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Synchrotron absorption in pulsar magnetospheres

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Abstract. Cyclotron absorption is known to be significant in pulsar magnetospheres. It is shown that when particles absorb significant fraction of pulsar radio emission, they acquire relativistic gyration. Therefore synchrotron absorption by relativistic plasma with small pitch-angles is considered. It is shown that both the frequency of the absorbed radiation and the absorption coefficient remain the same as in the cyclotron case provided that the angle the absorbed radiation makes with plasma velocity exceeds the pitch-angle. This condition holds if the absorbed energy is less than initial energy of plasma. Spontaneous reemission of the absorbed energy is considered. In short-period pulsars, a significant fraction of the absorbed energy is reemitted in the far infrared band.

Key words: plasmas – radiation mechanisms: non-thermal – stars: pulsars: general

1. Introduction

The radio emission observed from pulsars is known to be highly directional. So it is widely accepted that the pulsar radio emission originates well within the magnetosphere of the neutron star within a narrow tube of open magnetic field lines. The electron-positron plasma streams relativistically along the tube therefore the emission is concentrated in a narrow beam directed along the magnetic axis. Where the radio emission is presumably generated, the magnetic field strength is extremely high, so that the electron gyro-frequency greatly exceeds the frequency of the originating radio waves in the plasma rest frame. The gyro-frequency decreases outwards due to dipolar character of the magnetic field. So at some radius the gyro-frequency becomes equal, in the plasma rest frame, to the frequency of the outgoing radiation, that is the cyclotron resonance takes place. Then the radiation is absorbed, the plasma particles being excited into higher Landau orbitals. Because particles on high Landau orbitals emit synchrotron radiation, some part of the absorbed energy is reemitted at larger frequencies.

Note that in the magnetized plasma the cyclotron resonance occurs at the negative harmonic of the gyro-frequency as well. Then the particle transitions into higher Landau orbitals result in the anomalous Doppler emission, if only the plasma allows corresponding subluminous waves. Since initially the particles populate the ground Landau orbital, these waves should be amplified (Tsytovich & Kaplan 1972; Kawamura & Suzuki 1977, Machabeli & Usov 1979, 1989; Lominadze *et. al.* 1983, Kazbegi *et. al.* 1991). The instability is known to be possible at very high plasma densities for waves propagating along the magnetic field. We do not consider here this process because waves emitted well within the magnetosphere come to the resonance region at large enough angle to the magnetic field; therefore these waves may be only absorbed but not amplified.

The cyclotron absorption of radiation within pulsar magnetospheres was first considered by Blandford & Scharlemann (1976). The authors found the process to be extremely effective. According to their result, in the standard pulsar models radio emission from the internal parts of the magnetosphere should be heavily absorbed. Later on Mikhailovskii *et.al.* (1982) treated the cyclotron absorption of radio waves at pulsar conditions in terms of the plasma theory and came to similar conclusions. Note that the problem of radio wave escape from pulsar magnetospheres may be avoided by assuming that plasma generation above the polar cap is unsteady. Then at any given moment the plasma do not fill the whole open magnetic field tube and radiation can freely escape from the magnetosphere when there is no plasma at the cyclotron resonance radius.

Both of the two papers mentioned above deal with the case of non-relativistic particle rotation in the guiding-center frame. Indeed the electron-positron plasma leaves the polar cap region moving along the magnetic field in the lowest Landau orbital because any transverse energy immediately radiates away in the superstrong magnetic field. However it will be shown below that far from the neutron star, where the cyclotron resonance condition is met for radio frequencies, the electrons and positrons readily acquire relativistic gyration energies through absorption of pulsar radio emission. The aim of our work is to examine the conditions in which radio absorption allows relativistic transverse motion of particles and to consider the absorption by the relativistically gyrating particles. In Sect. 2 we estimate some essential parameters determinant for the cyclotron absorption process. Only the cyclotron resonanse within the light cylinder is considered, which is justified for ”normal” pulsars with $P \gtrsim 0.1$s and for low-field recycled pulsars with $P \gtrsim 10$ms. Sect. 3 deals with the kinematics of absorption. It is shown that when the particles absorb significant amount of pulsar radio emission, they acquire relativistic gyration energies. A general consideration of the synchrotron absorption and emission in terms of Einstein's coefficient method is given in Sect. 4. Sect. 5 is devoted to an analysis of spontaneous emission resulting from particle radiative de-excitation. In Sect. 6 we make some summary. The Appendix contains the calculation of the synchrotron absorption coefficient on the basis of plasma theory. The condition on which the plasma influence is negligible is given there as well.

