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Tidal torques and the clusters of galaxies evolution

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Abstract. We study the effect of tidal torques on the collapse of density peaks through the equations of motion of a shell of barionic matter falling into the central regions of a cluster of galaxies. We calculate the time of collapse of the perturbation taking into account the gravitational interaction of the quadrupole moment of the system with the tidal field of the matter of the neighbouring proto-clusters. We show that within high-density environments, such as rich clusters of galaxies, tidal torques slow down the collapse of low-ν peaks producing an observable variation in the time of collapse of the shell and, as a consequence, a reduction in the mass bound to the collapsed perturbation. Moreover, the delay of the collapse produces a tendency for less dense regions to accrete less mass, with respect to a classical spherical model, inducing a *biasing* of over-dense regions toward higher mass. Finally we calculate the bias coefficient using a selection function properly defined showing that for a Standard Cold Dark Matter (SCDM) model this $bias$ can account for a substantial part of the total bias required by observations on cluster scales.

Key words: cosmology: theory – cosmology: large scale structure of Universe – galaxies: formation

1. Introduction

It has long been speculated on the fundamental role that the angular momentum could play in determining the fate of collapsing proto-structures and several models have been proposed to correlate the galaxy type with the angular momentum per unit mass of the structure itself (Faber 1982; Kashlinsky 1982; Fall 1983). Some authors (see Barrow & Silk 1981; Szalay & Silk 1983 and Peebles 1990) have proposed that non-radial motions would be expected within a developing proto-cluster due to the tidal interaction of the irregular mass distribution around them, typical of hierarchical clustering models, with the neighbouring proto-clusters. The kinetic energy of these non-radial motions prevents the collapse of the proto-cluster, enabling the same to reach statistical equilibrium before the final collapse (the so-called previrialization conjecture by Davis & Peebles 1977, Peebles 1990). This effect may prevent the increase in the slope of the mass autocorrelation function at separations given by $\xi(r,t) \simeq 1$, expected in the scaling solution for the rise in $\xi(r,t)$ but not observed in the galaxy two-point correlation function. The role of non-radial motions has been pointed by several authors (see Davis & Peebles 1983: Gorski 1988; Groth et al. 1989; Mo et al. 1993; van de Weygaert & Babul 1994; Marzke et al. 1995 and Antonuccio-Delogu & Colafrancesco 1995). Antonuccio-Delogu & Colafrancesco derived the conditional probability distribution $f_{pk}(\mathbf{v}|\nu)$ of the peculiar velocity around a peak of a Gaussian density field and used the moments of the velocity distribution to study the velocity dispersion around the peak. They showed that regions of the proto-clusters at radii, r, greater than the filtering length, R_f, contain predominantly non-radial motions.

Non-radial motions change the energetics of the collapse model by introducing another potential energy term. In other words one expects that non-radial motions change the characteristics of the collapse and in particular the *turn around* epoch, t_m, and consequently the critical threshold, δ_c, for collapse. Here, we want to remind that t_m is the time at which the linear density fluctuations, that generate the cosmic structures, detach from the Hubble flow. The turn-around epoch is given by:

$$t_m = \left[\frac{3\pi}{32G\rho_b}(1+\overline{\delta})\right]^{1/2}(1+z)^{3/2} \tag{1}$$

where ρ_b is the mean background density, z is the redshift and $\overline{\delta}$ is the mean over-density within the non-linear region. After the *turn around* epoch, the fluctuations start to recollapse. As known for a spherical top hat model, the perturbation of the density field is completely collapsed when

$$\overline{\delta} = \delta_c = (3/5)(\frac{3\pi T_c}{4t_m})^{2/3} = 1.68 \tag{2}$$

where T_c is the time of collapse which is twice the turn around epoch. One expects that non-radial motions produce firstly a change in the turn around epoch, secondly a new functional form for δ_c, thirdly a change in the mass function calculable with the Press-Schechter (1974) formula and finally a modification of the two-point correlation function. As we shall show in a forthcoming paper (Del Popolo & Gambera 1997b) non-radial motions can reduce several discrepancies between the SCDM

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model and observations: the strong clustering of rich clusters of galaxies ($\xi_{cc}(r) \simeq (r/25h^{-1}Mpc)^{-2}$) far in excess of CDM predictions (Bahcall & Soneira 1983), the X-ray temperature distribution function of clusters over-producing the observed cluster abundances (Bartlett & Silk 1993).

For the sake of completeness, we remember that alternative models with more large-scale power than SCDM have been introduced in order to solve the latter problem. Several authors (Peebles 1984; Efstathiou et al. 1990; Turner 1991; White et al. 1993) have lowered the matter density under the critical value ($\Omega_m < 1$) and have added a cosmological constant in order to retain a flat universe ($\Omega_m + \Omega_\Lambda = 1$) . The spectrum of the matter density is specified by the transfer function, but its shape is affected because of the fact that the epoch of matter-radiation equality is earlier, $1 + z_{eq}$ being increased by a factor $1/\Omega_m$. Around the epoch z_Λ the growth of the density contrast slows down and ceases after z_Λ. As a consequence the normalisation of the transfer function begins to fall, even if its shape is retained.

Mixed dark matter models (MDM) (Bond et al. 1980; Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Holtzman 1989; Schaefer 1991; Schaefer & Shafi 1993; Holtzman & Primack 1993) increase the large-scale power because free-streaming neutrinos damp the power on small scales. Alternatively changing the primeval spectrum several problems of SCDM are solved (Cen et al. 1992). Finally, it is possible to assume that the threshold for galaxy formation is not spatially invariant but weakly modulated ($2\% - 3\%$ on scales $r > 10h^{-1}Mpc$) by large scale density fluctuations, with the result that the clustering on large-scale is significantly increased (Bower et al. 1993).

