Astron. Astrophys. 337, L9–L12 (1998)

ASTRONOMY
AND
ASTROPHYSICS

LETTER

Letter to the Editor

On a source of Alfvén waves heating the solar corona

Alexander Ruzmaikin[1] **and Mitchell A. Berger**[2]

[1] Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA (e-mail: aruzmaikin@jplsp.jpl.nasa.gov)
[2] Department of Mathematics, University College London, London, WC1E 6BT, UK (e-mail: mberger@math.ucl.ac.uk)

Received 29 June 1998 / Accepted 14 July 1998

Abstract. Studies of the origin of coronal heating and acceleration of the solar wind invoke high-frequency Alfvén waves. Here we suggest a source for such waves associated with twisted magnetic loops emerging on the solar surface and reconnecting with the open field. We identify the loops with the ephemeral regions (small-scale bipoles) observed by ground-based instruments and by SOHO. To characterize the loops we employ the concept of a minimum energy state for topologically complex fields. Emerging loops release energy relaxing to the minimum state. Relaxation along the minimum state–due to a competition between footpoint twisting by photospheric motions and reconnections inside the loops– releases blinks of energy into the solar atmosphere. We estimate the power released and the range of wave frequencies.

Key words: Solar corona heating: magnetic fields

1. Introduction

The heating of the solar corona and acceleration of the solar wind have a common origin related to magnetic fields (Hollweg 1986). Models of these processes prescribe the source of the heating and its power (c.f. Esser et al. 1997). The physical nature of the mechanism of the build-up and release of energy remains a problem, although a number of crucial insights have been made in the study of this problem. It has been suggested by many authors that the mechanism involves twisting of flux tubes by convective motions, reconnections inside the flux tubes, and reconnections of closed flux tubes with open ones (Sturrock, Uchida 1981, van Ballegooijen 1986, Parker 1990, Feldman et al. 1993, Berger 1994, Galsgard, Nordlund 1996, Shibata 1997). An interesting idea about the form of energy release has been put forward by Axford and McKenzie (1992) and recently developed by Tu and Marsch (1997) and Marsch and Tu (1997ab). According to these authors, a source generates high-frequency ($\geq$ 1 Hz) Alfvén waves, that dissipate by ion-cyclotron-resonance damping in the inner corona and thus provide the energy for heating of the low corona and the pressure for initial acceleration of the solar wind. The origin of these waves was associated with small-scale magnetic activity (such as reconnections) in the chromospheric network but has not been discussed in any detail.

Here we suggest a specific mechanism for the generation of the high-frequency Alfvén waves by the magnetic activity. Our approach is based on the concept of *the minimum state* for a topologically complex field, introduced by Freedman and He (1991) and Berger (1993) (see the section below). We use the highly fragmented nature of the solar magnetic field and utilize the ideas of twisting of flux tubes by random motions and reconnections of closed magnetic loops with the open field. Although the energy basically comes from convective motions and associated magnetic fields, the topological constraints, imbedded here in the form of the minimum energy state, help us to understand what portion of the magnetic energy can be released.

The solar loops can release magnetic energy in two primary ways. First, the magnetic flux emerging to the solar surface may have a high level of topological complexity acquired from shear motions inside the convection zone (for observational support of this point see Leka et al. 1997) and lose the energy by relaxing toward the minimum state. Second, the already emerged flux tubes can be further entangled due to random photospheric shear motions and lose their energy and topological complexity due to reconnections inside the flux tubes (Sturrock, Uchida 1981, van Ballegooijen 1986, Berger 1994) and due to reconnections with open structures (Axford, McKenzie 1992, Feldman et al. 1993, Shibata 1997). We show that relaxation along the minimum state contributes to coronal heating–this gives a new interpretation to the well-known dynamic balance between the twisting of footpoints and reconnections. We argue that reconnections of closed loops (emerging to the solar surface and those in the minimum state) with the open field release energy in the form of high-frequency Alfvén waves. To estimate the power released in the relaxation and the wave power we identify the magnetic loops with the observed small-scale bipoles (ephemeral regions) emerging on the Sun and interacting with the open network magnetic field.

