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ASTRONOMY AND ASTROPHYSICS

Galactic disc tidal action and observability of the Oort cloud comets

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Abstract. We study qualitatively the effect of the galactic disk potential on the comets from the Oort cloud. The problem is examined in the Hamiltonian formalism. A simple pre-selection criterion for calculating the minimal heliocentric distance of the Oort cloud comets is found. We can use this criterion in a simulation to select a sub-population of potentially observable comets. The proposed criterion is very effective—it rejects about 90% of orbits randomly chosen from the initial population of comets.

Key words: celestial mechanics, stellar dynamics – comets: general

1. Introduction

Long-period comets and some of Halley-type comets are believed now to have come from a nearly spherical cloud of comets (the so called Oort cloud, Oort 1950) extending over tens of thousands astronomical units around the planetary system. The comets from the Oort cloud, moving in very long-period orbits, are affected by cumulative perturbations of passing stars, molecular clouds and the galactic tide. Both random and systematical action of all these forces change angular momenta of such comets and may direct them into orbits with perihelia small enough to make these comets observable. The tidal influence of the galactic disk is considered the most important one, dominating over perturbations coming from the remaining effects of the galactic centre, stellar and molecular clouds (Heisler and Tremaine, 1986; Bailey et al., 1990). A simple model of galactic perturbations approximated by the gravitational force of a flat infinite disk with smooth matter distribution, proposed by Heisler and Tremaine (1986), is used in many studies of crucial problems such as the origin of comets, formation and evolution of the Oort cloud or evolution of comets from the Oort cloud to near parabolic observable orbits or to short-period orbits. This model or its generalizations are also applied to study other and more general problems such as the distribution of dark matter in the solar neighborhood, the origin of the planetary system or cometary showers and mass extinctions on the Earth. In many cases the problem is investigated by a computer simulation based on modelling the Oort cloud with Monte Carlo method and numerical integration of equations of cometary motion over the time comparable even with the age of the Solar System (Duncan et al., 1987; Dybczyński and Prętka, 1996). One of the main purposes of these studies is to determine the distribution of observable comets coming from the Oort cloud. Massive Monte Carlo simulations with long time integration are time consuming. To study the problem more efficiently, it was proposed to use averaged equations of motion (Matese and Whitman, 1992; Breiter et al., 1996). This approach is not always well justified. It is especially the case when we study the motion of comets from the outer Oort cloud with semi-major axes of order 10^4 AU. In this case the galactic disk tidal force is dominant and the averaged equations do not properly describe the evolution of a cometary orbit.

The aim of this paper is to present a new approach to this problem. In order to improve the efficiency of Monte Carlo simulations we analyze the problem qualitatively. In most studies we are interested in comets that can appear in the neighborhood of the planetary system. Thus, it is important to select from the original population of comets in the Oort cloud a sub-population of those comets that can potentially pass through the prescribed vicinity of the Sun. We find a very simple criterion of selection. As it will be shown, the criterion is very efficient—it reduces the number of orbits we have to integrate numerically to less than 10% of the total number of orbits. We also present how to use the criterion to establish initial conditions for computer simulations of the observable comet population.

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2. Equation of motion

When the tidal force of the galactic disk is taken into account, Newton's equation of motion of a comet from the Oort cloud can be written in the following form

$$\ddot{x} = -\frac{x}{r^3}, \quad \ddot{y} = -\frac{y}{r^3}, \quad \ddot{z} = -\frac{z}{r^3} + \alpha^2 z, \tag{1}$$

where $r = \sqrt{x^2 + y^2 + z^2}$ and (x, y, z) are the Cartesian coordinates in the reference frame with the center in the Sun and z-axis perpendicular to the galactic disk (see Heisler and Tremaine, 1986, for a detailed derivation). As a unit of distance we choose 10000 AU and such a unit of time that the Sun Gravitational constant is one. In (1) parameter α^2 is proportional to the mean density of stars in the galactic disk ρ, and is given by $\alpha^2 = 4\pi G\rho$. We assume that $\rho = 0.185 \mathrm{M}_\odot \mathrm{pc}^{-3}$ (Bahcall 1984b) and for this value we find $\alpha^2 = 0.00026$.

Instead of system (1) it is more convenient to put the problem into the frame of Hamiltonian formalism. In spherical coordinates the Hamiltonian function for the model considered can be written in the following form

$$\tilde{H} = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\varphi^2}{r^2\cos^2\theta}\right) - \frac{1}{r} + \frac{1}{2}\alpha^2 r^2 \sin^2\theta, \qquad (2)$$

where $(r, \theta, \varphi) \in \mathbb{R}_+ \times (-\pi/2, \pi/2) \times \mathbb{S}^1$ are spherical coordinates and $(p_r, p_\theta, p_\varphi) \in \mathbb{R}^3$ are the corresponding momenta ($\mathbb{R}_+, \mathbb{S}^1$ denote the positive real axis and the unit circle, respectively). Coordinate φ is cyclic, and thus p_φ is the first integral of the system. Using this fact, we can reduce by one the number of degrees of freedom considering p_φ a parameter of the problem. Thus, we will study a Hamiltonian system with two degrees of freedom and with the following Hamiltonian function

$$H = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) + \frac{\gamma^2}{r^2\cos^2\theta} - \frac{1}{r} + \frac{1}{2}\alpha^2 r^2 \sin^2\theta, \qquad (3)$$

where $\gamma = p_\varphi$ is a fixed value of integral p_φ.