Moreover, this study of the role of non-radial motions in the collapse of density perturbations can help us to give a deeper insight in to the so-called problem of biasing. As pointed out by Davis et al. (1985), unbiased CDM presents several problems: pairwise velocity dispersion larger than the observed one, galaxy correlation function steeper than that observed (see Liddle & Lyth 1993 and Strauss & Willick 1995). The remedy to these problems is the concept of biasing (Kaiser 1984), i.e. that galaxies are more strongly clustered than the mass distribution from which they originated. The physical origin of such biasing is not yet clear even if several mechanisms have been proposed (Rees 1985; Dekel & Silk 1986; Dekel & Rees 1987; Carlberg 1991; Cen & Ostriker 1992; Bower et al. 1993; Silk & Wyse 1993). Recently Colafrancesco et al. (1995, hereafter CAD) and Del Popolo & Gambera (1997a) have shown that dynamical friction delays the collapse of low-ν peaks inducing a bias of dynamical nature. Because of the dynamical friction, under-dense regions in clusters (the clusters outskirts) accrete less mass than that accreted in absence of this dissipative effect and as a consequence over-dense regions are biased toward higher mass (Antonuccio-Delogu & Colafrancesco 1994 and Del Popolo & Gambera, 1996). Non-radial motions acts in a similar way to dynamical friction: they delay the shell collapse consequently inducing a dynamical bias similar to that produced by dynamical friction. This dynamical bias can be evaluated defining a selection function similar to that given in CAD and using Bardeen et al. (1986, hereafter BBKS) prescriptions.

The methods used in this paper are fundamentally some results of the statistics of Gaussian random fields, the biased galaxy formation theory and the spherical model for the collapse of density perturbations. In particular, we calculate the specific angular momentum acquired by protoclusters and the time of collapse of protoclusters using the Gaussian random fields theory and the spherical collapse model following Ryden's (1988a, hereafter R88a) approach. The selection function that we introduce is general and obtained by the only hypothesis of Gaussian density field. The approach and the final result is totally different from BBKS selection function and similar to that of Colafrancesco, Antonuccio & Del Popolo (1995). Only the biasing parameter is obtained from a BBKS approximated formula. This choice will be clarified in the following sections of the paper.

The plan of the paper is the following: in Sect. 2 we obtain the total specific angular momentum acquired during expansion by a proto-cluster. In Sect. 3 we use the calculated specific angular momentum to obtain the time of collapse of shells of matter around peaks of density having $\nu_c = 2, 3, 4$ and we compare the results with Gunn & Gott's (1972, hereafter GG) spherical collapse model. In Sect. 4 we derive a selection function for the peaks giving rise to proto-structures while in Sect. 5 we calculate some values for the bias parameter, using the selection function derived, on three relevant filtering scales. Finally in Sect. 6 we discuss the results obtained.

2. Tidal torques

The explanation of galaxies spins gain through tidal torques was pioneered by Hoyle (1949) in the context of a collapsing protogalaxy. Peebles (1969) considered the process in the context of an expanding world model showing that the angular momentum gained by the matter in a random comoving *Eulerian* sphere grows at the second order in proportion to $t^{5/3}$ (in a Einstein-de Sitter universe), since the proto-galaxy was still a small perturbation, while in the non-linear stage the growth rate of an oblate homogeneous spheroid decreases with time as t^{-1}.

More recent analytic computations (White 1984; Hoffman 1986 and R88a) and numerical simulations (Barnes & Efstathiou 1987) have re-investigated the role of tidal torques in originating galaxies angular momentum. In particular White (1984) considered an analysis by Doroshkevich (1970) that showing as the angular momentum of galaxies grows to first order in proportion to t and that Peebles's result is a consequence of the spherical symmetry imposed to the model. White showed that the angular momentum of a Lagrangian sphere does not grow either in the first or in the second order while the angular momentum of a non-spherical volume grows to the first order in agreement with Doroshkevich's result.

Hoffman (1986) has been much more involved in the analysis of the correlation of the growth of angular momentum with the density perturbation $\delta(r)$. He found an angular momentum-density anticorrelation: high density peaks acquire less angular momentum than low density peaks. One way to study the variation of angular momentum with radius in a galaxy is that followed by R88a. In this approach the protogalaxy is divided into a series of mass shells and the torque on each mass shell is computed separately. The density profile of each proto-structure is approximated by the superposition of a spherical profile, $\delta(r)$, and a random CDM distribution, $\varepsilon(\mathbf{r})$, which provides the quadrupole moment of the protogalaxy. To the first order, the initial density can be represented by:

$$\rho(\mathbf{r}) = \rho_b \left[1 + \delta(r)\right] \left[1 + \varepsilon(\mathbf{r})\right] \quad (3)$$

where ρ_b is the background density and $\varepsilon(\mathbf{r})$ is given by:

$$\langle |\varepsilon_k|^2 \rangle = P(k) \quad (4)$$

being $P(k)$ the power spectrum, while the density profile is (Ryden & Gunn 1987):

$$\langle \delta(r) \rangle =$$

$$\frac{\nu \xi(r)}{\xi(0)^{1/2}} - \frac{\vartheta(\nu\gamma, \gamma)}{\gamma(1-\gamma^2)} \left[\gamma^2 \xi(r) + \frac{R_*^2}{3} \nabla^2 \xi\right] \cdot \xi(0)^{-1/2} \quad (5)$$

where ν is the height of a density peak, $\xi(r)$ is the two-point correlation function, γ and R_* are two spectral parameters (BBKS, Eq. 4.6a, 4.6d) while $\vartheta(\gamma\nu, \gamma)$ is a function given in BBKS (Eq. 6.14). As shown by R88a the net rms torque on a mass shell centered on the origin of internal radius r and thickness δr is given by:

$$\langle |\tau|^2 \rangle^{1/2} =$$

$$\sqrt{30} \left(\frac{4\pi}{5} G\right) \left[\langle a_{2m}(r)^2 \rangle \langle q_{2m}(r)^2 \rangle - \langle a_{2m}(r) q_{2m}^*(r) \rangle^2\right]^{1/2} \quad (6)$$

