic age dispersion exists among the clusters, over that due to the errors in the single age determinations, we have performed the F-test (Press et al. 1992) in the way described by Chaboyer et al. (1996) and SW97. It compares the *observed* age distribution with the *expected* one under the assumption of no intrinsic age dispersion, and provides an estimate of the probability that the two distributions are actually the same, taking into account the errors on the individual ages. If this test indicates that the sample is not coeval, the size of the true age dispersion ($\sigma_{\rm real}$) among the clusters can be estimated according to $\sigma^2_{\rm real} = (\sigma^2_{\rm obs} - \sigma^2_{\rm exp})$, where $\sigma_{\rm obs}$ and $\sigma_{\rm exp}$ are, respectively, the 1σ dispersion in the observed age distribution and in the expected one. When considering the calibration of the semi-empirical method given by our Eqs. (3a-b), we find that the complete sample of GC (with the exclusion of the clusters with [Fe/H]$<$-2.0, for which we could not calibrate the method) shows a real 1σ age spread by 0.9 Gyr. Performing the same test with the calibration of the method given in BCP for the same sample of GC, one obtains a 1σ spread by ≈ 2 Gyr.

To conclude, we find that both the theoretical and the semi-empirical approach yield the same relative ages for the cluster sample of BCP when using the isochrones by SW98. In particular, the $\triangle(B-V)^{\rm RGB}_{\rm TO}$ values as a function of age and metallicity predicted by these models are fully consistent with the observations. We also showed how the empirical method and results deduced from it (e.g. the amount of the age spread among GC) depend on the absolute age of the coeval sample and pointed out that the definition of this sample also depends on the GC absolute ages. The consistency of relative ages derived from the isochrones (Figs. 1 and 2) and the agreement with those from the empirical relation is achieved only for the lower ages determined in SW97 and SW98. This result therefore further substantiates the lower ages for GC as found recently in several papers.

Acknowledgements. The work of M.S. was carried out as part of the TMR programme financed by the EC.

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