2. Characteristic parameters of cyclotron absorption

Let radiation propagate through a medium of relativistically moving electrons and positrons embedded in a magnetic field. The cyclotron resonance condition for radio frequencies is met at the distances from the neutron star comparable to the light cylinder radius; the plasma density there falls considerably because the open field lines diverge. Hence, the refractive index of any wave mode at the cyclotron resonance radius is close to unity (for more details see Appendix), so that one can regard the waves undergoing the cyclotron resonance to be approximately transverse electromagnetic ones. If a wave reaches (by any way) the cyclotron resonance region, it is already transverse electromagnetic wave independently of what type of wave was radiated well within the magnetosphere.

We consider absorption of linearly polarized transverse electromagnetic waves. Electrons and positrons absorb linearly polarized radiation identically, therefore below we consider only electrons. Although in pulsar conditions electron and positron distribution functions are known to be slightly different, the contribution of the effect to absorption process is negligible. Note that the two-stream instability, which may be caused by this difference, does not affect the cyclotron absorption process because the instability generates underluminous waves in the Cherenkov resonance with particles whereas we consider absorption of waves whose frequencies meet the condition of cyclotron resonance (see Appendix).

The optical depth to cyclotron absorption of radiation may be easily obtained as follows (Blandford & Scharlemann 1976). The absorption cross-section in the rest frame of an absorbing electron is given by a well-known formula:

$$\sigma = 2\pi^2 \frac{e^2}{mc}\delta(\omega' - \omega_H). \tag{2.1}$$

Here e is the electron charge, m the electron mass, c the speed of light, ω' the frequency of incident radiation, ω_H the cyclotron frequency; $\omega_H = \frac{eB}{mc}$, with B being the magnetic field strength. If the electron is moving at the angle θ to the flux of radiation, the probability of absorbing a photon per unit length is $\sigma(1-\beta\cos\theta)$, where β is the electron velocity in units of c. Therefore the optical depth to cyclotron absorption in the laboratory frame is written as

$$\Gamma_c = \int \sigma(1-\beta\cos\theta)N dl, \tag{2.2}$$

where N is the plasma number density and integration is over the ray trajectory.

We assume the magnetic field inside the pulsar magnetosphere to be dipolar one: $B = B_\star(r_\star/r)^3$, where $r_\star$ is the neutron star radius, $B_\star$ the field strength at the stellar surface. Continuity of the plasma flow within the open field line tube requires the plasma number density to follow the magnetic field strength:

$$N = N_\star \left(\frac{r_\star}{r}\right)^3.$$

The number density $N_\star$ at the bottom of the open field line tube is conveniently normalized by the Goldreich-Julian charge density

$$N_\star = \frac{\kappa B_\star}{Pce},$$

where κ is the plasma multiplicity factor, P the pulsar period. Note that the standard pulsar models (*e.g.*, Ruderman & Sutherland 1975, Arons 1983) suggest $\kappa \sim 10^3 - 10^4$, however because of significant uncertainties in the available models, one can also assume κ is outside this range. Making use of the above formulas, the cyclotron optical depth may be estimated as

$$\Gamma_c \sim \kappa \frac{r_c}{r_L}\theta_c^2, \tag{2.3}$$

where $r_L = cP/2\pi$ is the light cylinder radius, r_c and θ_c are referred to the point of cyclotron resonance.