where q_{lm}, the multipole moments of the shell and a_{lm}, the tidal moments, are given by:

$$\langle q_{2m}(r)^2 \rangle = \frac{r^4}{(2\pi)^3} M_{sh}^2 \int k^2 dk P(k) j_2(kr)^2 \quad (7)$$

$$\langle a_{2m}(r)^2 \rangle = \frac{2\rho_b^2 r^{-2}}{\pi} \int dk P(k) j_1(kr)^2 \quad (8)$$

$$\langle a_{2m}(r) q_{2m}^*(r) \rangle = \frac{r}{2\pi^2} \rho_b M_{sh} \int k dk P(k) j_1(kr) j_2(kr) \quad (9)$$

where M_{sh} is the mass of the shell, $j_1(r)$ and $j_2(r)$ are the spherical Bessel function of first and second order while the power spectrum $P(k)$ is given by:

$$P(k) = A k^{-1} [\ln(1 + 4.164k)]^2 \cdot (192.9 + 1340k$$

$$+ 1.599 \cdot 10^5 k^2 + 1.78 \cdot 10^5 k^3 + 3.995 \cdot 10^6 k^4)^{-1/2} \quad (10)$$

(Ryden & Gunn 1987). The normalization constant A can be obtained, as usual, by fixing that the mass variance at $8h^{-1}Mpc$, that is σ_8, be equal to unity. Filtering the spectrum on cluster scales, $R_f = 3h^{-1}Mpc$, we have obtained the rms torque, $\tau(r)$, on a mass shell using Eq. (6) then we obtained the total

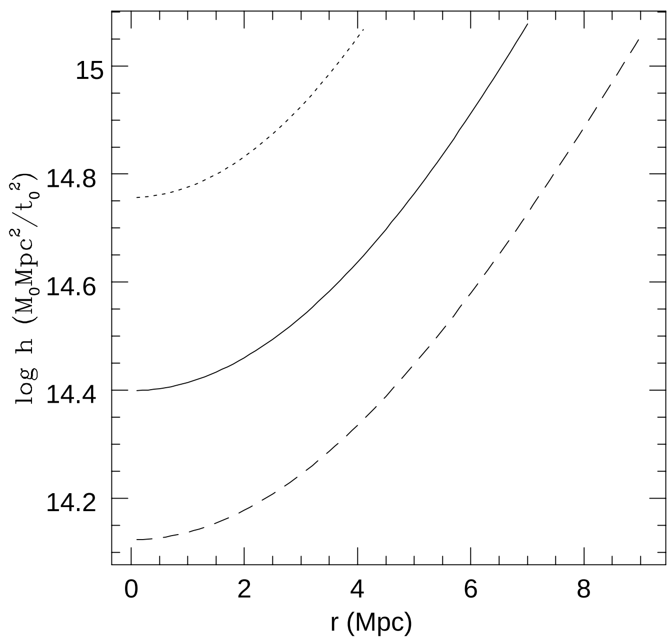

Fig. 1. The specific angular momentum, in units of $M_\odot$, Mpc and the Hubble time, t_0, for three values of the parameter ν ($\nu = 2$ dotted line, $\nu = 3$ solid line, $\nu = 4$ dashed line) and for $R_f = 3h^{-1}Mpc$.

specific angular momentum, $h(r, \nu)$, acquired during expansion integrating the torque over time (R88a Eq. 35):

$$h(r, \nu) = \frac{\tau_o t_o \overline{\delta}_o^{-5/2}}{\sqrt[3]{48} M_{sh}} \int_0^\pi \frac{(1-\cos\theta)^3}{(\vartheta - \sin\vartheta)^{4/3}} \frac{f_2(\vartheta) \cdot d\vartheta}{f_1(\vartheta) - f_2(\vartheta) \frac{\delta_o}{\overline{\delta}_o}} \quad (11)$$

where the functions $f_1(\vartheta)$, $f_2(\vartheta)$ are given by R88a (Eq. 31) while the mean over-density inside the shell, $\overline{\delta}(r)$, is given by (R88a):

$$\overline{\delta}(r, \nu) = \frac{3}{r^3} \int_0^\infty d\sigma \sigma^2 \delta(\sigma) \quad (12)$$

In Fig. 1 we show the variation of $h(r, \nu)$ with the distance r for three values of the peak height ν. The rms specific angular momentum, $h(r, \nu)$, increases with distance r while peaks of greater ν acquire less angular momentum via tidal torques. This is the angular momentum-density anticorrelation showed by Hoffman (1986). This effect arises because the angular momentum is proportional to the gain at turn around time, t_m, which in turn is proportional to $\overline{\delta}(r, \nu)^{-\frac{3}{2}} \propto \nu^{-3/2}$.

3. Shell collapse time

One of the consequences of the angular momentum acquisition by a mass shell of a proto-cluster is the delay of the collapse of the proto-structure. As shown by Barrow & Silk (1981) and Szalay & Silk (1983) the gravitational interaction of the irregular mass distribution of proto-cluster with the neighbouring proto-structures gives rise to non-radial motions, within the protocluster, which are expected to slow the rate of growth of the density contrast and to delay or suppress the collapse. According to

Davis & Peebles (1977) the kinetic energy of the resulting non-radial motions at the epoch of maximum expansion increases so much to oppose the recollapse of the proto-structure. Numerical N-body simulations by Villumsen & Davis (1986) showed a tendency to reproduce this so-called previrialization effect. In a more recent paper by Peebles (1990) the slowdown in the growth of density fluctuations and the collapse suppression after the epoch of the maximum expansion were re-obtained using a numerical action method.