From Eqs. (1) it follows that every plane passing through the z-axis is invariant with respect to this system. For orbits lying in such a plane we have $\gamma = 0$. For such orbits, it is natural to assume that in Hamiltonian (3) angle θ is the polar angle, i.e., $\theta \in \mathbb{S}^1$.

The dynamics described by Hamiltonian (3) is complicated. 'Typical' Poincaré cross-sections for regular and chaotic orbits are shown in Fig. 1 and Fig. 2 respectively.

In this work we investigate only the galactic disk perturbation on the motion of comets. We neglect the effects of stellar and giant molecular clouds passages as well as the influence of more realistic, not only vertical, potential of the Galaxy. In a few papers (Heisler and Tremaine, 1986; Morris and Muller, 1985; Prętka, 1998) it was shown that the galactic disk influence dominates over the galactic centre and stellar perturbations when we consider the evolution of comets from the Oort cloud. However, it is necessary to remember that the Galaxy acts continuously on comets while close passing stars and interstellar clouds are randomizing the effects which may produce intense cometary showers in the planetary system (Morris and Muller, 1985) and fill up regions of the outer part of the Oort cloud emptied due to galactic tides. Those stochastic effects should be included in more realistic studies to obtain a complete characteristic of perturbations on the Oort cloud comets.

3. Criterion for minimal distance

Let us consider all orbits with a prescribed energy E and with a given value of the third component of the angular momentum γ. In the phase space of our problem they fill out a three dimensional region defined by

$$\mathcal{M}(\gamma, E) = \{(r, \theta, p_r, p_\theta) \,|\, H(r, \theta, p_r, p_\theta) = E\}. \qquad (4)$$

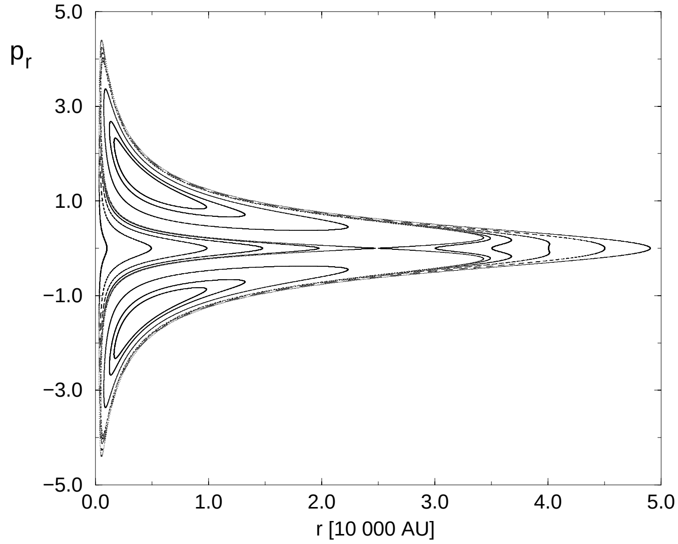

Fig. 1. Poincaré cross-section for regular orbits with energy $E = -0.5$ and $\gamma^2 = 0.05$

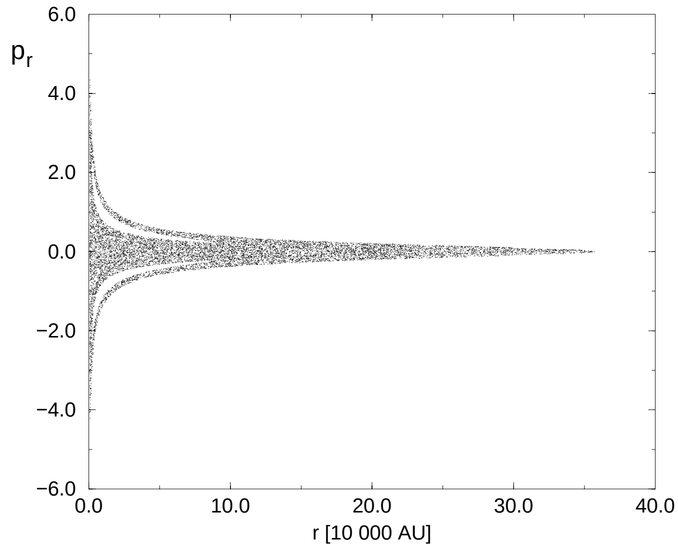

Fig. 2. Poincaré cross-section for chaotic orbits with energy $E = -0.02$ and $\gamma^2 = 0.09$

We ask how close a comet can approach the Sun when moving in one of these orbits. Our question can be stated formally in the following way. We look for $r_{\min}$ defined as follows

$$r_{\min} = \min_{(r,\theta,p_r,p_\theta)\in\mathcal{M}} g(r, \theta, p_r, p_\theta), \qquad (5)$$

where $g(r, \theta, p_r, p_\theta) = r$. Thus, our problem is equivalent to searching a global minimum of function $g(r, \theta, p_r, p_\theta)$ in a domain $\mathcal{M}(\gamma, E)$. Let us notice that the last problem can be formulated also as searching the global constrained minimum of function $g(r, \theta, p_r, p_\theta)$ with $F(r, \theta, p_r, p_\theta) = H(r, \theta, p_r, p_\theta) - E$ as a constraint. To solve the last problem, we introduce the following Lagrange function

$$L = L(r, \theta, p_r, p_\theta, \lambda) = g(r, \theta, p_r, p_\theta) + \lambda F(r, \theta, p_r, p_\theta). \qquad (6)$$

