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Research Note

Study of transient double-mode behaviour of nonlinear models of Cepheids

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Abstract. Nonlinear pulsation models of Cepheids pulsating in first and second overtone modes have been constructed in order to study their transient double-mode behaviour. The purposes were to: (1) verify the possible nonlinear effects on period ratios; (2) compare the general nonlinear characteristics of the theoretical light curves with the observed ones; (3) check the plausibility of the hypothesis of very low metallicity of CO Aur. The analysis of the transient behaviour indicates that the difference between linear and nonlinear period ratios is always negligible, and hence period ratios of linear models can be used for comparisons with observed values. Moreover, the Fourier analysis suggests that, for a given order of fit, some combination frequency terms should be preferentially detected in real stars, and this is confirmed by observations. Finally, the models suggest that first and second overtone double-mode Cepheids with very low metallicity must have much shorter period than CO Aur; this supports the view that CO Aur is compatible only with a standard mass-luminosity relation.

Key words: stars: individual: CO Aur – stars: oscillations – Cepheids

1. Introduction

Hydrodynamical models of Cepheids were successful in explaining several features of observed stars pulsating in just one mode. Up to now, however, no Cepheid model has been found to have stable double-mode pulsation and the cause for beat behaviour remains a puzzle at the present time (Buchler 1996; see however Kolláth et al. 1998 for recent progress). In the Galaxy there are thirteen double-mode Cepheids (DMC) pulsating in the fundamental and first overtone ($F/1O$) modes, and one star, CO Aur, pulsating in the first and second overtone ($1O/2O$) modes. Thanks to projects for the baryonic dark matter search such as MACHO, many $F/1O$ and $1O/2O$ DMC have been discovered in the Magellanic Clouds (e.g. Alcock et al. 1995). For reasonable physical parameter values the linear models are able to predict periods and period ratios of DMC in agreement with observations (e.g. Christensen-Dalsgaard & Petersen 1995), but an uncertainty remains about the possible effects of nonlinear pulsation on periods and period ratios. Some studies have shown that generally DMC are not able to discriminate between different mass-luminosity $M-L$ relations (Buchler et al. 1996), the only exception being CO Aur (Antonello et al. 1997). In the latter case, it seems that a standard M-L relation is the best choice, while a relation valid for 'overshooting' models would imply very low metallicity values. Such problems urged us to attempt a study of hydrodynamical models pulsating in two modes, and this was performed by analysing the transient phase of multimode pulsation before the convergence of the model to the stable single-mode pulsation. We recall that some years ago Buchler & Kovács (1987) made a similar survey of double-mode RR Lyrae models.

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2. Models

We selected four models so that their periods and period ratios were similar to those of CO Aur, and the effective temperature T_e was similar to that indicated by observed colour and spectrum of the star (Mantegazza 1983; see also Antonello et al. 1996). The standard $M-L$ relation (Becker et al. 1977) was assumed for models A-C, while a $M-L$ relation derived from evolution calculation with convective overshooting (Fagotto et al. 1994; see Antonello et al.1997) was adopted for models D and E. We also assumed different chemical compositions to verify their effects on nonlinear behavior. The adopted physical parameters and the linear nonadiabatic results are reported in Table 1; PR indicates the period ratio P_2/P_1 in the adiabatic (*ad*), nonadiabatic (*nad*) and nonlinear case (*NL*), respectively.

The nonlinear models were computed with the code TGRID (Simon and Aikawa 1986) using the opacity values supplied by OP project (Seaton et al. 1994), and the OPFIT code (Seaton 1993) for fitting and smoothing. We ignored convection entirely in static model construction and in linear and nonlinear analyses. For nonlinear simulations, we used the Richtmyer-Morton formula for artificial viscosity (Stellingwerf 1975) with parameters $c_Q = 10.0, c_{Qcut} = 0.005$; the number of zones was about 100.