In the central regions of a density peak ($r \leq 0.5 R_f$) the velocity dispersion attains nearly the same value (Antonuccio-Delogu & Colafrancesco 1995) while at larger radii ($r \geq R_f$) the radial component is lower than the tangential component. This means that motions in the outer regions are predominantly non-radial and in these regions the fate of the infalling material could be influenced by the amount of tangential velocity relative to the radial one. This can be shown writing the equation of motion of a spherically symmetric mass distribution with density $n(r)$ (Peebles 1993):

$$\frac{\partial}{\partial t} n\langle v_r\rangle + \frac{\partial}{\partial r} n\langle v_r^2\rangle + \left(2\langle v_r^2\rangle - \langle v_\vartheta^2\rangle\right)\frac{n}{r} + n(r)\frac{\partial}{\partial t}\langle v_r\rangle = 0 \quad (13)$$

where $\langle v_r\rangle$ and $\langle v_\vartheta\rangle$ are, respectively, the mean radial and tangential streaming velocity. Eq. (13) shows that high tangential velocity dispersion ($\langle v_\vartheta^2\rangle \geq 2\langle v_r^2\rangle$) may alter the infall pattern. The expected delay in the collapse of a perturbation may be calculated solving the equation for the radial acceleration (Peebles 1993):

$$\frac{dv_r}{dt} = \frac{L^2(r,\nu)}{M^2 r^3} - g(r) \quad (14)$$

where $L(r,\nu)$ is the angular momentum and $g(r)$ the acceleration. Writing the proper radius of a shell in terms of the expansion parameter, $a(r_i,t)$:

$$r(r_i,t) = r_i a(r_i,t) \quad (15)$$

remembering that

$$M = \frac{4\pi}{3}\rho_b(r_i,t)a^3(r_i,t)r_i^3 \quad (16)$$

and that $\rho_b = \frac{3H_0^2}{8\pi G}$, where H_0 is the Hubble constant and assuming that no shell crossing occurs so that the total mass inside each shell remains constant, that is:

$$\rho(r_i,t) = \frac{\rho_i(r_i,t)}{a^3(r_i,t)} \quad (17)$$

Eq. (14) may be written as:

$$\frac{d^2a}{dt^2} = -\frac{H^2(1+\overline{\delta})}{2a^2} + \frac{4G^2L^2}{H^4(1+\overline{\delta})^2 r_i^{10} a^3} \quad (18)$$

or integrating the equation once more:

$$\left(\frac{da}{dt}\right)^2 = H_i^2\left[\frac{1+\overline{\delta}}{a}\right] + \int \frac{8G^2L^2}{H_i^2 r_i^{10}\left(1+\overline{\delta}\right)^2}\frac{1}{a^3}da - 2C \quad (19)$$

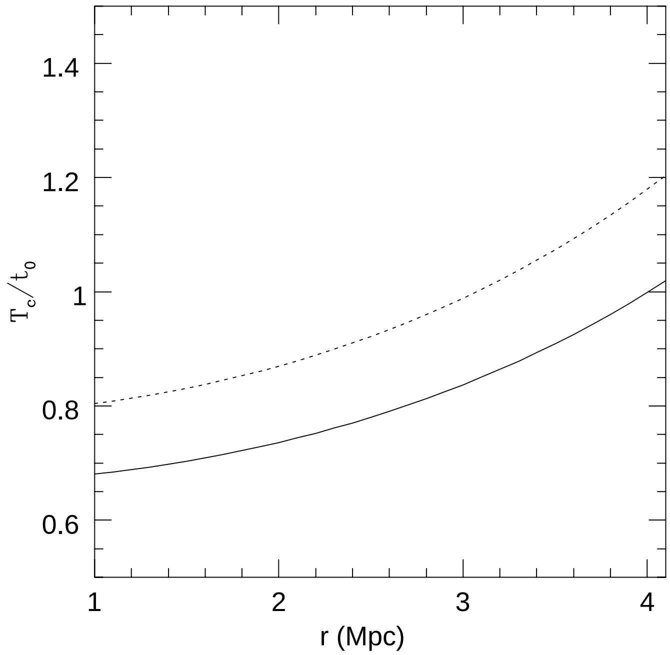

Fig. 2. The time of collapse of a shell of matter in units of the age of the Universe t_0 for $\nu = 2$ (dotted line) compared with GG's model (solid line).

where C is the binding energy of the shell. The value of C can be obtained using the condition for turn around $\frac{da}{dt} = 0$ when $a = a_{max}$ leading to the new equation:

$$\left(\frac{da}{dt}\right)^2 =$$

$$H_i^2\left[\frac{1+\overline{\delta}}{a} - \frac{1+\overline{\delta}}{a_{max}}\right] + \int_{a_{max}}^{a} \frac{8G^2L^2}{H_i^2 r_i^{10}\left(1+\overline{\delta}\right)^2}\frac{1}{a'^3}da' \quad (20)$$

Eq. (14) or equivalently Eq. (18) may be solved using the initial conditions: $(\frac{da}{dt}) = 0$, $a = a_{max} \simeq 1/\overline{\delta}$ and using the function $h(r,\nu) = L(r,\nu)/M_{sh}$ found in Sect. 2 to obtain the time of collapse, $T_c(r,\nu)$.

In Figs. 2 $\div$ 4 we compare the results for the time of collapse, T_c, for $\nu = 2, 3, 4$ with the time of collapse of the classical GG spherical model:

$$T_{c0}(r,\nu) = \frac{\pi}{H_i}[\overline{\delta}(r,\nu)]^{-3/2} \quad (21)$$

As shown the presence of non-radial motions produces an increase in the time of collapse of a spherical shell. The collapse delay is larger for a low value of ν and becomes negligible for $\nu \geq 3$. This result is in agreement with the angular momentum-density anticorrelation effect: density peaks having low value of ν acquire a larger angular momentum than high ν peaks and consequently the collapse is more delayed with respect to high ν peaks. Given $T_c(r,\nu)$ we also calculated the total mass gravitationally bound to the final non-linear configuration. There are at least two criteria to establish the bound region to a perturbation $\delta(r)$: a statistical one (Ryden 1988b), and a dynamical