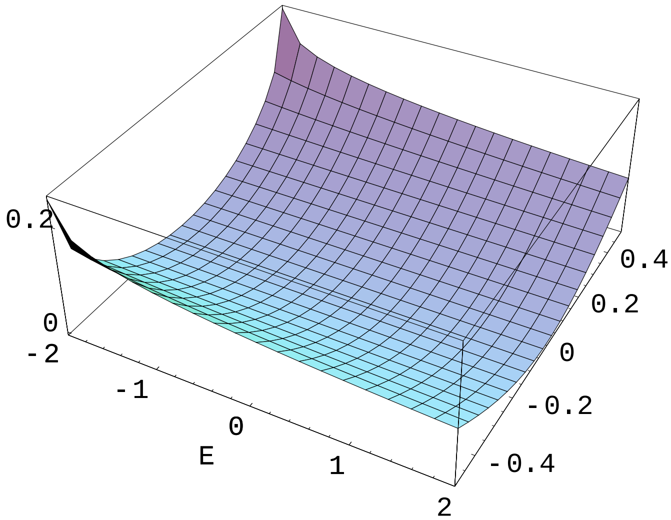

Fig. 3. Function $r_{\min}(\gamma, E)$.

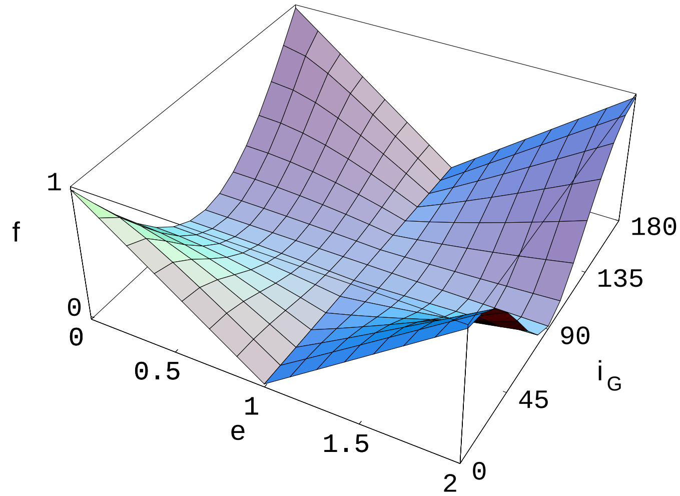

Fig. 4. Function $f(e, i)$.

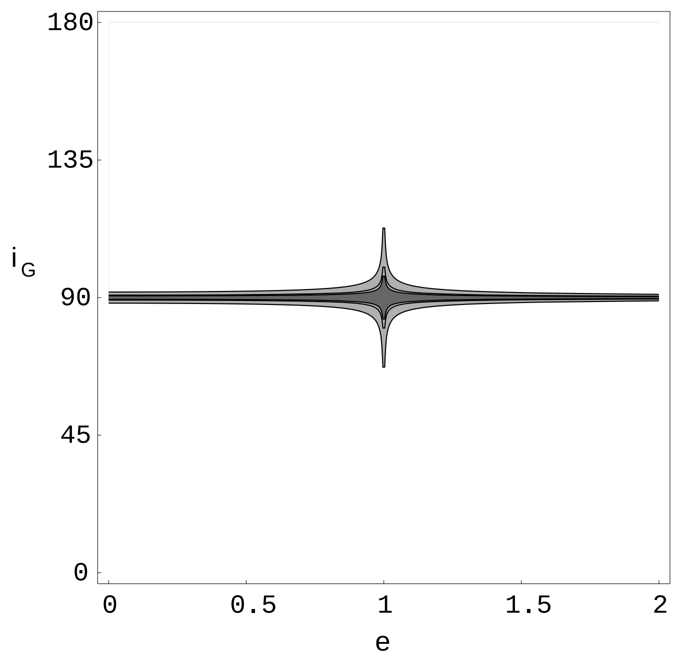

Fig. 5. Constant value contours of f corresponding to $f = 5 \cdot 10^{-4}$, $f = 10^{-4}$ and $f = 5 \cdot 10^{-5}$.