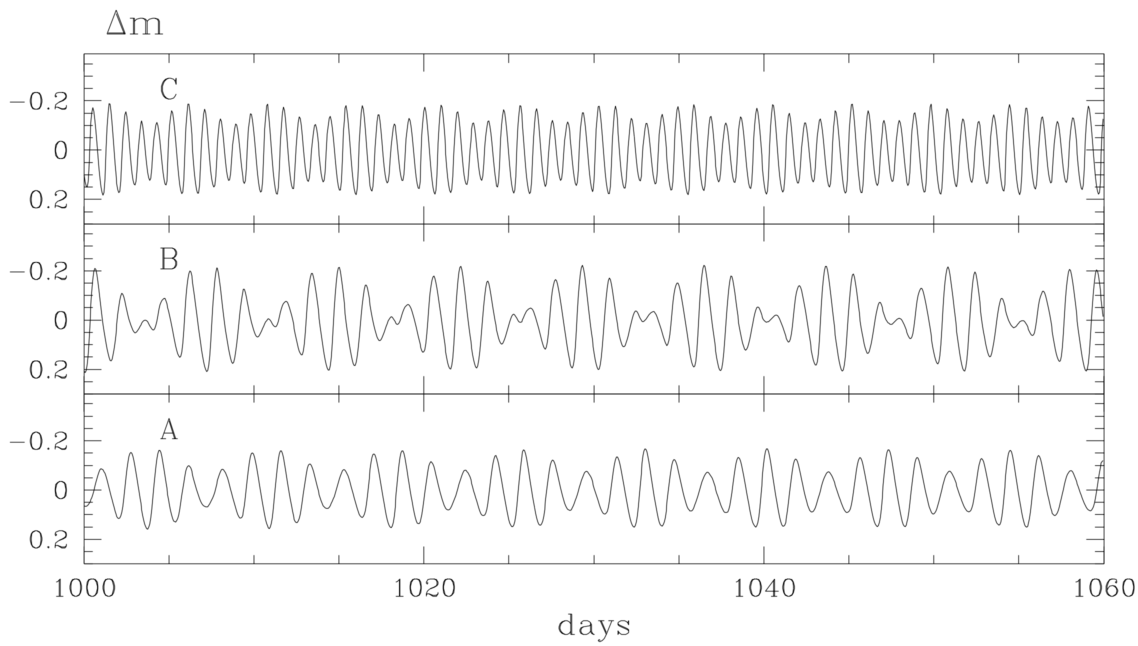

Fig. 1. Light curves of the models A, B and C during the transient phase

Table 1. Models

Model	$M/M_{\odot}$	$L/L_{\odot}$	T_e	Z	P_0	P_1	η_1	P_2	η_2	PR_{ad}	PR_{nad}	PR_{NL}
A	5.06	1122	6300	.020	2.54364	1.78487	5.6E-3	1.43260	9.8E-3	.8021	.8026	.8025
B	5.06	1122	6300	.010	2.49865	1.78472	1.3E-2	1.43569	2.3E-2	.8038	.8044	.8046
C	3.30	543	6500	.001	1.54657	1.14887	1.8E-2	.92909	4.1E-3	.8076	.8087	.8086
D	4.00	1070	6500	.005	2.47069	1.77857	9.2E-3	1.41480	3.4E-3	.7968	.7955	-
E	4.00	1070	6500	.001	2.40086	1.76788	1.4E-2	1.41914	1.4E-2	.8005	.8027	-

The initial static models were perturbed with an initial velocity given by

$$v(x) = a\xi_1(x) + b\xi_2(x) \tag{1}$$

where $\xi_1(x)$ and $\xi_2(x)$ are the first and second overtone eigenfunctions, respectively, and x is the fractional stellar radius. The coefficients a and b were taken to be 0.5 for all of the models. The subsequent analysis showed the presence also of some components related to the fundamental mode, even if this mode was linearly stable and was not excited. All models eventually settled into the first overtone limit cycle. The time intervals of the transient phase to the limit cycle, however, were longer in models A-C than in models D and E. After 500 days from start, models D and E were almost settled into the limit cycle, while the transient interval lasted for more than 2000 days in models A-C. It seems that models D and E are more dissipative than A-C because of their higher luminosities (for a given mass), and the suppression of minor modes by the dominant mode should be more effective in this case.