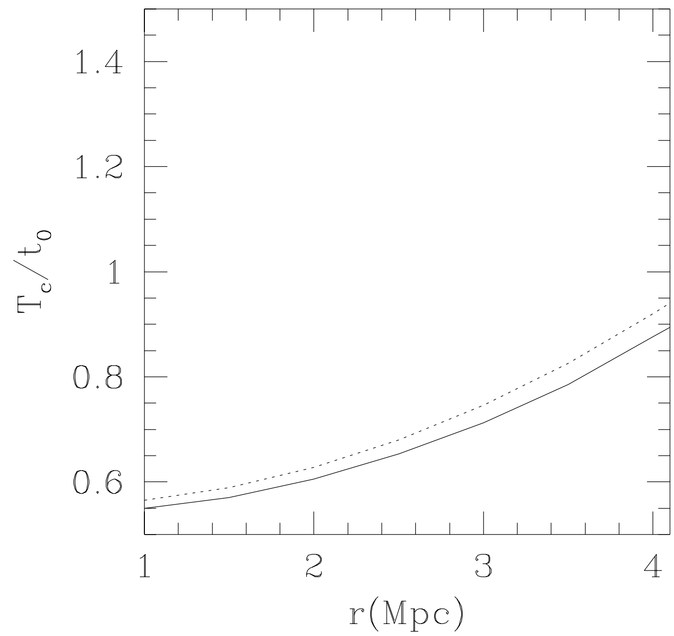

Fig. 3. The time of collapse of a shell of matter in units of the age of the Universe t_0 for $\nu = 3$ (dotted line) compared with GG's model (solid line).

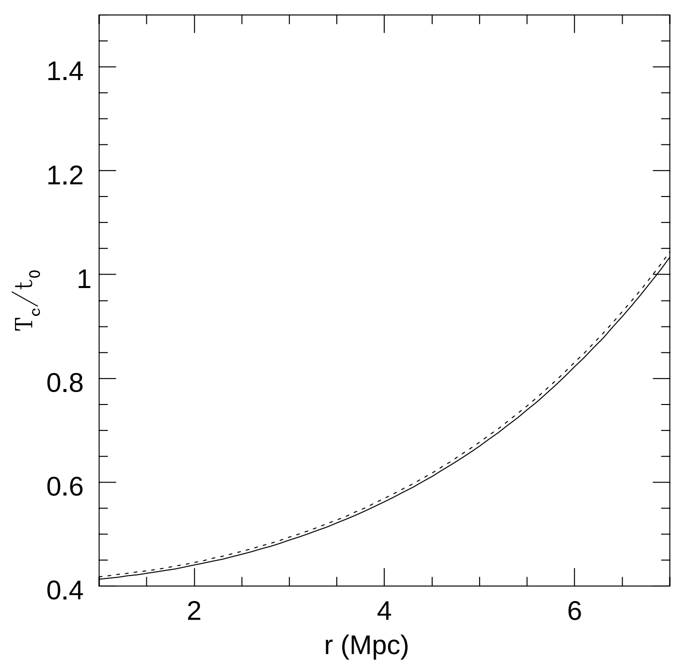

Fig. 4. The time of collapse of a shell of matter in units of the age of the Universe t_0 for $\nu = 4$ (dotted line) compared with GG's model (solid line).

one (Hoffman & Shaham 1985). The dynamical criterion, that we have used, assumes that the binding radius is given by the condition that a mass shell collapse in a time, T_c, smaller than the age of the universe t_0:

$$T_c(r, \nu) \leq t_0 \tag{22}$$

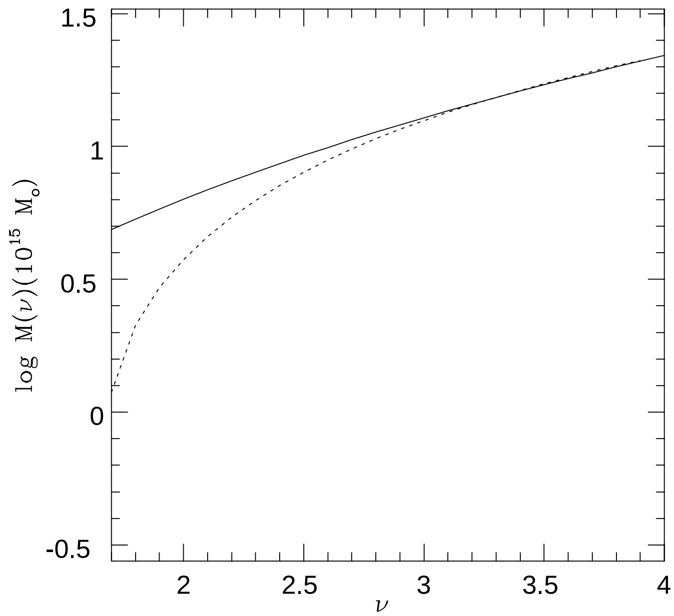

Fig. 5. The mass accreted by a collapsed perturbation, in units of $10^{15} M_\odot$, taking into account non-radial motions effect (dotted line) compared to GG's mass (solid line).

We calculated the time of collapse of GG spherical model, $T_{c0}(r, \nu)$, using the density profiles given in Eq. (5) for $1.7 < \nu < 4$ and then we repeated the calculation taking into account non-radial motions obtaining $T_c(r, \nu)$. Then we calculated the binding radius, $r_b(\nu)$, for a GG model solving $T_{c0}(r, \nu) \leq t_0$ for r and for several values of ν, while we calculated the binding radius of the model that takes into account non-radial motions, $r_b(\nu)$, repeating the calculation, this time with $T_c(r, \nu) \leq t_0$. We found a relation between ν and the mass of the cluster using the equation: $M = \frac{4\pi}{3} r_b^3 \rho_b$.