Now, the necessary conditions for a constrained extremum of g are equivalent to the necessary conditions for an ordinary extremum of L, i.e., they have the form

$$\frac{\partial L}{\partial r} = 0, \quad \frac{\partial L}{\partial \theta} = 0, \quad \frac{\partial L}{\partial p_r} = 0, \quad \frac{\partial L}{\partial p_\theta} = 0, \quad \frac{\partial L}{\partial \lambda} = 0. \tag{7}$$

In an explicit form Eqs. (7) read

$$0 = 1 - \frac{\lambda}{r}\left[\frac{p_\theta^2}{r^2} + \frac{\gamma^2}{r^2\cos^2\theta} - \frac{1}{r} - \alpha^2 r^2 \sin^2\theta\right], \tag{8}$$

$$0 = \lambda\left[\frac{\gamma^2}{r^2\cos^3\theta} + \alpha^2 r^2 \cos\theta\right]\sin\theta, \tag{9}$$

$$0 = \lambda p_r, \qquad 0 = \lambda\frac{p_\theta}{r^2}, \qquad H(r, \theta, p_r, p_\theta) = E. \tag{10}$$

From Eq. (8) it follows that the Lagrange multiplier λ is necessarily different from zero. Thus, we have $p_r = p_\theta = 0$. In Eq. (9) the term in the square bracket cannot be equal zero, thus $\sin\theta = 0$. This means that $\theta = 0$. Now, the last equation in (10) has the form

$$E = \frac{\gamma^2}{2r^2} - \frac{1}{r}. \tag{11}$$

This is equivalent to the following quadratic equation

$$h(r) = 2Er^2 + 2r - \gamma^2 = 0. \tag{12}$$

If we assume that $\Delta = 4(1 + 2E\gamma^2) \geq 0$, we find two solutions of this equation $r_\pm$. We can suspect that the smallest positive solution of Eq. (12), i.e.,

$$r_{\min} = \frac{\gamma^2}{1 + \sqrt{1 + 2E\gamma^2}}, \tag{13}$$

gives the desired minimal distance. To show formally that it is true one has to demonstrate that the Hessian of L is positive defined at point $(r_{\min}, 0, 0, 0, \lambda_*)$, where

$$\lambda_* = -\frac{r_{\min}^3}{\gamma^2 - r_{\min}}.$$

Instead of doing this, it is more instructive to analyze the topology of the constant energy manifold $\mathcal{M}(\gamma, M)$. Depending on values of γ and E this manifold can be bound or unbound or it can be an empty set. To analyze this, we write equation $H(r, \theta, p_r, p_\theta) = E$ in the following form

$$2Er^2 + 2r - \gamma^2 = r^2 p_r^2 + p_\theta^2 + \gamma^2\tan^2\theta + \alpha^2 r^4 \sin^2\theta. \tag{14}$$

Now, we make the following change of variables

$$C : \mathbb{R}_+ \times [-\theta_0, \theta_0] \times \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}_+ \times I \times \mathbb{R} \times \mathbb{R},$$

$$C : (r, \theta, p_r, p_\theta) \longrightarrow (r, \hat{\theta}, \hat{p}_r, p_\theta),$$

defined by the following relations

$$\hat{\theta}^2 = \gamma^2\tan^2\theta + \alpha^2 r^4 \sin^2\theta, \quad \operatorname{sgn}\hat{\theta} = \operatorname{sgn}\theta; \quad \hat{p}_r = r p_r. \tag{15}$$

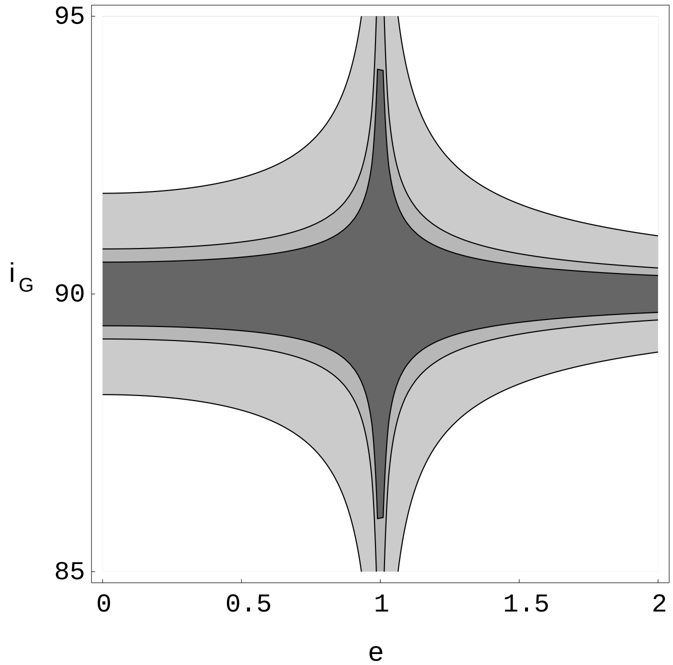

Fig. 6. Magnification of Fig. 5.

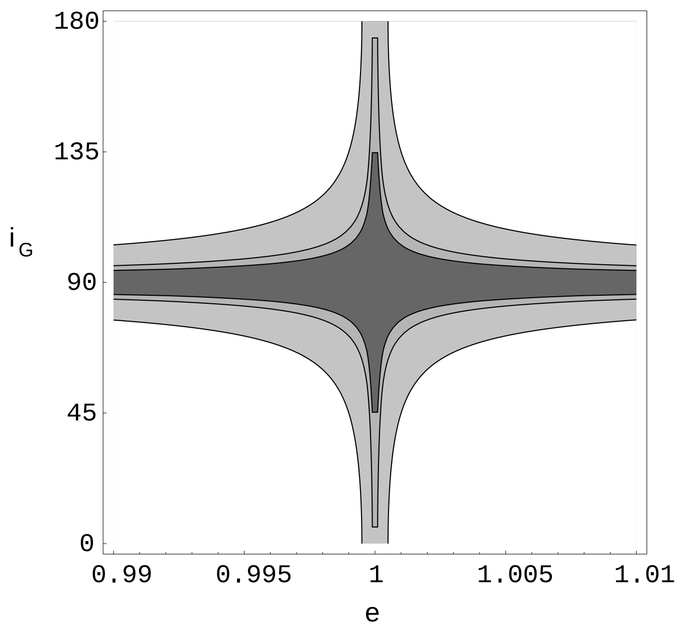

Fig. 7. Magnification of Fig. 5.