Now we turn to magnetosphere geometry in order to find the cyclotron resonance radius r_c and the angle θ_c (see also Barnard 1986). The cyclotron resonance condition may be written as

$$\omega\gamma(1-\beta\cos\theta) = \omega_H, \tag{2.4}$$

where γ is the plasma Lorentz-factor. First we assume that $r_c \ll r_L$. Let a ray be emitted along the magnetic field line at a radius r_0. In the case of non-rotating dipolar magnetic field the ray would make the angle $3\theta_0(1-r_0/r)/4$ with the field line at a radius r; here θ_0 is the polar angle of the emission point. Due to rotation the magnetic axis turns through the angle $(r-r_0)\sin\vartheta/r_L$, with ϑ being the angle between the rotational and magnetic axes of the pulsar. Then the angle between the ray and magnetic line at the radius $r \gg r_0$ is $r\sin\vartheta/2r_L \pm 3\theta_0/4$. One can neglect θ_0, since the ray is emitted along the magnetic field well inside the open field line tube $\theta_0 \lesssim \sqrt{r_0/r_L} \lesssim 0.1$. The magnetic field structure is known to deviate from the dipolar one, the field lines bending away from the rotational direction. However, the corresponding angle, $\sim (r/r_L)^{3/2}$, does not exceed the rotation angle and can be neglected. In the laboratory frame electrons move along the magnetic lines with an ultra-relativistic velocity and take part in the magnetosphere rotation with the linear velocity $cr\sin\vartheta/r_L$. So to the first order in r/r_L we find the angle θ between the ray and electron velocity to be

$$\theta = \frac{r}{2r_L}\sin\vartheta. \tag{2.5}$$

At the cyclotron resonance radius $\theta\gamma \gg 1$; the cyclotron resonance condition (2.4) then yields:

$$\frac{r_c}{r_L} = \frac{0.3B_{\star 12}^{1/5}}{(P^3\nu_9\gamma_2\sin^2\vartheta)^{1/5}}, \tag{2.6}$$

where $B_{\star 12} \equiv \frac{B_\star}{10^{12}\text{G}}$, $\nu_9 \equiv \frac{\nu}{10^9\text{Hz}}$, $\gamma_2 \equiv \frac{\gamma}{10^2}$. The star radius here and hereafter is taken as $r_\star = 10^6$cm. Thus the resonance for radio frequencies occurs inside the light cylinder on condition that

$$P > 0.13 B_{\star 12}^{1/3} \nu_9^{-1/3} \gamma_2^{-1/3} \sin^{-2/3}\vartheta \quad \text{s} \tag{2.7}$$

So for long-period pulsars the cyclotron resonance takes place well within the light cylinder at any reasonable conditions. In that case one can evaluate the optical depth to cyclotron absorption using Eqs. (2.3)–(2.6):

$$\Gamma_c = \frac{2\kappa_3 B_{\star 12}^{3/5} \sin^{4/5}\vartheta}{(P^3 \nu_9 \gamma_2)^{3/5}}. \tag{2.8}$$

Apparently, the cyclotron absorption inside the light cylinder is efficient if the plasma multiplicity factor $\kappa_3 \equiv \frac{\kappa}{10^3} \gtrsim 1$. In agreement with Eq. (2.7), if $P \sim 0.1$s the resonance takes place close to the light cylinder; this is also true for the recycled pulsars with $P \gtrsim 10$ms because their magnetic field is low. Given that $r_c \approx r_L$ the angle θ is of order unity. So the cyclotron depth for pulsars with $P \sim 0.1$s may be estimated as

$$\Gamma_c \sim 10^2 \kappa_3, \tag{2.9}$$

which is rather high at any reasonable κ. Because the magnetic field structure beyond the light cylinder is badly known, we do not consider fast pulsars ($P < 0.1$s for normal pulsars and $P < 10$ms for recycled ones).