In Fig. 5 we compare the peak mass obtained from GG model, using Hoffman & Shaham's (1985) criterion, with that obtained from the model taking into account non-radial motions. As shown for high values of ν ($\nu \geq 3$), the two models give the same result for the mass while for $\nu \leq 3$ the effect of non-radial motions produces less bound mass with respect to GG model.

Before concluding this section we want to discuss the applicability of the spherical model and consequently the use of spherical shells in our model. The question of the applicablity of the idealized spherical collapse model and that of the secondary infall model (SIM) to realistic systems and initial conditions is almost as old as the model itself (GG; Gunn 1977; Filmore & Goldreich 1984; Quinn et al. 1986; Zurek et al. 1988; Quinn & Zurek 1988; Warren et al. 1991; Crone et al. 1994). In a recent paper Zaroubi et al. (1996) investigate the applicability of the quoted model to realistic systems and initial conditions by comparing the results obtained using the spherical model and SIM with that of a set of simulations performed with different N-body codes (Treecode and a "monopole term" code). They introduced a numerical model to trace the evolution of density peaks under the assumption of aspherically symmetric force and realistic initial conditions instead of the spherically symmetric force and initial conditions as assumed in the SIM. They obtained a good agreement between the SIM and the simulations.

4. Tidal field and the selection function

According to biased galaxy formation theory the sites of formation of structures of mass $\sim M$ must be identified with the maxima of the density peak smoothed over a scale R_f ($M \propto R_f^3$). A necessary condition for a perturbation to form a structure is that it goes non-linearly and that the linearly extrapolated density contrast reaches the value $\overline{\delta}(r) \geq \delta_c = 1.68$ or equivalently that the threshold criterion $\nu_t > \delta_c/\sigma_0(R_f)$ is satisfied, being $\sigma_0(R_f)$ the variance of the density field smoothed on scale R_f. When these conditions are satisfied the matter in a shell around a peak falls in toward the cluster center and virializes. In this scenario only rare high ν peaks form bright objects while low ν peaks ($\nu \approx 1$) form under-luminous objects. The kind of objects that forms from non-linear structures depends on the details of the collapse. Moreover, if structures form only at peaks in the mass distribution they will be more strongly clustered than the mass.

Several feedback mechanisms have been proposed to explain this segregation effect (Rees 1985; Dekel & Rees 1987). Even if these feedback mechanisms work one cannot expect they have effect instantaneously, so the threshold for structure formation cannot be sharp (BBKS). To take into account this effect BBKS introduced a threshold or selection function, $t(\nu/\nu_t)$. The selection function, $t(\nu/\nu_t)$, gives the probability that a density peak forms an object, while the threshold level, ν_t, is defined so that the probability that a peak forms an object is 1/2 when $\nu = \nu_t$. The selection function introduced by BBKS (Eq. 4.13), is an empirical one and depends on two parameters: the threshold ν_t and the shape parameter q:

$$t(\nu/\nu_t) = \frac{(\nu/\nu_t)^q}{1 + (\nu/\nu_t)^q} \tag{23}$$

If $q \to \infty$ this selection function is a Heaviside function $\vartheta(\nu - \nu_t)$ so that peaks with $\nu > \nu_t$ have a probability equal to 100% to form objects while peaks with $\nu \leq \nu_t$ do not form objects. If q has a finite value sub-ν_t peaks are selected with non-zero probability. Using the given selection function the cumulative number density of peaks higher than ν is given, according to BBKS, by:

$$n_{pk} = \int_{\nu}^{\infty} t(\nu/\nu_t) N_{pk}(\nu) d\nu \tag{24}$$

where $N_{pk}(\nu)$ is the comoving peak density (see BBKS Eq. 4.3). A form of the selection function, physically founded, can be obtained following the argument given in CAD. In this last paper the selection function is defined as:

$$t(\nu) = \int_{\delta_c}^{\infty} p\left[\overline{\delta}, \langle\overline{\delta}(r_{Mt}, \nu)\rangle, \sigma_{\overline{\delta}}(r_{Mt}, \nu)\right] d\delta \tag{25}$$

where the function

$$p\left[\overline{\delta}, \langle\overline{\delta}(r)\rangle\right] = \frac{1}{\sqrt{2\pi}\sigma_{\overline{\delta}}} \exp\left(-\frac{|\overline{\delta} - \langle\overline{\delta}(r)\rangle|^2}{2\sigma_{\overline{\delta}}^2}\right) \tag{26}$$

gives the probability that the peak overdensity is different from the average, in a Gaussian density field. The selection function depends on ν through the dependence of $\overline{\delta}(r)$ on ν. As displayed, the integrand is evaluated at a radius r_{Mt} which is the typical radius of the object that we are selecting. Moreover, the selection function $t(\nu)$ depends on the critical overdensity threshold for the collapse, δ_c, which is not constant as in a spherical model (due to the presence, in our analysis, of non-radial motions that delay the collapse of the proto-cluster) but it depends on ν. The dependence of δ_c on ν can be obtained in several ways, for example according to Peebles (1980) the value of δ_c depends on the ratio T_c/t_m between the perturbation collapse time, T_c, and its turn around time, t_m:

$$\delta_c = \frac{3}{5}\left(\frac{3\pi}{4}\frac{T_c}{t_m}\right)^{2/3} \tag{27}$$