In our definition of transformation C we take: $\theta_0 \in (-\pi/2, \pi/2)$ for a case when $\gamma \neq 0$ or $\theta_0 = \pi/2$ when $\gamma = 0$, and we define $I = C([-\theta_0, \theta_0])$. It can be shown that transformation C is a diffeomorphism (or homomorphism when $\gamma = 0$) of respective domains. Thus, we can study the problem in new variables.

In terms of the new variables Eq. (14) has the form

$$2Er^2 + 2r - \gamma^2 = \hat{p}_r^2 + p_\theta^2 + \hat{\theta}^2. \tag{16}$$

Now, to describe $\mathcal{M}(\gamma, E)$ we cut it by a hypersurface $r = r_* > 0$. From (16) it follows that function $h(r) = 2Er^2 + 2r - \gamma^2$

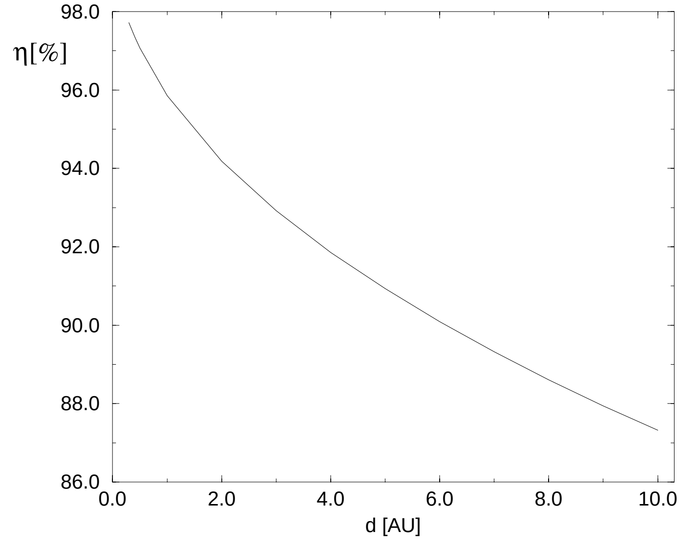

Fig. 8. Percentage of comets randomly chosen from the initial distribution rejected by criterion $r_{\min}(\gamma, E) < d$.

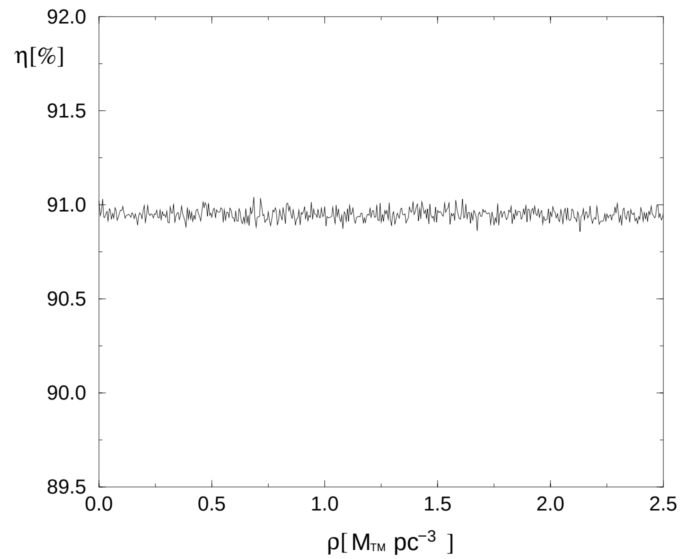

Fig. 9. Efficiency of criterion $r_{\min}(\gamma, E) < 5$ AU for different values of density of stars in the solar neighbourhood.

decides what this cut looks like. We have three possibilities: (i) if $h(r_*) < 0$ then this cut is empty, (ii) if $h(r_*) = 0$ then it is one point $(r_*, 0, 0, 0)$, (iii) when $h(r_*) > 0$ then it is a two dimensional sphere $\mathbb{S}^2$. Using these simple considerations we can describe manifold $\mathcal{M}(\gamma, E)$.

1. If $E \geq 0$ then $\Delta = 4(1+2E\gamma^2) > 0$ and equation $h(r) = 0$ has one non-negative root $r_{\min}$ given by (13); every cut of $\mathcal{M}(\gamma, E)$ by hypersurface $r = r_*$ with $r_* < r_{\min}$ is empty, thus $r_{\min}$ is the minimal value of r in $\mathcal{M}(\gamma, E)$. In this case $\mathcal{M}(\gamma, E)$ is diffeomorphic with $\mathbb{R}^3 \backslash \{0\}$ (when $\gamma = E = 0$) or is diffeomorphic with $\mathbb{R}^3$ (when $\gamma \neq 0$ and $E > 0$).
2. If $E < 0$ then either $\Delta < 0$ and $\mathcal{M}(\gamma, E) = \emptyset$ or $\Delta \geq 0$. In the last case equation $h(r)$ has two positive roots $r_{\min}$ and $r_{\max}$. Because a cut of $\mathcal{M}(\gamma, E)$ by a hypersurface $r = r_*$