2. Minimum energy of topologically complex magnetic fields

Of all field configurations with a given normal component at the solar surface, the potential field has the minimum energy. Any

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Send offprint requests to: A. Ruzmaikin

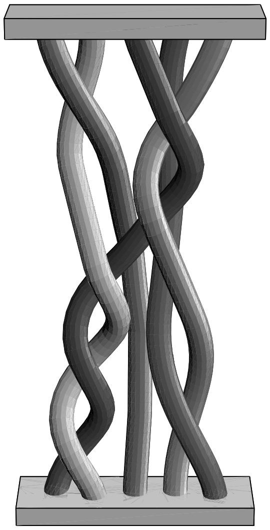

Fig. 1. A topologically complicated magnetic loop configuration can be represented by a braid. The figure shows an example of three flux tubes braided with 12 crossings.

equilibrium field with a current has higher energy. A magnetic configuration of a general type tends to relax to the minimum energy state; in a trivial case to a potential field. The minimum magnetic energy in a non-trivial case depends on topological invariants of the configuration, which do not change in the course of magnetic relaxation (Moffatt 1990). An example of such relaxation is the transition of a twisted flux tube into a writhed (coiled) tube, as happens to telephone cords (for a detailed study of this transition see Ricca (1995)). It is the current that induces topological complexity, such as twist, writhe and linking of field lines–usually described by the invariant called "magnetic helicity". The magnetic helicity does not characterize all of the topological complexity of the field: there are an infinite number of high-order invariants (c.f. Ruzmaikin, Akhmetiev 1994). A simple measure of complex magnetic line entanglement of general nature, called "crossing number", was introduced by Freedman and He (1991). Berger (1993) derived a lower bound for the energy of braided magnetic fields as a function of the crossing number and extended the concept of the crossing number to continuous fields.

A braid is defined to be a collection of curves stretching between two parallel planes (Fig. 1). A one-string braid is topologically trivial. Two-string braids can simulate the twist of magnetic lines around each other. With three or more strings the braid can simulate topologically complicated configurations. We identify the strings with thin magnetic flux tubes and their positions at the lower and upper plane with the positive and negative footpoints of the flux tubes. This represents magnetic loops in the solar atmosphere assuming that the positive footpoints are well separated from the negative ones. What is neglected in such representation is the curvature effects of loops. The complexity of a braid is measured by the number of times the strings cross each other as seen in projection. Because the braids are three-dimensional, this crossing number depends on the viewing point. However, the crossing number averaged over the viewing angle (say, a polar angle in the low plane in Fig. 1) is angle independent (Berger 1993). The averaged crossing number is not in itself a topological invariant. It has, however, a positive minimum (called here K) which is, as well as the minimum energy E, a topological invariant (Freedman, He 1991).

We employ K as a measure of topological complexity inside a magnetic loop of length L and radius r. To simulate solar conditions we assume that the axial field B is much stronger than the perpendicular field. Let ℓ be a typical coherence length of the perpendicular field, in the radial direction. Then the loop can contain up to $n = r^2/\ell^2$ flux tubes (strings) braided about each other. The invariant K tells us the minimum number of times that flux tubes wrap around each other between the two ends.

The magnetic energy E stored in the transverse field B_t is constrained by the complexity K

$$E \geq aK^2, \quad a \approx 10^{-1} L^{-1} n^{-4} \Phi^2 \tag{1}$$

(Berger 1993), where $\Phi = \pi r^2 B$ is the magnetic flux of the braid. To qualitatively understand this relationship, let $\mu = B_t/B$ measure the typical transverse field strength, and $L_t \approx L\mu$ be the transverse length of a (curved) tube. Then $E \approx L\pi r^2 B_t^2/8\pi = L\mu^2\Phi^2/(8\pi^2 r^2)$. Now it takes a transverse distance of about $\pi\ell$ for one tube to wrap around another tube. In projection, however, the tube will be seen to cross $n^{1/2}$ other tubes. One tube of length L_t then contributes $n^{1/2}L_t/\pi\ell$ crossings; and for n tubes

$$K \approx \frac{n^{3/2} L_t}{\pi\ell} = \frac{n^{3/2} L\mu}{\pi\ell}. \tag{2}$$

If we express μ in terms of K then $E \approx \Phi^2\ell^2K^2/(8r^2Ln^3)$, consistent with Eq. (1).