3. Variations of plasma parameters in the course of absorption process

In the previous section the optical depth to cyclotron absorption of radio waves in pulsar magnetospheres was found to be significant. So the plasma particles absorb an essential part of the energy of incident radiation, the energy of their transverse motion increasing. The treatment of the absorption process carried out above is valid until the particles absorb enough energy to perform relativistic transverse motion in the guiding-center frame. In the present section we deduce the condition under which the transverse motion becomes relativistic and examine whether it is the case in pulsar magnetospheres.

It is convenient to investigate the absorption process in the corotating frame, in which the electric field is absent and initially electrons stream along the magnetic field at the relativistic velocity. Consider absorption by an electron moving at an angle θ to incident radiation. Conservation of energy and momentum parallel to the field implies

$$\begin{aligned} \gamma mc^2 + \alpha\gamma mc^2 &= \gamma_1 mc^2, \\ \gamma mv + \alpha\gamma mc\cos\theta &= \gamma_1 m v_\parallel, \end{aligned} \tag{3.1}$$

where γ and γ_1 are initial and final electron Lorentz-factors, respectively, $\alpha\gamma mc^2$ is the energy absorbed. Hence, one can obtain:

$$\beta_\parallel = \frac{\beta + \alpha\cos\theta}{1+\alpha}, \tag{3.2}$$

$$\beta_\perp = \frac{\sqrt{\alpha^2\sin^2\theta + 2\alpha(1-\beta\cos\theta)}}{1+\alpha}, \tag{3.3}$$

where $\beta_\parallel$, $\beta_\perp$ are the components of the final electron velocity, parallel and perpendicular to the field, respectively, in units of c, β the initial velocity in units of c. Let the absorbed energy be much less than the initial energy of the electron, $\alpha \ll 1$, then $\beta_\perp \approx \theta\sqrt{\alpha}$. The transverse electron momentum, $p_\perp \equiv \beta_\perp \gamma mc$, remains unchanged by Lorentz transformation to the guiding-centre frame. So the electron gyration is relativistic when

$$\alpha > (\theta\gamma)^{-2}, \tag{3.4}$$

i.e., when the absorbed energy is still very small as compared to the initial electron energy. As soon as the inequality (3.4) becomes true the Lorentz-factor of longitudinal motion, $\gamma_\parallel \equiv (1-\beta_\parallel^2)^{-1/2}$, decreases approaching $(\theta\sqrt{\alpha})^{-1}$. Note that, in accordance with Eqs. (3.2), (3.3), when the absorbed energy exceeds the initial electron energy, $\alpha > 1$, the electron pitch-angle approaches θ, if only $(\pi/2 - \theta) > 1/\alpha$. In the laboratory frame the longitudinal energy always significantly exceeds the transverse one.

Given the efficient cyclotron absorption the plasma absorbs an essential part of the energy of passing radiation. So the maximum α may be estimated as the ratio of the initial energies of radiation and the absorbing plasma. Radiation is supposed to be emitted along the open magnetic lines and concentrated into a beam of angular width ψ ($\psi \sim 0.1$). First we consider long-period pulsars. At the cyclotron resonance radius r_c ($r_c \ll r_L$), the scale length of lightened area in the direction perpendicular to magnetosphere rotation is ψr_c while in the direction of rotation it is $\sim r_c^2 \sin\vartheta / r_L$. Then the cross-section of lightened area may be estimated as

$$S = \frac{\psi r_c^3}{r_L}\sin\vartheta. \tag{3.5}$$

Note that it turns out to be much less than the open field line tube cross-section, which is $\pi r_c^3/r_L$. Therefore only a small part of the plasma occupying the open field line tube takes part in absorbing radiation. In agreement with the definition of α_{max} one can write:

$$\alpha_{max} = \frac{L}{NS\gamma mc^3}, \tag{3.6}$$

where L is the pulsar luminosity at a given frequency. Using Eqs. (2.6), (3.5) Eq. (3.6) may be reduced to the form:

$$\alpha_{max} = \frac{3 L_{28} P^2}{\kappa_3 \gamma_2 \psi_{-1} B_{\star 12} \sin\vartheta}, \tag{3.7}$$

where $L_{28} \equiv \frac{L}{10^{28}\text{erg}\cdot\text{s}^{-1}}$, $\psi_{-1} \equiv \frac{\psi}{0.1}$. It is easy to see that α_{max} is independent on r_c since $N \propto r_c^{-3}$, $S \propto r_c^3$. Therefore in case of short-period pulsars ($r_c \sim r_L$) α_{max} is also given by Eq. (3.7). Note that the value of α_{max} appears to vary within orders of magnitude due to the essential dependence on L, P and κ. The inequality (3.4) with α given by Eq. (3.7) is true if

$$\kappa_3 < 700 L_{28} P^{4/5} \nu_9^{-2/5} B_{\star 12}^{-3/5} \gamma_2^{3/5} \psi_{-1}^{-1} \sin^{-9/5}\vartheta.$$

In accordance with Eq. (2.8), if $\kappa_3 \lesssim P^{9/5}$ the efficiency of cyclotron absorption is small. Then the energy absorbed is characterized by $\alpha = \alpha_{max}\Gamma_c$. Using the latter in Eq. (3.4) yields that relativistic gyration energies are achieved at

$$L_{28} \geq 10^{-3} P \nu_9 \psi_{-1} \sin\vartheta.$$

These inequalities hold for all pulsars, therefore absorption by the relativistically gyrating electron should be considered.

4. Synchrotron absorption coefficient

As soon as an electron acquires relativistic gyration energy in the guiding-centre frame the character of absorption alters, so that the cross-section for cyclotron absorption given by Eq. (2.1) is expected to be no longer valid. Now we proceed to the more general consideration. The only restriction assumed is that in the corotating frame the electron energy is chiefly longitudinal; in another words, the electron pitch-angle is much less than unity. Absorption and emission of small-pitch-angle synchrotron radiation will be investigated in terms of Einstein's coefficients.

Einstein's coefficients A_l^j, B_l^j, B_j^l are related to each other by well-known expressions:

$$B_l^j = B_j^l; \qquad B_l^j = \frac{8\pi^3 c^3}{n^2 \hbar\omega^3 \left|\dfrac{\partial(n\omega)}{\partial\omega}\right|} A_l^j, \tag{4.1}$$

where n is the refractive index of the incident wave. In the above relations the refractive index deviation from unity can be ignored. Note that the power emitted spontaneously by gyrating electron, and consequently A_l^j, depends on n as well; there $|1-n|$ can be ignored on more stringent condition: $|1-n| \ll 1/\gamma^2$ (see, *e.g.*, Ginzburg & Syrovatskii 1969), with γ being the electron Lorentz-factor. Further on we suppose this condition is fulfilled. A more general treatment on the basis of the dispersion relations for the wave modes in the magnetized plasma is carried out in Appendix.

The absorption coefficient, μ, may be written as

$$\mu = \frac{\hbar\omega}{c}\sum (N_j B_j^l - N_l B_l^j) = \frac{8\pi^3 c^3}{\omega^2}\sum A_l^j (N_j - N_l). \tag{4.2}$$

Here N_j, N_l are the numbers of electrons on the j-th and l-th Landau orbitals, respectively, and summation is over all transitions between the states whose energies differ by $\hbar\omega$. The difference $(N_j - N_l)$ may be expressed in terms of the electron distribution function $f(p_\perp, p_\parallel)$, where $p_\perp$ and $p_\parallel$ are the components of momentum parallel and perpendicular to the magnetic field. The normalization is assumed to be as follows: $2\pi \int f(p_\perp, p_\parallel) p_\perp dp_\perp dp_\parallel = 1$. Then

$$N_j - N_l = -N\left(\frac{\partial f}{\partial p_\perp}\Delta p_\perp + \frac{\partial f}{\partial p_\parallel}\Delta p_\parallel\right). \tag{4.3}$$

