Astron. Astrophys. 337, 39–42 (1998)

ASTRONOMY AND ASTROPHYSICS

Young massive stellar populations in M 33

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Received 7 October 1997 / Accepted 27 March 1998

Abstract. In this paper we estimate the ratio OB stars to red supergiants (OB/R) as a function of the distance from the centre of M 33. We found a gradient of the ratio (OB/R) really exists. Our study is based on a new approach in estimating the errors on the ratio (OB/R). On the other hand, by applying a new correlation technique between OB star candidates and red supergiant candidates in the central region of M 33, the gradient becomes pronounced. Our results confirm the existence of such a gradient and we consider this fact as a consequence of a similar gradient of the chemical abundance in M 33.

Key words: methods: data analysis – stars: supergiants – galaxies: M 33 – galaxies: stellar content

1. Introduction

The gas component and the stellar distribution of M 33 have been well studied. Catalogues of HII regions have been published by Boulesteix et al. (1974), Viallefond et al. (1986) and Courtes et al. (1987). CCD surveys of this galaxy have been undertaken by Freedman (1984), Wilson et al. (1990), Wilson (1991) and Regan and Wilson (1993). Extensive photographic catalogues of blue and red stars in M 33 have been published by Humphreys and Sandage (1980) and Ivanov et al. (1993) (hereafter IFM). A list of the Wolf-Rayet stars has been presented by Massey and Conti (1983) and Massey et al. (1987). Massey et al. (1995) have found seven new WN stars and later Massey et al. (1996) (hereafter MBHS96) published a list of previously unknown 28 WR stars. Nowadays the number of known WR stars in M 33 amounts to 168. In the present study we combine those surveys to compare the distributions of massive stars in M 33. We also discuss the ratios of the number of OB star to red-supergiant candidates (OB/R) as a function of distance from the centre of M 33. Maeder et al. (1980) (hereafter MLA) explained the gradient in these ratios as a consequence of the chemical gradient. However serious doubts exist about the presence of the gradient of the ratio (OB/R) in M 33 because of measurement errors (Freedman, 1984). The main goal of this paper is to find observational evidences for a radial gradient of the ratio (OB/R). First, by applying an improved, more realistic technique to determine the error counts, the gradient becomes pronounced. Second, we propose a new correlation technique for comparison of the stellar populations in M 33 which supports the existence of a radial gradient in the ratio OB/R.

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2. Correlation technique

Let N1 stars of one population in M 33 have a surface density δ_1 while another population of N2 stars has a surface density δ_2 and $\mathrm{d_k}$ be the two-dimensional angular distance between the stars of the k-th stellar couple as defined in the Appendix. Supposing a random distribution of the stellar populations, the distance $\mathrm{d_k}$ between the two stars of the k-th couple has a probability (see the Appendix):

$$\mathrm{P_{12}(k)} = \left[1 - \exp\left(-\pi \mathrm{d_k^2} \delta_1\right)\right] \left[1 - \exp\left(-\pi \mathrm{d_k^2} \delta_2\right)\right] \ . \qquad (1)$$

The quantity $\mathrm{P_{12}(k)}$ gives the probability to find one star of population 1 and another star of population 2 within a radius equal to $\mathrm{d_k}$ in case both populations are randomly distributed. The associated stars of two different populations form couples with $\mathrm{d_k} \to 0$ and consequently $\mathrm{P_{12}(k)} \to 0$. The couples of foreground stars have large mutual distances $\mathrm{d_k}$ and $\mathrm{P_{12}(k)} \to 1$. Small values of $\mathrm{P_{12}(k)}$ can be used as a good characteristic for associated couples. We chose upper and lower limits of the probability, $\mathrm{P_{min}}$ and $\mathrm{P_{max}}$. $For \mathrm{P_{12}(k)} < \mathrm{P_{min}}$, couples of associated stars are selected. Those stars for which $\mathrm{P_{12}(k)} > \mathrm{P_{max}}$ as “foreground couples” N_{bgr} are defined. Further in Sect. 4.1 will see that $\mathrm{P_{min}} = 0.05$ and $\mathrm{P_{max}} = 0.95$. When the associated stars are selected by the criterion $\mathrm{P_{12}(k)} < 0.05$ the number of associated couples is indicated as N5. A stronger criterion for selecting the associated stars assumes $\mathrm{P_{12}(k)} < 0.01$ and the foreground couples $\mathrm{P_{12}(k)} > 0.99$. Then the number of associated couples is denoted as N1. A simple way to evaluate the correlation between two populations is to obtain the percentage of associated objects

$$\mathrm{R5 = N5/N} \ , \quad \text{or} \quad \mathrm{R1 = N1/N} \ . \qquad (2)$$