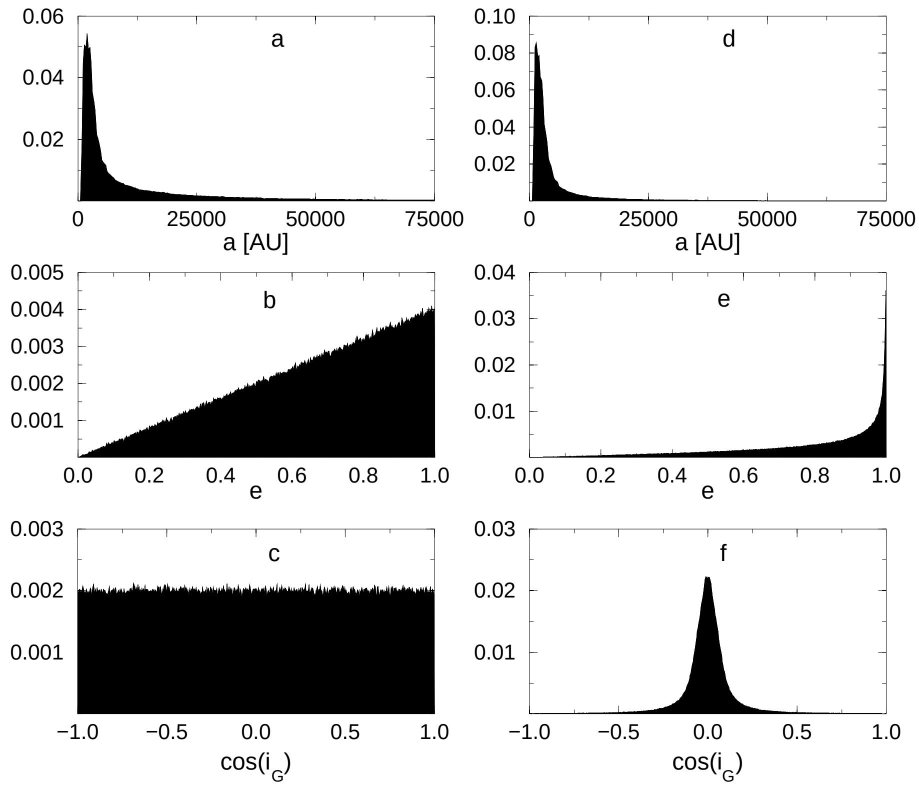

Fig. 10a–f. Initial distributions of a, e and $\cos i$ for comets from the Oort cloud (panels **a**, **b**, **c**, respectively,) and distributions of these elements after rejecting comets which do not satisfy the criterion $r_{\min}(\gamma, E) < 5$ AU (panels **d**, **e**, **f**, respectively).

with $r_* \notin [r_{\min}, r_{\max}]$ is empty, thus the minimal value of r in $\mathcal{M}(\gamma, E)$ is $r_{\min}$ defined by (13). In this case, manifold $\mathcal{M}(\gamma, E)$ is diffeomorphic with a three dimensional sphere $\mathbb{S}^3$.

Thus, formula (13) gives the desired minimal distance from the Sun for comets moving in orbits with the prescribed energy E and the value of z-th component of the angular momentum γ. Fig. 3 shows $r_{\min}$ as a function of E and γ. It is more informative to express $r_{\min}$ in terms of osculating Keplerian elements $\{a, e, i_{\rm G}, \omega_{\rm G}, \Omega_{\rm G}, T\}$ (angular elements are taken with respect to the galactic reference frame). However, the total energy of a comet is a sum of its Keplerian energy and the potential of the disk. This potential depends on the position in the orbit. Thus, in general the criterion for minimal distance expressed in Keplerian elements will be complicated. However, it is possible to obtain a very simple criterion involving only three Keplerian elements $\{a, e, i_{\rm G}\}$ if we make a reasonable assumption concerning the initial distribution of comets. In fact, let us assume that for a given shape of a randomly chosen orbit we can find comets from the total population with an arbitrary position in the chosen orbit. Let us note additionally that the value of γ does note depend on α. Thus, we can ask which of comets with the fixed value of γ and a approach the Sun the closest. This is equivalent to asking about the minimum of the function

$$k(z) = \frac{\gamma^2}{1 + \sqrt{1 + 2E(z)\gamma^2}}, \quad \text{where}$$
$$E(z) = -\frac{1}{2a} + \frac{1}{2}\alpha^2 z^2 . \tag{17}$$

As it is easy to verify it happens when $z = 0$, i.e., when the energy of the comet E is equal to its Keplerian energy $E_{\rm K}$. Thus, in formula (13) defining r_{min} instead of E we can take $E_{\rm K}$. Because of this it is convenient to introduce the following function

$$f = \left(\frac{r_{\min}}{|a|}\right)_{E=E_{\rm K}} = \operatorname{sgn}(1-e)\left[1 - \sqrt{1 - (1-e^2)\cos^2 i}\right] . \tag{18}$$

It is shown in Fig. 4. A typical value of a for long-period comets from the Oort cloud is from 10^4 AU to 10^5 AU. Taking $r_{\min}$ equal to 5 AU we obtain $5 \cdot 10^{-5}$, 10^{-4}, $5 \cdot 10^{-4}$ as the corresponding values of f. In Fig. 5 we show the contours of constant values of f corresponding to $\{5 \cdot 10^{-5}, 10^{-4}, 5 \cdot 10^{-4}\}$. As one can see only comets with the initial inclination very close to 90° can approach the Sun. A magnification of this figure around the line $i = 90^\circ$ is shown in Fig. 4. In Fig. 5 we show a magnification of a very close neighborhood of line $e = 1$. It is remarkable that, potentially, a comet with an arbitrary inclination can approach the Sun, but its initial eccentricity must be very close to 1.