Here the variations $\Delta p_\perp$, $\Delta p_\parallel$ may be found from the conservation laws:

$$\begin{aligned} \Delta p_\parallel &= \tfrac{\hbar\omega}{c}\cos\theta, \\ \varepsilon\Delta\varepsilon &= p_\parallel \Delta p_\parallel c^2 + p_\perp \Delta p_\perp c^2, \end{aligned} \tag{4.4}$$

where θ is the angle the incident wave makes with the magnetic field, ε the electron energy, $\Delta\varepsilon = \hbar\omega$.

The coefficient A_l^j is related to the synchrotron energy loss rate at the l-th harmonic of gyro-frequency, $P_l(\omega,\theta) = \hbar\omega A_j^{j+l}(\omega,\theta)$. Making use of the well-known formula for the power of synchrotron emission (see, e.g., Eq.(6.15) in Bekefi 1966) and taking into account that the observed power, $\eta_l(\omega,\theta)$, is related to $P_l(\omega,\theta)$ as $P_l = (1-\beta_\parallel\cos\theta)\eta_l$, one can write

$$A_j^{j+l}(\omega,\theta) = \frac{e^2\omega}{2\pi c\hbar}\left\{\left(\frac{\cos\theta - \beta_\parallel}{\sin\theta}\right)^2 J_l^2(\xi) + \beta_\perp^2 J_l'^2(\xi)\right\}\delta[l\omega_H/\gamma - \omega(1-\beta_\parallel\cos\theta)], \tag{4.5}$$

Here $J_l(\xi)$ is the Bessel function, $J_l'(\xi)$ the derivative with respect to argument,

$$\xi = \frac{l\beta_\perp \sin\theta}{1-\beta_\parallel\cos\theta}. \tag{4.6}$$

The first term in the braces of Eq. (4.5) describes an ordinary wave polarized in one plane with the wave vector and the magnetic field while the second term corresponds to an extraordinary wave polarized perpendicularly to this plane.

Note that the expression for $P_l(\omega,\Omega)$ written above refers to the corotating frame where the electric field is absent and there is no particle drift. Being transformed into the laboratory frame the angle θ changes due to aberration by the correction of $r\sin\vartheta/r_L$ becoming equal to $-r\sin\vartheta/2r_L$ while all the other quantities remain unchanged to the first order in r/r_L (see Sect. 2). Since θ turns out to be the same in the absolute value and emission is certainly symmetrical with respect to the magnetic field then to the first order in r/r_L Eq. (4.5) is appropriate in the laboratory frame as well.

Using Eqs. (4.3)-(4.6) in Eq. (4.2) one can obtain:

$$\mu_i = \frac{8\pi^3 e^2 N}{mc}\int \frac{dp_\perp dp_\parallel}{\gamma}\left[\left(-\frac{\partial f}{\partial p_\perp}\right)(1-\beta_\parallel\cos\theta) + \left(-\frac{\partial f}{\partial p_\parallel}\right)\frac{p_\perp}{\gamma mc}\cos\theta\right]\sum_l R_l^{(i)}\delta[l\omega_H/\gamma - \omega(1-\beta_\parallel\cos\theta)], \tag{4.7}$$

where $i = 1, 2$, with the indices 1 and 2 referring to ordinary and extraordinary modes, respectively;

$$R_l^{(1)} = \left(\frac{\cos\theta - \beta_\parallel}{\sin\theta}\right)^2 p_\parallel^2 J_l^2(\xi), \quad R_l^{(2)} = p_\perp^2 J_l'^2(\xi). \tag{4.8}$$

So we found the coefficients of synchrotron absorption and emission for both ordinary and extraordinary waves.