The light curves of models A-C during the transient phase are shown in Fig. 1. The data in the time interval 1000 - 1020 d of each model were analysed with similar methods to those adopted in the analysis of observational data (e.g. Pardo and Poretti 1997). We used the least-squares power spectrum method (Vanicek 1971) since it allows to detect one by one the constituents of the curves. Then we fitted the (bolometric) magnitudes by the formula

$$m(t) = m_0 + \sum A_{ijk} \cos[2\pi(if_2 + jf_1 + kf_0)(t - T_0) + \phi_{ijk}]. \tag{2}$$

As an example, the solution for the model A is reported in Table 2.

In principle it would be possible to detect a large number of components, but we limited the analysis only to terms with amplitudes larger than few tenths of millimag. The solutions for models A and B are good and there is no appreciable sign of changing amplitudes. In the case of model C the analysis shows the presence of a spurious component with a frequency close to that of f_1, which is related to the amplitude change during the 20 d time span. Models D and E were not characterized by a sufficiently long transition phase, and therefore it was not possible to get reliable information on the pulsation components.

The trend of the amplitudes of the pulsation modes in the time interval between 0 and 2500 days for the three models A, B and C is shown in Fig. 2.

Table 2. Coefficients of least squares fit of Model A light curve shown in Fig. 1; the frequencies of the modes are f_2=.69756, f_1=.55991 and f_0=.3923 c/d

order	$i(f_2)$	$j(f_1)$	$k(f_0)$	A_{ijk}	ϕ_{ijk}
1	0	1	0	.11311	-2.757
	1	0	0	.04373	2.571
	0	0	1	.00246	-2.972
2	0	2	0	.01307	-1.297
	1	1	0	.00915	-2.026
	1	-1	0	.00335	-2.149
	2	0	0	.00108	-2.763
	0	1	1	.00063	-1.577
3	0	3	0	.00172	0.019
	1	2	0	.00105	-0.332
	-1	2	0	.00070	-2.198
	0	2	-1	.00034	-1.410
	2	-1	0	.00030	0.009
	2	1	0	.00024	0.192
	3	0	0	.00022	-0.167
4	1	3	0	.00079	2.748
	2	2	0	.00051	2.260
	3	1	0	.00021	2.302
5	1	4	0	.00072	0.075
	0	5	0	.00052	-2.147
	2	3	0	.00043	-1.770
	-2	3	0	.00021	2.500
6	0	6	0	.00027	-0.589
	1	5	0	.00026	2.438
	2	4	0	.00022	0.858
7	2	5	0	.00030	2.176
	3	4	0	.00021	1.299