3. Relaxation along the minimum state: the source of "nanoflares"

The dynamics of solar magnetic loops is defined by the rate of energy input and energy loss. The velocity of photospheric motions V, twisting flux tubes inside a loop, is relatively small (1 km/s) compared to the Alfvén speed (100 km/s or higher in the low corona). For twisting a thin flux tube of radius $\ell << L$ however, the characteristic time ℓ/V can be comparable with the characteristic magnetic relaxation time L/V_A. The relaxation typically involves reconnection which proceeds at the rate of tens or more Alfvén times.

Consider the dynamics of a loop (braid) in the minimum state (Fig. 2). If the crossing number of the loop structure is high enough then internal reconnections will move the configuration down the minimum curve releasing the energy in small portions. According to estimates by Berger (1994) for a three string braid, the reconnections become effective when the ratio between the transverse and axial components of the field in the braid, B_t/B, exceeds 0.3, which corresponds to about a 30 degree angle between the directions of neighboring field

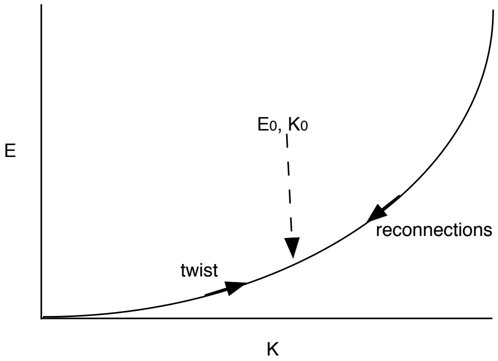

Fig. 2. A magnetic loop emerging with arbitrary values of E_0 and K_0 evolves fast (in a reconnection time) to the minimum state defined by the magnetic energy and crossing number. Reconnections destroy crossings and thus move the configuration down along the minimum state curve. Random photospheric motions increase topological complexity and thus move the configuration up along the curve.

lines. If the transverse field is smaller there is enough time between reconnections for random convective motions to entangle the braid, thus increasing the crossing number and moving the structure up along the minimum state. This leads to a stationary situation in which the rate of magnetic energy input through the random motions is balanced by the rate of energy release through the reconnections. The energy per unit time released in a loop due to moving along the minimum state can be estimated from Eq. (1): $dE/dt = 2aKdK/dt$. Due to convective motions of a random-walk type (with root-mean-square velocity V) the flux tubes become more entangled. If the stepsize λ is large enough, e.g. $\lambda \approx \ell$, then the entanglement prevents cancellation between subsequent steps going in opposite directions. Thus the rate of change of L_t, and hence μ and K, will be proportional to V. From Eq. (2), $dK/dt = (\epsilon V/\ell)n^{3/2}$, where ϵ measures the efficiency of braiding due to the random motions. Berger (1994) found $\epsilon \approx 0.5$ for a three-string braid. The value of μ at which energy loss through reconnection balances energy supply from the twisting will be assumed to be 0.3. If N is the total number of such loops on the solar surface then the power per unit area for the whole Sun is

$$P_b = \frac{N}{4\pi R_\odot^2}\frac{dE}{dt} \approx \frac{0.1\epsilon\mu N V}{6\pi R_\odot^2}\left(\frac{\Phi}{r}\right)^2 . \tag{3}$$

Note that it does not depend on n. To evaluate the power (3) we identify the braids with closed loops evolving in the solar magnetic network. The network magnetic field is mostly open, the only polarity mixing (closed loops) is provided by small bipoles of typical flux $\Phi = 10^{19}$ Mx called ephemeral regions. Ephemeral regions have been extensively studied by K. Harvey (Ph.D. thesis Utrecht Univ., 1993) and others whose ground-based results are summarized and extended by the new SOHO/MDI results in a recent paper by Schrijver et al. 1997. The emergence rate of ephemeral regions estimated from the ground observations is about $dN/dt = 10^3$ per day on the entire Sun. The rate estimated from the MDI magnetograms is about 10 times higher due to the improved spatial resolution.