The ratios R1 and R5 are very suitable measures of the correlation between the stellar populations. If all the stars between two populations are associated, then $\mathrm{R5} = 1$ or $\mathrm{R1} = 1$. In the opposite case there are no associated stars between the populations

($\mathrm{R5} = 0$ or $\mathrm{R1} = 0$). The ratios of Eq. 2 are analogous to the conventional coefficient of correlation in the statistics. Another way to evaluate the correlation between two stellar populations is to calculate the ratio of the number of associated objects to the expected number in a random distribution:

$$\mathrm{RN5} = \mathrm{N5}/\mathrm{N_{bgr}}\ , \quad \text{or} \quad \mathrm{RN1} = \mathrm{N1}/\mathrm{N_{bgr}}\ . \tag{3}$$

3. Correlation between stellar populations in M 33

3.1. Correlation between OB stars and other stellar populations

Using UBV-photometry and spectral determinations, MBHS96 have selected 89 OB stars. This is probably only a small fraction of all OB stars in M 33. On the basis of spectroscopy the membership in M 33 of these stars with masses $\approx 40\mathrm{M}_\odot$ is confirmed. Compared with the CCD photometry of Regan and Wilson (1993), the photometry of MBHS96 is more suitable for applying the present method and nearly covers the whole area of M 33. The data presented in Table 1 can be interpreted as an evidence of tight correlations between OB stars, WC stars and the bright HII regions (BR). About 60% of the OB stars are associated with bright HII regions and WC stars. The first correlation is expected since the source for ionization of the gas in a bright HII region are stars earlier than B2. Hence the OB stars selected by CCD photometry should correlate with bright HII regions. The tight correlation between OB stars and bright HII regions is a check of the present criterion is suitable for studying the correlation between stellar populations in galaxies. A correlation between OB stars and WC also exists. The ratios $\mathrm{R5} \approx 0.6$ given by Eq. 2 for WC stars and $\mathrm{RN5} \approx 4$ given by Eq. 3 are high. This can be expected because the progenitors of WC stars have masses $\mathrm{M} \geq 40\mathrm{M}_\odot$, and massive OB stars as well as WC stars must originate from the same sites of star formation.

3.2. Photographic UBV-photometry

Ivanov, Freedman and Madore (1993) (IFM) presented a UBV-photometry of the blue and red stars in M 33 based on plates obtained with the 3.6 m Canada-France-Hawaii Telescope and the 2.0 m telescope of the Bulgarian National Observatory in the Rhodopa Mountains. The completeness limit of this catalogue reaches V = 19.5 mag. IFM have selected 389 red stars with $\mathrm{B-V} > 1.8$. Judging from colour criteria, most of them are probable member of M 33 and are red-supergiant candidates (RSG). Some results based on these data have already been published (Freedman 1985a, 1985b; Ivanov 1991; Georgiev and Ivanov 1997). The photographic UBV-photometry of IFM is not very suitable for selecting massive stars due to its poor accuracy for determining $\mathrm{M_{bol}}$. However it allows to study the correlation between OB stars and red supergiants for different galactocentric distance. This can be seen from the data in Table 2. The photographic data also include a large number OB star-candidates and therefore enable the study of the distribution of probabilities $\mathrm{P_{12}(k)}$. Following Massey et al. (1987), FitzGerald (1970) and Flower (1977), a reddening-free parameter for OB stars is

$$\mathrm{Q} = (\mathrm{U}-\mathrm{B}) - 0.72(\mathrm{B}-\mathrm{V})$$

from which one derives

$$\begin{aligned} E_{B-V} &= (B-V) - 0.33Q + 0.017, \\ \log T_{eff} &= 3.994 - 0.267Q + 0.367Q^2 \\ BC &= 23.493 - 5.926\log T_{eff}\ . \end{aligned} \tag{4}$$

Using Eqs. 4 we have selected 206 stars with $(\mathrm{B-V})_\mathrm{o} < -0.2$, $(\mathrm{U-V})_\mathrm{o} < -1$, $-8.8 < \mathrm{M_{bol}} < -6$, and $\mathrm{E_{B-V}} < 0.34$. Hereafter we call them OB star candidates whereas the stars selected from the CCD UBV photometry are called simply OB stars. The distributions of OB star candidates, OB stars and other stellar populations as a function of the galactocentric distance are given in Table 2.