In Sect. 3 it was pointed out that as long as the energy absorbed remains less than the initial electron energy, $\alpha \ll 1$, the electron Lorentz-factor is nearly constant and $\beta_\perp \ll \theta$. Then the resonance condition given by the argument of delta-function in Eq. (4.7) may be approximately written as

$$\omega = \frac{2l\omega_H}{\theta^2\gamma}.$$

So in case $\beta_\perp \ll \theta$, the frequency of the basic harmonic suffering synchrotron absorption is the same as the frequency absorbed in the cyclotron case (cf. Eq. (2.4)) although electron gyration is already relativistic.

Note that the second term in the square brackets of Eq. (4.7) is $\beta_\perp^2/\theta^2$ less than the first one, so it can be neglected. The small-pitch-angle condition implies that $p_\parallel \equiv \beta_\parallel \gamma mc = \gamma mc$. Then the first term in Eq. (4.7) may be easily integrated by parts yielding:

$$\mu_1 = \frac{16\pi^3 e^2 N}{mc} \int \frac{p_\parallel^2 dp_\perp dp_\parallel}{p_\perp \gamma} \left(\frac{\cos\theta - \beta_\parallel}{\sin\theta}\right)^2 \sum_l \xi J_l(\xi) J_l'(\xi)$$
$$\times (1-\beta_\parallel \cos\theta) f \delta[l\omega_H/\gamma - \omega(1-\beta_\parallel\cos\theta)], \qquad (4.9)$$

$$\mu_2 = \frac{16\pi^3 e^2 N}{mc} \int \frac{p_\perp dp_\perp dp_\parallel}{\gamma} \sum_l \frac{l^2-\xi^2}{\xi} J_l(\xi) J_l'(\xi)$$
$$\times (1-\beta_\parallel \cos\theta) f \delta[l\omega_H/\gamma - \omega(1-\beta_\parallel\cos\theta)]. \qquad (4.10)$$

In the last expression we used the relation:

$$\xi J_l''(\xi) + J_l'(\xi) = \frac{l^2-\xi^2}{\xi} J_l(\xi)$$

resulting from the Bessel equation. Recall that the above formulas are obtained for electrons. Positrons absorb linearly polarized radiation independently in the same way. Therefore, if distribution functions of electrons and positrons are the same, one can use Eqs. (4.9, 4.10) with N being the total number of particles. In general case one should take sum of the expressions (4.9) (or, correspondingly, (4.10)) for both these species with proper distribution functions. Note that in pulsar magnetospheres distribution functions of electrons and positrons do differ however the difference is not large, the characteristic energies of most of particles are the same.

To the first order in $\beta_\perp/\theta$ the Bessel function argument (4.6) becomes:

$$\xi = \frac{2l\beta_\perp}{\theta}. \qquad (4.11)$$

Then at $l \ll \theta/\beta_\perp$ the Bessel function can be approximately presented in the form:

$$J_l(\xi) \sim \frac{1}{l!}\left(\frac{\xi}{2}\right)^l. \qquad (4.12)$$

So induced absorption of the first harmonic is the most effective. Substituting Eqs. (4.11), (4.12) into Eqs. (4.9), (4.10) it is easy to obtain to the first order in $\beta_\perp/\theta$:

$$\mu_1 = \mu_2 = \frac{2\pi^2 N e^2}{mc\omega} F\left(\frac{2\omega_H}{\omega\theta^2}\right), \qquad (4.13)$$

with

$$F(\gamma) = 2\pi \int f(p_\perp, p_\parallel) p_\perp dp_\perp.$$

Above it is assumed that $\theta \gg 1/\gamma$. The absorption coefficient given by Eq. (4.13) is exactly the same as the cyclotron one. In agreement with Eq. (2.2), $\mu = N\sigma\theta^2/2$. Then taking $f(\gamma) = \delta(\gamma - \tilde\gamma)$ we immediately come to $\sigma = 2\pi^2 \frac{e^2}{mc}\delta(\omega\tilde\gamma\theta^2/2 - \omega_H)$, which is the customary cross-section for cyclotron absorption. Thus the latter is appropriate in the synchrotron case as well, provided that $\beta_\perp \ll \theta$.