Finally, let us make one remark concerning the case of polar orbits. For such orbits we always have $r_{\min} = 0$ independently of E (see (13)). The reason for this is the fact that for an arbitrary energy E there exists a collisional orbit passing through the origin, e.g. a straight line orbit along the z-axis.

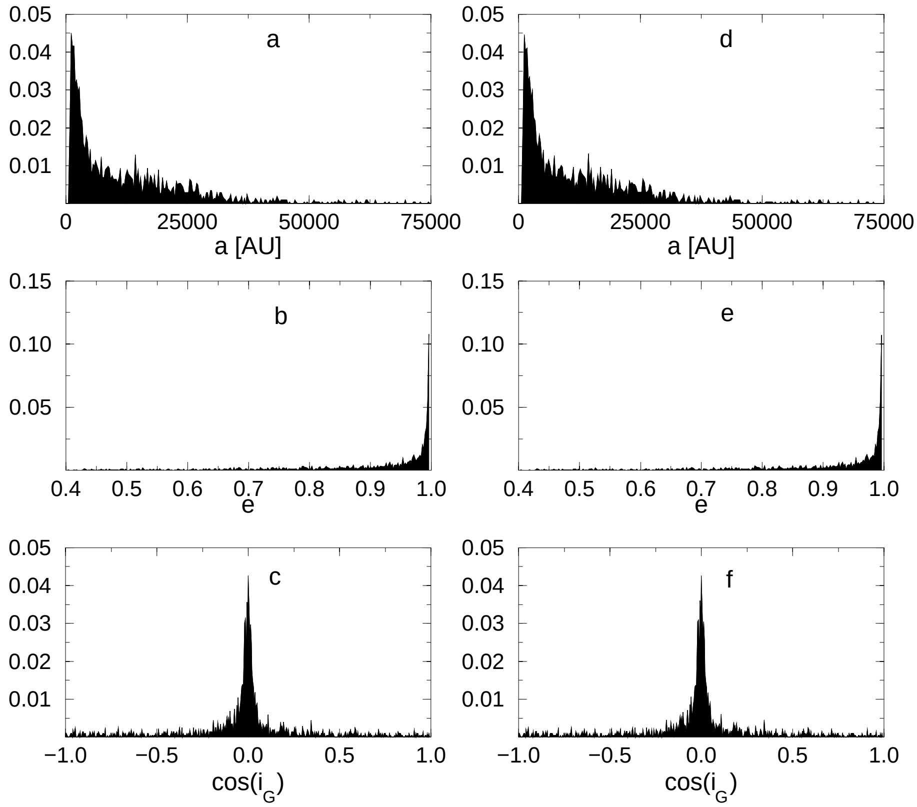

Fig. 11a–f. Resulting initial distributions of a, e and $\cos i$ for observable comets from the Oort cloud obtained from simulations started with elements chosen from Fig. 10a, b and c (panels **a**, **b** and **c** respectively) and from Fig. 10d,e,f (panels **d**, **e**, **f**, respectively).