Table 1. Correlation parameters between OB stars of MBHS96 and other stellar populations in M 33

Correlation parameter	BR	WR	WN	WC	RSG
R5	0.61	042	0.17	0.61	0.45
R1	0.31	0.33	0.12	0.54	0.25
RN5	3.9	1.2	0.24	3.1	1.7
RN1	2.6	0.9	0.18	2.7	1.0
Number of couples	89	89	89	41	89

The contents of the table are as follows:
Column 1 gives the correlation parameters.
Columns 2 give the correlation parameters between OB stars and bright HII regions defined by Eq. 2 and Eq. 3.
Columns 3–5 give the correlation parameters between OB stars and WR stars.
Column 6 gives the correlation parameters between OB stars and RSGs.

Table 2. The number of stars for different galactocentric distances R

R(kpc)	$\mathrm{N_{OB}}$	$\mathrm{N_{MBHS}}$	$\mathrm{N_{WR}}$	$\mathrm{N_{RSG}}$	$\mathrm{N_{WN}}$	$\mathrm{N_{WC}}$
0–1	27	16	10	4	7	4
1–2	44	27	49	31	28	16
2–3	26	14	18	34	5	6
3–4	48	16	31	80	18	2
4–5	24	7	14	74	6	4
5–6	17	7	7	78	2	2
6–7	19	1	7	41	0	2

The contents of table are as follows:
Column 2 gives the number of OB star candidates of IFM.
Column 3 gives the number of OB stars of MBHS96.
Column 4–7 give the number of RSGs and WR stars.

4. Discussion

4.1. Limited probabilities for selection of associated and foreground couples

The individual probabilities $\mathrm{P_{12}(k)}$ can be used for distinguishing associated couples of OB and WR stars from foreground

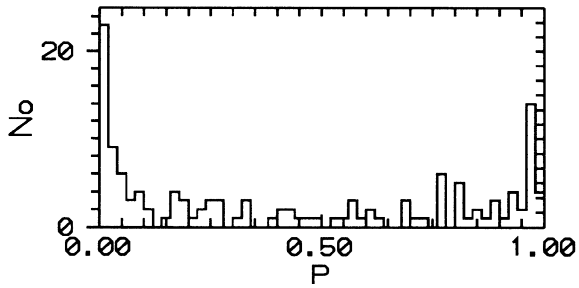

Fig. 1. The distribution of the probabilities $P_{12}(k)$ of couples of OB and WR stars.

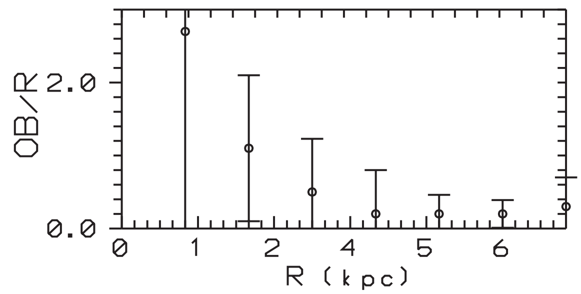

Fig. 2. The ratio OB stars-to-red supergiants as a function of galactocentric distance.

stars. The distribution of these quantities for 206 couples is shown in Fig. 1. The associated stars of the two stellar populations constitute couples with substantially smaller distances d_k than those of the foreground ones. From Fig. 1 we see that associated couples can be selected when $P_{12}(k) < 0.05$. On the other hand, $P_{12}(k) > 0.95$ corresponds to foreground couples. Taking $P_{min} = 0.01$ and $P_{max} = 0.99$ would make a stronger criterion but would mean the lost of many associated couples. We keep the first criterion as a compromise for our purpose.