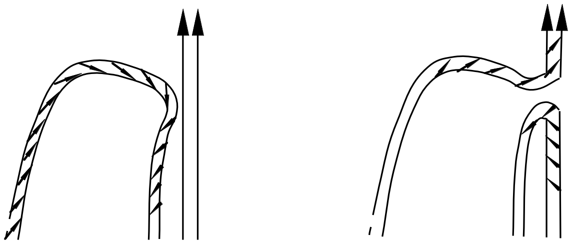

Fig. 3. The reconnection of a topologically complex closed magnetic loop with an open flux tube (left picture) releases a train of circularly polarized Alfvén waves into the solar corona. The new closed loop (right picture) formed in this process continues the blinking "nanoflare" activity.

The mean life-time of the ephemeral regions is 4.4 hours (the dispersion is high, there are ephemeral regions living 12 hours). Hence about $N = 10^4 \times (4.4/24) \approx 2 \times 10^3$ of these regions, which we identify with our braided loops are present on the Sun at any time. An ephemeral region is composed of many unresolved flux tubes. We conservatively estimate the radius associated with a loop to be about the width of the network, $r_* \approx 10^3$ km. The speed of motions in the network is about 1 km/s. Substituting these numbers into Eq. (3) we obtain $P_b \approx 3 \times 10^5 \mathrm{erg/cm^2 s}$. This power is released in small portions, intermittently, and may be associated with "nanoflares" envisaged by Parker (1990). Although this power is insufficient to heat the whole corona it can be important source of heating of the lower part of the corona, see the next section, and can explain the phenomenon of "blinkers" recently observed by SOHO/CDS (see http://solg2.bnsc.rl.ac.uk/cds/main.html).

4. Generation of waves by reconnections of braided loops with open field lines

There is another, more sporadic, effect of reconnection: From time to time, randomly, a topologically complex loop meets an open field of opposite direction and reconnects, releasing a train of Alfvén waves up into the corona (Fig. 3, first suggested by Axford and McKenzie 1992). Because these waves arise from a twisted configuration, they are circularly polarized. Since K scales as $n^{3/2}$, the number $Kn^{-3/2}$ measures how many times one flux tube inside the loop wiggles about its neighbors. The size of this wiggling can be associated with the minimum wavelength of the emitted waves: $\lambda = L/(Kn^{-3/2}) \approx 3l/\mu$. The corresponding upper-bound frequency $V_A/\lambda \approx \mu V_A/3l$ is defined by the Alfvén speed and the typical radius of the flux tubes $\ell = rn^{-1/2}$ braided in the loop of radius r. The lower-bound frequency is about V_A/L.

At present, the numerical values of these frequencies can only be estimated on model grounds. For example, Marsch and Tu (1997b) use $B = 130$ G at $L = 3000$ km where the density is $5 \times 10^8 \mathrm{cm}^{-3}$ and $B = 10$ G at $L = 12700$ km where the density is $6 \times 10^6 \mathrm{cm}^{-3}$. At both levels it gives the Alfvén speed about 10^4 km/s. Because the size ℓ is below the present spatial resolution of observations (0.2 arcsec or about 150 km), we estimate the minimum of this size by the order of magnitude as a thickness of a skin-layer determined by plasma resistivity, i.e. $\ell \approx LR_m^{-1/2} \approx 1$ km, where $R_m \approx 10^8$ is the magnetic Reynolds number on the solar surface. With this minimum size and $V_A = 10^4$ km/s the upper frequency reaches about a thousand Hz required in these models. The lower-bound frequency is about 1 Hz. It is worth noting, however, that even with this extreme value of ℓ, these estimates are very crude already because there are no measurements of magnetic field in the solar corona. In view of the importance of these frequency bounds, especially the upper one, a more detailed study is needed.