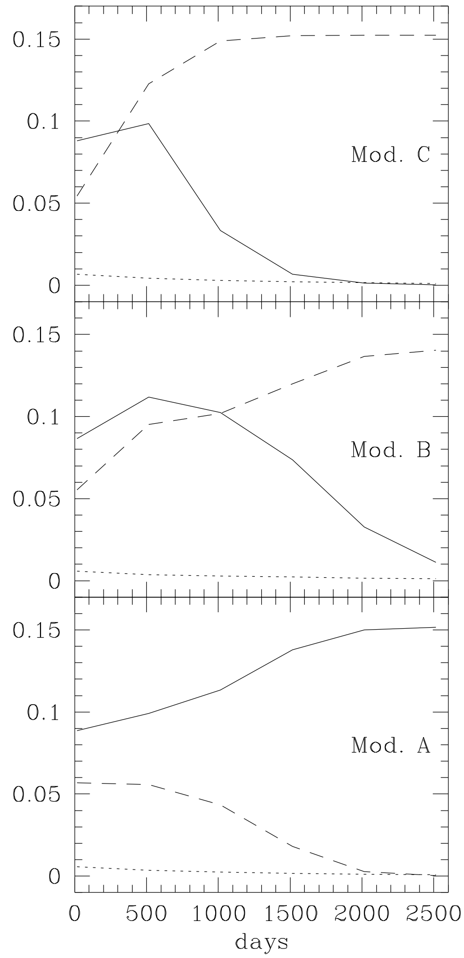

Fig. 2. Amplitudes of the light curve components as functions of time for the models A, B and C. *Continuous line*: first overtone; *dashed line*: second overtone; *dotted line*: fundamental mode

3. Discussion and conclusion

Since the analysis of the theoretical light curves has shown the presence of some components related to the fundamental mode, the models actually represent triple mode pulsators. The amplitude of the fundamental mode however is small, and this mode will not be considered any longer in this discussion. We just remark that significant triple combination frequencies ($i, j, k \neq 0$) are not shown by the analysis.

In general the pulsation amplitude is in reasonable agreement with that of observed DMC. In model A the first overtone has the largest amplitude, in model B the amplitude of the two modes is similar and in model C the amplitude of the second overtone is the largest. It is interesting to note from Table 1 that in all cases the nonlinear period ratio is not significantly different from the nonadiabatic period ratio. This characteristics is similar to that already noted in RR Lyrae models by Kovács & Buchler (1993). Moreover, since the observed period ratios of all $1O/2O$ DMC in Galaxy and LMC are in the narrow range .800 - .807, accurate comparisons should be performed using linear nonadiabatic rather than just linear adiabatic models.

Nonlinear pulsation favours the combination frequency terms where both i and j are positive. For a given order, a combination frequency term can have larger amplitude than the harmonic of the modes, in particular when both modes have comparable amplitudes. These appear to be rather general properties, and they are found in observed DMC, irrespective of the pulsation mode (e.g. Pardo & Poretti 1997).

Regarding the phases ϕ_{ijk}, we have checked whether the generalized phase differences

$$\phi_{ijk} - (i\phi_{100} + j\phi_{010} + k\phi_{001}) \tag{3}$$

(Antonello 1994) have similar regularities to those shown by very simple models and by observed DMC (see also Pardo & Poretti 1997). According to these works, the generalized phase differences of the various combination terms for a given order do not differ too much from each other. The present models indicate that second order terms have indeed generalized phase difference values between 4 and 5 rad as expected, but the higher order terms do not show similar regularities. The only plausible mechanism which can change significantly the phases is a resonance between pulsation modes, but this cannot explain all lacks of regularity. Therefore the question is if stable double-mode pulsation is strictly required for having 'regular' values.

It was not possible to get long-lasting transient double-mode behaviours for the models D and E, while it was obtained for model C. This is probably related to the fact that, owing to lower nonadiabatic characteristics, models with standard $M-L$ have significantly longer transition phases than models with overshooting $M-L$. Therefore we can derive an interesting suggestion: $1O/2O$ DMC with relatively long period should be found only if they have a large enough Z value and a relatively low luminosity. A comparison with the results of the simple linear models (Antonello et al. 1997) shows quite clearly that a model for the galactic DMC CO Aur cannot have a luminosity as large as that given by overshooting-type $M-L$ relations. Therefore the present study confirms the discrepancy about the $M-L$ relations already remarked by these authors. The other implication is that in galaxies with lower metal content such as the Magellanic Clouds we should expect DMC with shorter periods; in other words, the observed period distribution of DMC in LMC should be also a product of the pulsational stability characteristics, and not only of the stellar formation and evolution. A cautionary note is of course needed at this point: the above conclusions should be considered just as interesting hints, and must be substantiated with better nonlinear models.

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