4.2. The ratio OB stars to RSGs (OB/R)

Humphreys and Sandage (1980) found that the ratio of the number of blue stars- to-red supergiants (B/R) decreases as a function of the galactocentric distance in M 33. Later on Freedman (1984) has taken into account errors on the ratio (B/R) and concluded the result has a low statistical weight in the centre of M 33. She inferred that the data of Humphreys and Sandage (1980) when plotted with their error bars are in apparent contradiction with their previous result. Freedman has calculated the error using the following relations: $s_1 = \frac{N_B+\sqrt{N_B}}{N_{RSG}-\sqrt{N_{RSG}}}$ and $s_2 = \frac{N_B-\sqrt{N_B}}{N_{RSG}+\sqrt{N_{RSG}}}$, where s_1 and s_2 are respectively the maximum and minimum fluctuations of the ratio (B/R). N_B and N_{RSG} are the numbers of blue and red stars respectively in each bin of galactocentric distance. The errors defined in this way are correct if N_B and N_{RSG} are independent statistical variables. But OB stars and RSGs correlate as it is the case M 33 then the errors bars on the ratio (OB/R) are

$$\sigma_i = \sqrt{s_1^2 + s_2^2 - 2r_{12}s_1s_2}\ , \tag{5}$$

where r_{12} is the coefficient of correlation between blue and red stars (see Eadie et al. (1971)). Here we take $r_{12} = R5$, where R5 is given by Eq. 2. This substitution seems approximately correct. Fig. 2 shows the ratio (OB/R) as a function of the galactocentric distance R in the deprojected plane of the galaxy. We used a position angle $PA = 22°$ and an inclination of the plane of the galaxy $i = 57°$. The error bars of the ratio (OB/R) in this figure are not crucial for assessing the existence (or non-existence) of a radial gradient of the ratio (OB/R). They just indicate the statistical weight of the ratios (OB/R) in each radial bin. We define the statistical weight in each bin as:

$$w_i = \overline{\sigma^2}/\sigma_i^2\ , \tag{6}$$

Table 3. The ratios (OB/R), R5 and RN5 between OB stars and RSGs

R(kpc)	(OB/R)	σ_i	(OB/R)	σ_i	R5	σ_{R5}	RN5	N
0–1	2.7	± 2.9	1.6	± 2.3	0.50	± 0.97	5.0	10
1–2	1.1	± 1.03	0.87	± 0.91	0.16	± 0.28	0.3	31
2–3	0.5	± 0.73	0.41	± 0.47	0.31	± 0.49	0.7	26
3–4	0.2	± 0.60	0.19	± 0.20	0.31	± 0.44	15.0	48
4–5	0. 2	± 0.26	0.09	± 0.11	0.58	± 0.78	7.0	24
5–6	0.2	± 0.19	0.09	± 0.10	0.76	± 0.19	13.0	17
6–7	0.3	± 0.40	0.09	± 0.10	0.42	± 0.68	1.6	19

The contents of the table are as follows:
Column 1 gives the bins in galactocentric distances.
Column 2 gives the ratio of the OB star candidates of IFM to RSGs.
Column 3 gives the error bar of the ratio (OB/R) obtained by Eq. 5.
Column 4 gives the ratio of OB stars of MBHS 96 to RSGs.
Column 5 gives the error bar of the ratio (OB/R) based on the data of MBHS96.
Column 6 gives the ratio R5 obtained by Eq. 2.
Column 7 gives the error bar of the ratio R5 obtained by Eq. 5.
Column 8 gives the ratio RN5 obtained by Eq. 3
Column 9 gives the number of couples containing OB star candidates and RSGs.

where $\overline{\sigma} = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\sigma_i^2}$, where n is the number of bins and σ_i are defined by Eq. 5 for each radial bin. We obtained a coefficient of correlation for data in Fig. 2 $r = -0.63 \pm 0.23$. This correlation is not very strong but is statistically significant. Therefore it appears that the observational evidence for a radial gradient of the ratio (B/R) found by Humphreys and Sandage (1980) is real.

4.3. Observational evidences of the evolution of massive stars

Table 1 indicates a tight correlation between WC stars and OB stars. We also obtain tight correlation between WC stars and RSGs ($R5 \approx 0.6$). This result suggests that the WC progenitors with higher masses $M \geq 40M_\odot$ evolve first to RSGs and then loose their envelopes. No other stellar population in M 33 shows a correlation with RSGs as strong as WC stars. If WN stars have a smaller initial mass than WC stars, we should find less WC than WN stars. Indeed in M 33 were found twice more WN (89) than WC stars (41). The small coefficient correlation between WN stars and RSGs (R5 = 0.33) calculated on the basis of MBHS96 data suggests that lower mass stars ($M \approx 30M_{\odot}$) rather evolve directly to WN stars.