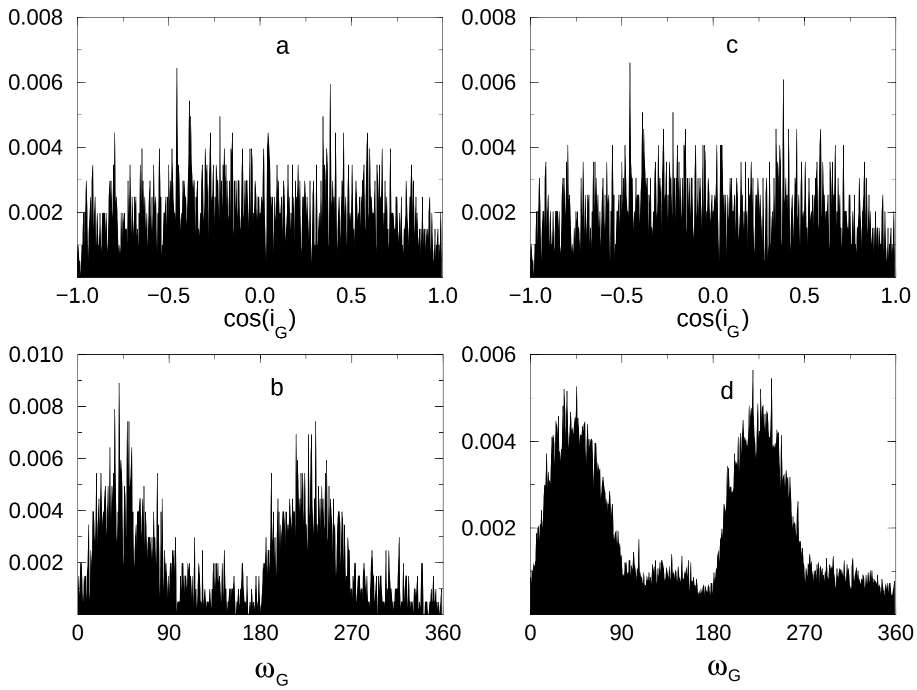

Fig. 12a–d. Resulting final distributions of $\cos i$ and argument of perihelion for observable comets from the Oort cloud obtained from simulations started with elements chosen from Fig. 10a,b and c (panels **a**, **b**) and from Fig. 10d,e,f (panels **c**, **d**).

4. Tests

The results from the previous section can be easily applied in many Monte Carlo simulations. In many works (Matese and Whitman, 1989; Heisler, 1990; Delsemme, 1987; Dybczyński and Prętka, 1997) a big effort was made to determine the distribution of elements of observable comets from the Oort cloud. In such simulations most of the computer time is devoted for integrating the motion of comets which are not observable. The application of our criterion allows to separate the sub-population of potentially observable comets from the rest of the non-observable Oort cloud population.

Below we present an example how to obtain, with the help of our criterion, such an observable sub-population of comets from any initial distribution of comets in the Oort cloud. Because we want to show only the applicability of our criterion, we neglect in simulations all stochastic effects (as stars or giant molecular clouds passing in the vicinity of the Sun) and concentrate on the tidal influence of the galactic disk.

First we estimated the efficiency of our criterion. For this purpose we randomly chose $5 \cdot 10^5$ comets with semi-major axes and eccentricities from distributions based on Duncan et al., (1987); the distribution of inclinations is uniform, see Fig. 10a, b, c. For each comet we calculated E and γ. For a given visibility radius d, we calculated the number of comets which do not satisfy condition $r_{\min}(\gamma, E) < d$. The results are presented in Fig. 8. The criterion is very efficient—for $d < 10$ AU more than 87% of comets do not satisfy the criterion. For $d = 5$ AU (a reasonable value, in many simulations taken as the observability limit for comets) it lets us reject from the simulation more than 90% of comets and, in this way, significantly reduce the time of integration.

Then we checked how the efficiency of the criterion depends on the value of the local matter density in the galactic disk we use in calculations. The problem of determining this value and its uncertainty is caused by the unknown distribution of the unseen dark matter in the Galaxy. In different papers it ranges from 0.05 to even 2.4 depending on a galactic model and the set of observational data (Bahcall et al., 1992; Matese et al., 1995) while the latest value based on Hipparcos data is $\rho = 0.076 \pm 0.015 \mathrm{M}_{\odot} \mathrm{pc}^{-3}$ (Crézé et al., 1998). We repeated the previous test, calculating the number of comets which do not satisfy condition $r_{\min}(\gamma, E) < 5$ AU for values of matter density in the solar neighbourhood from 0 to 2.5 using formulae from Eq. (13). Fig. 9 shows that it is a constant close to 91%. This precisely confirms our observation that for a large class of cometary elements distributions our criterion does not depend on the value of ρ of the model.

Now, we determine the distribution of the sub-population of comets from the Oort cloud which are potentially observable, i.e., which satisfy condition $r_{\min}(\gamma, E) < 5$ AU. To this end, we rejected from initial distributions of a, e and $\cos i$ (Fig. 10a,b and c respectively) comets which do not satisfy the criterion $r_{\min}(\gamma, E) < 5$ AU. As the result we obtained distributions of these elements shown in Fig. 10 panels d, e and f respectively. Fig. 10e and f confirm the previous conclusions that from the whole Oort cloud mainly comets with initial inclination close to $90°$ or initial eccentricity very close to one may approach the Sun to be observable. Comparing original and resulting distributions we may conclude that the sub-population of comets from the Oort cloud which may be observed due to the galactic disk action is very small.

The obtained distributions can be used as initial ones in further simulations of the observable population of the Oort cloud. To check practical applications of our criterion for statistical investigations we made two following simulations. For both of them, as initial conditions, we randomly chose six orbital elements (semi-major axis, eccentricity, inclination, argument of perihelion, longitude of the ascending node and the mean anomaly) and then integrated numerically equations of motion of each comet until it entered the observability region defined as a sphere with the radius equal to 5 AU. If a comet does not approach the Sun during the time span of 500 My (which is comparable with the period of long-term galactic perturbations for comets from the outer part of the Oort cloud), we consider it a non-observable comet. The difference between these simulations was in initial distributions of a, e and $\cos i$. For the first simulation we used the distributions from Fig. 10a, b and c; for the second we applied distributions depicted in Fig. 10d, e, f. The rest of elements in both simulation was chosen from uniform distributions.

The results are presented as the distribution of chosen osculating elements of observable comets. The initial distributions, i.e., when the osculating epoch is the initial time of integration, are shown in Fig. 11; the final distribution, i.e., when the osculating epoch is the time of the close encounter with the Sun, are shown in Fig. 12. For both distributions results of two simulations are in perfect agreement, but, the time of calculation was ten times shorter for the second simulation.

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