On the other hand, there is no special need for very high frequency waves damped close to the solar surface. This heat release closest to the surface can instead be provided by the relaxation along the minimum energy state, described in the previous section. The waves with lower frequencies, say ≤ 100 Hz, can still be an effective source of heat and momentum higher in the solar atmosphere provided they carry a sufficient energy flux.

The energy flux of the waves depends on the energy content of the closed loop, $E(K)$ when it is in the minimum state, and the rate of its energy release contacts with the open configuration. To evaluate the energy flux we identify, as above, the closed loops with small-scale bipoles (ephemeral regions) and assume that the loops are destroyed through reconnections with the open field. Then the minimum rate of contacts with the open field is the rate of emergence of ephemeral regions. However during its life-time, a closed loop can come into contact with open configurations many times acquiring energy due to twisting its footpoints in between. The power released into the waves for entire Sun can be estimated as

$$P_w \approx \nu \frac{\delta E(K)}{4\pi R_\odot^2} \frac{dN}{dt} \approx 0.2 \times 10^5 \frac{\mathrm{erg}}{\mathrm{cm^2 s}} \nu\delta \qquad (4)$$

where ν is the rate of contact, δ is a portion of energy released and the numerical values $\Phi = 10^{19}\mathrm{Mx}, r = 10^3\mathrm{km}, L = 10^4\mathrm{km}, dN/dt = 10^4/\mathrm{day}$ are used. The estimate (4) reduces to the value $5 \times 10^5 \mathrm{erg/cm^2 s}$ used in the model by Marsch and Tu (1997) when $\nu\delta = 25$. Because the mean life-time of ephemeral regions is 4.4 hours and the mean time of gaining energy through twisting, proportional to ℓ/V, is short, the number of times the energy can be released by a closed loop into the open configuration can be high. For example, if the relaxation to the level of balance between twisting and reconnections on the minimum energy state takes $100\ell/V$, the loop can come into contact with the open field up to $\nu \approx 1.6 \times 10^2$ times. In this case, the power $5 \times 10^5 \mathrm{erg/cm^2 s}$ is achieved when 10% of contacts occur with about 15% energy released in a single contact. These values are not unreasonable.

The estimated power (4) is increased when we take into account the energy released due to reconnection of open field with newly emerging loops that are not in the minimum energy state. In this case the energy is higher than that, $E(K)$, used in Eq. (4), at least in the first collision. The difference can be estimated as follows. Assume that the magnetic loops within the convection zone are in their minimum state. In fact, it follows from the equipartition argument that the velocity of motions and the Alfvén speed in the convection zone are of the same order. Hence, the balance between increase and decrease of topological complexity (twisting and reconnections), can easily be achieved and sustained. Because the magnetic flux Φ and topological complexity K in Eq. (1) are conserved, the energy of an emerged loop is different from the energy the loop had inside the convection zone due only to the change of the loop size. Let L_0 be the loop size beneath the solar surface and L is the size above the surface, as used before. Then, it follows from Eq. (1), $\Delta E = E(K)(L/L_0 - 1)$. The expansion factor L/L_0, expected $\gg 1$, is unknown, although in principle can be found from MHD models of magnetic field generation, emergence and subsequent expansion the emerged loops into the solar atmosphere. In any case the emerging loop (not in the minimum state) can release L/L_0 times larger power than the loop in the minimum state. After that the loop will relax to the minimum state and the estimate (4) is valid.

We conclude that our estimates, based on the use of the concept of the minimum energy state, support models of coronal heating and acceleration of the solar wind by high-frequency Alfvén waves. The severe requirements on the upper-bound frequency, used in these models, can be relaxed by taking into account the energy released due to reconnections within magnetic loops whose footpoints are twisted by surface convective motions.

Acknowledgements. We thank the referee and Joan Feynman for helpful critical comments. This research was conducted in part at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautic and Space Administration, and it was supported in part by International Division of National Science Foundation.

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