The gradient in the number of blue to red stars seen by Humphreys & Sandage (1980) and confirmed by the present study can be interpreted by a chemical abundance gradient effect (Henry & Howard, 1995) as explained by MLA.

5. Summary

The present study explains a correlation technique for comparison of different stellar populations in M 33. The radial gradient of the ratio blue to red stars (B/R) obtained by Humphreys & Sandage (1980) and predicted by the evolutionary models of MLA is confirmed. The present study confirms the existence of the radial gradient of the ratio (OB/R) in M 33. Using a new approach in estimating the errors on the ratio (OB/R) and, on the other hand, the correlation between OB star-candidates and RSGs in the central region of M 33. Considering the results, we believe that the observational evidence for a radial gradient of the ratio (OB/R) is real.

Acknowledgements. This project was supported partially by the grant A-02-049 of ESO C&EE Programme and the grant of the Bulgarian National Foundation. I am grateful to Dr. A. Aparicio and Dr. Yu. Efremov for their constructive suggestions which improved this paper.

Appendix A: derivation of Eq. (1)

Let N_1 objects of one population have a surface density δ_1 while N_2 objects of another population have a surface density δ_2. Both populations occupy the same area of a galaxy. We assume the distribution of the stellar populations in the galaxy to be Poissonian. The coordinates of population 1 objects x_i, y_i (i = 1,2,..., N1) and those of population 2 objects are x_j, y_j (j = 1, 2,..N2). The two-dimensional angular distances are

$$\begin{aligned} d_{ij} &= \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \ , \\ i &= 1, 2, \ldots, N1 \ , \quad j = 1, 2, \ldots, N2 \ . \end{aligned} \quad \text{(A1)}$$

The total number of stellar distances is $N = N_1 x N_2$. The quantities d_{ij} are used in order to identify the couples of closest neighbours between the two populations. The distance of the first couple constituted from the first population 1 star and its nearest neighbour of population 2 is:

$$\begin{aligned} d_1 &= min\{d_{ij}\} \ , \\ i &= 1, 2, \ldots, N1 \ , \quad j = 1, 2, \ldots, N2 \ , \end{aligned} \quad \text{(A2)}$$

The stars of this couple are excluded from the further analysis. Then the distances $d_2, d_3, \ldots, d_k$ are obtained in the same way and the stars of these couples are consecutively excluded also. The distance between the stars of the k-th couple is:

$$\begin{aligned} d_k &= min\{d_{ij}\} \ , \quad i = 1, 2, \ldots, N1 - (k-1) \ , \\ j &= 1, 2, \ldots, N2 - (k-1) \ , \end{aligned} \quad \text{(A3)}$$

In this way a series of increasing distances $d_k, \mathrm{fork} = 1, 2, \ldots, N_{couple}$ are obtained. The maximum possible number of couples is $N_{couple} = N1$ (if $N1 < N2$) and $N_{couple} = N2$ (if $N2 < N1$).

The probability to find at least one object of population 1 within a radius d_k from its closest neighbour of population 2 can be defined by Eq. (see Appendix in Ivanov, 1996):

$$P_1 = 1 - \exp\left(-\pi d_k^2 \delta_1\right) \ , \quad \text{(A4)}$$

Similarly, the probability to find at least one object of population 2 within a radius d_k from its nearest neighbour of population 1 is:

$$P_2 = 1 - \exp\left(-\pi d_k^2 \delta_2\right) \ . \quad \text{(A5)}$$

Then the probability that two neighbours - one from population 1 and another from population 2-fall within a radius d_k from one of them is:

$$\begin{aligned} P_{12}(k) &= P_1 P_2 \\ &= \left[1 - exp\left(-\pi d_k^2 \delta_1\right)\right] \left[1 - exp\left(-\pi d_k^2 \delta_2\right)\right] \ . \end{aligned} \quad \text{(A6)}$$

This is Eq. 1 of the paper).

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