Non-radial motions slow down the collapse of the mass shell with respect to the GG collapse time changing the value of T_c. Using the calculated time of collapse for a given shell, and its dependence on ν, $\delta_c(\nu)$ can be calculated using Eq. (27). An analityc determination of $\delta_c(\nu)$ can be obtained following a technique similar to that used by Bartlett & Silk (1993). Using Eq. (19) it is possible to obtain the value of the expansion parameter of the turn around epoch, a_{max}, which is characterized by the condition $\frac{da}{dt} = 0$. Using the relation between v and δ_i, in linear theory (Peebles 1980), we can find C that substituted in Eq. (19) gives at turn around:

$$\delta_c(\nu) = \delta_{c0}\left[1 + \frac{8G^2}{\Omega_o^3 H_o^6 r_i^{10} \overline{\delta}(1+\overline{\delta})^2}\int\frac{L^2 da}{a^3}\right] \tag{28}$$

where $\delta_{c0} = 1.68$ is the critical threshold for GG's model. In Fig. 6 we show the overdensity threshold as a function of ν. As shown, $\delta_c(\nu)$ decreases with increasing ν: when $\nu > 3$ the threshold assumes the typical value of the spherical model. This means, according to the cooperative galaxy formation theory, (Bower et al. 1993) that structures form more easily if there are other structures nearby, i.e. the threshold level is a decreasing function of the mean mass density. Known $\delta_c(\nu)$ and chosen a spectrum, the selection function is immediately obtainable through Eq. (25) and Eq. (26). The result of the calculation, plotted in Fig. 7, for two values of the filtering radius, ($R_f = 3, 4$ $h^{-1} Mpc$), shows that the selection function, as expected, differs from an Heaviside function (sharp threshold). The shape of the selection function depends on the values of the filtering length R_f and on non-radial motions. The value of ν at which the selection function $t(\nu)$ reaches the value 1 ($t(\nu) \simeq 1$) increases for growing values of the filtering radius, R_f. This is due to the smoothing effect of the filtering process. The effect of non-radial motions is, firstly, that of shifting $t(\nu)$ towards higher values of ν, and, secondly, that of making it steeper. The selection function is also different from that used by BBKS (Table 3a). Finally it is interesting to note that the selection function defined by Eq. (25) and Eq. (26) is totally general, it does not depend on the presence or absence of non-radial motions. The latter influences the selection function form through the change of δ_c induced by non-radial motions itself.

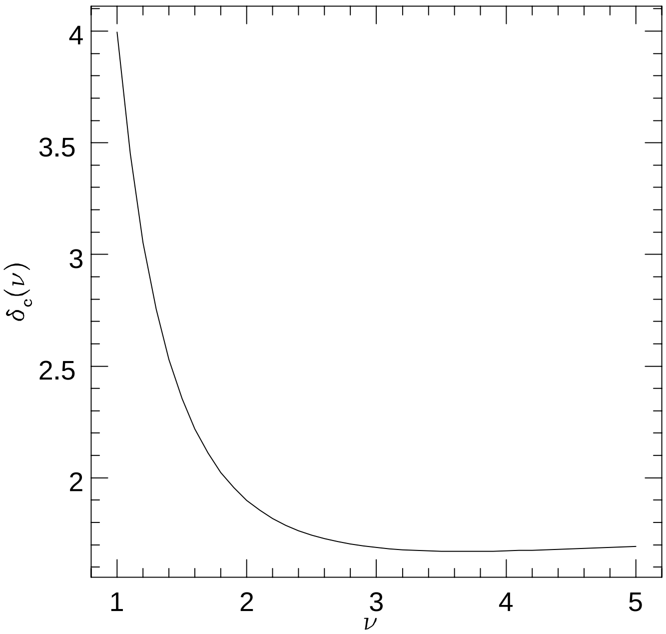

Fig. 6. The critical threshold, $\delta_c(\nu)$ versus ν

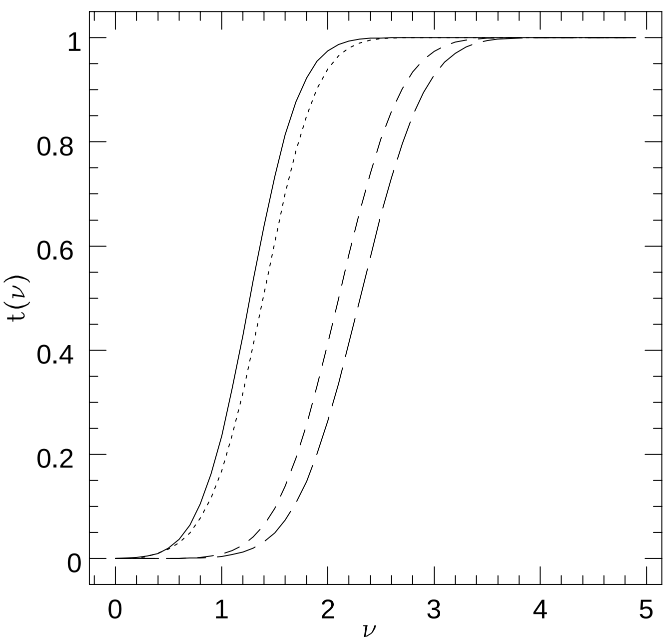

Fig. 7. The selection function, $t(\nu)$, for $R_f = 3h^{-1}Mpc$ (the solid line plots the selection function obtained without taking into account the effects of non-radial motions; the dotted line plots the selection function obtained taking into account the effects of non-radial motions) and for $4h^{-1}Mpc$ (the short dashed line plots the selection function obtained without taking into account the effects of non-radial motions; the long dashed line plots the selection function obtained taking into account the effects of non-radial motions).

5. The bias coefficient

A model of the Universe in which light traces the mass distribution accurately (unbiased model) is subject to several problems. As pointed out by Davis et al. (1985) an unbiased CDM produces a galaxy correlation function which is steeper than that observed and a pairwise velocity dispersion larger than that deduced from redshift surveys. A remedy to this problem can be found if we do not assume that light traces mass and adopt the biasing concept, i.e., galaxies are more clustered than the distribution of matter in agreement with the concept of biasing inspired by Kaiser (1984). The observations show that clusters of galaxies cluster more strongly than galaxies, in the sense that the dimensionless two-point correlation function, $\xi_{cc}(r)$, is much larger than the galaxy two-point function, $\xi_{gg}(r)$. The galaxy two-point correlation function $\xi_{gg}(r)$ is a power-law:

$$\xi_{gg}(r) = \left(\frac{r}{r_{0,g}}\right)^{\gamma} \tag{29}$$

with a correlation length $r_{0,g} \simeq 5h^{-1}$ Mpc and a slope $\gamma \simeq 1.8$ for $r \leq 10h^{-1}$ Mpc (Davis & Peebles 1983; Davis et al. 1985; Shanks et al. 1989), (some authors disagree with this values; for example Strauss et al. 1992 and Fisher et al. 1993 find $r_{0,g} \simeq 3.79h^{-1}$ Mpc and $\gamma \simeq 1.57$). As regards the clusters of galaxies the form of the two-point correlation function, $\xi_{cc}(r)$, is equal to that given by Eq. (29). Only the correlation length is different. In the case of clusters of galaxies the value of $r_{0,c}$ is uncertain (see Bahcall & Soneira 1983; Postman et al. 1986; Sutherland 1988; Bahcall 1988; Dekel et al. 1989; Olivier et al. 1990 and Sutherland & Efstathiou 1991) however it lies in the range $r_{0,c} \simeq 12 \div 25h^{-1}$ Mpc in any case larger than $r_{0,g}$. One way of defining the bias coefficient of a class of objects is that given by (BBKS):

$$b(R_f) = \frac{\langle\tilde{\nu}\rangle}{\sigma_0} + 1 \tag{30}$$

where $\langle\tilde{\nu}\rangle$ is:

$$\langle\tilde{\nu}\rangle = \int_0^{\infty} \left[\nu - \frac{\gamma\theta}{1-\gamma^2}\right] t\left(\frac{\nu}{\nu_t}\right) N_{pk}(\nu) d\nu \tag{31}$$

We want to remember that, as shown by Coles (1993), the biasing parameter can be also estimated by means of the ratio of the amplitudes of the correlation function, $\xi(r)$, and the matter auto-covariance function, $\Gamma(r)$:

$$b^2(r) = \frac{\xi(r)}{\Gamma(r)} \tag{32}$$

or by means of the ratio of the cumulative integral of the two-points correlation function ($K_3(r) = \int_0^r \xi(r)r^2 dr$) and that of the auto-covariance function ($J_3(r) = \int_0^r \Gamma(r)r^2 dr$):

$$b^2(r) = \frac{K_3(r)}{J_3(r)} \tag{33}$$

or finally the ratio of galaxy, $Q(k)$, to mass, $P(k)$, power spectra:

$$b^2(r) = \frac{Q(k)}{P(k)} \tag{34}$$

As stressed by Coles (1993) a local bias generally produces a different response in each of these descriptors. Even the qualitative behaviour of the limit of large scales can be different, i.e. $b(r)$ can increase or decrease. So one should decide very carefully which one of these definitions must be used, when discussing the behaviour of the biasing parameter. Being conscious of these difficulties we have chosen one of the most popular descriptor of the biasing parameter (see also Lilje 1990, Liddle & Lith 1993, Croft & Efstathiou 1994, CAD) in order to make comparisons with other models. From Eq. (31) it is clear that the bias parameter can be calculated once a spectrum, $P(k)$, is fixed. The bias parameter depends on the shape and normalization of the power spectrum. A larger value is obtained for spectra with more power on large scale (Kauffmann et al. 1996). In this calculation we continue to use the standard CDM spectrum ($\Omega_0 = 1$, $h = 0.5$) normalized imposing that the rms density fluctuations in a sphere of radius $8h^{-1}Mpc$ is the same as that observed in galaxy counts, i.e. $\sigma_8 = \sigma(8h^{-1}Mpc) = 1$. The calculations have been performed for three different values of the filtering radius ($R_f = 2, 3, 4\ h^{-1}Mpc$). The values of b, that we have obtained, are respectively, in increasing order R_f, 1.6, 1.93 and 2.25.

As shown, the value of the bias parameter tends to increase with R_f due the filter effect of $t(\nu)$. As shown $t(\nu)$ acts as a filter, increasing the filtering radius, R_f, the value of ν at which $t(\nu) \simeq 1$ increases. In other words when R_f increases, $t(\nu)$ selects density peaks of a larger height. The reason for this behavior must be searched in the smoothing effect that the increasing of the filtering radius produces on density peaks. When R_f increases the density field smooths and $t(\nu)$ has to shift towards a higher value of ν in order to select a class of object of fixed mass M.

6. Conclusions

In this paper we have studied the role of non-radial motions on the collapse of density peaks solving numerically the equations of motion of a shell of barionic matter falling into the central regions of a cluster of galaxies. We have shown that non-radial motions produce a delay in the collapse of density peaks having a low value of ν while the collapse of density peaks having $\nu > 3$ is not influenced. A first consequence of this effect is a reduction of the mass bound to collapsed perturbations and an increase in the critical threshold, δ_c, which now is larger than that of the top-hat spherical model and depends on ν. This means that shells of matter of low density have to be subject to a larger gravitational potential, with respect to the homogeneous GG's model, in order to collapse.

The delay in the proto-structures collapse gives rise to a dynamical bias similar to that described in CAD whose bias parameter may be obtained once a proper selection function is defined. The selection function found is not a pure Heaviside function and is different from that used by BBKS to study the statistical properties of clusters of galaxies. Its shape depends on the effect of non-radial motions through its dependence on $\delta_c(\nu)$. The function $t(\nu)$ selects higher and higher density peaks with increasing value of R_f due to the smoothing effect of the density field produced by the filtering procedure. Using this selection function and BBKS prescriptions we have calculated the coefficient of bias b.

On clusters scales for $R_f = 4h^{-1}Mpc$ we found a value of $b = 2.25$ comparable with that obtained from the mean mass-to-light ratio of clusters, APM survey, or from N-body simulations combined with hydrodynamical models (Frenk et al. 1990). Morever, the value of the coefficient of biasing b that we have calculated is comparable with the values of b given by Kauffmann et al. (1996). This means that non-radial motions play a significant role in determining the bias level. In our next paper (Del Popolo & Gambera, in preparation) we make a more detailed analysis on the problem of the bias.

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