In the previous section we found out that as soon as the energy absorbed exceeds the initial electron energy, $\alpha > 1$, $\beta_\perp$ approaches θ. Then the cross-section for cyclotron absorption is no longer valid. In this case Eqs. (4.7)–(4.8) for the absorption coefficients cannot be simplified without using a specific form of the distribution function. Since the distribution function formed after absorption of large amount of energy may be rather complicated detailed consideration of the case $\beta_\perp \sim \theta$ is beyond the scope of the present paper.

As long as $\beta_\perp \ll \theta$ the optical depth for synchrotron absorption certainly coincides with that given by Eq. (2.8) in the cyclotron case. Hence, at

$$\kappa_3 > 0.5 \nu_9^{3/5} P^{9/5} B_{\star 12}^{-3/5} \gamma_2^{3/5} \sin^{-4/5}\vartheta \qquad (4.14)$$

pulsar radiation should be heavily affected by synchrotron absorption. Note that the shorter the pulsar period the more effective the synchrotron absorption process; given that $P \leq 0.1$ s the absorption depth appears to be large at any conceivable pulsar parameters. Then inhomogeneity of the plasma flow can only account for the escape of pulsar radiation from the magnetosphere.

5. Spontaneous reemission

In the above consideration spontaneous reemission was supposed to be sufficiently weak to alter the electron gyrational energy essentially. The spontaneous power found by Blandford & Scharlemann (1976) for the cyclotron case proved to be negligible. However, the synchrotron power is sure to be larger than the cyclotron one.

Note that while synchrotron absorption occurs predominantly at the first harmonic of gyro-frequency, spontaneous emission peaks at the essentially higher frequency, ω_s. The latter corresponds to the harmonic with the number of $\sim \gamma_0^3$, where γ_0 is the electron Lorentz-factor in the guiding-centre frame, so that

$$\omega_s \gamma_\parallel (1 - \beta_\parallel \cos\theta_s) = \omega_H \gamma_0^2,$$

where θ_s is the angle which the emission makes with the magnetic field. Spontaneously emitted radiation is mainly directed along the electron velocity, $\theta_s \sim 1/\gamma_\parallel$, within the interval $\Delta\theta_s \sim 1/\gamma$. Then ω_s is related to the frequency of radiation absorbed at the angle θ to the field, ω, as

$$\omega_s = \omega \frac{\theta^2\gamma^2}{2}. \qquad (5.1)$$

First let us consider long-period pulsars ($P \sim 1$s). By means of Eqs. (2.5), (2.6) eq.(5.1) may be readily reduced to the form:

$$\nu_s = 10^{11} \frac{\nu_9^{3/5} \gamma_2^{8/5} B_{\star 12}^{2/5}}{P^{6/5} \sin^{4/5}\vartheta} \quad \text{Hz}. \qquad (5.2)$$

Pulsar luminosity provided by the spontaneous emission in the frequency band of $\sim \nu_s$ may be estimated as

$$L_s = \frac{2e^2\beta_\perp^2\gamma^2\omega_H^2}{3c}NSr_c, \tag{5.3}$$

where N is the plasma number density, S the lightened area given by Eq. (3.5). Assuming $\beta_\perp$ to be $\sim \theta$ one can obtain:

$$L_s = 7.4\cdot 10^{22}\kappa_3\psi_{-1}\left(\frac{\nu_9^3\gamma_2^{13}B_{\star 12}^{12}\sin^{11}\vartheta}{P^{26}}\right)^{1/5} \quad \mathrm{erg}\cdot\mathrm{s}^{-1}. \tag{5.4}$$

Although the luminosity provided by the synchrotron spontaneous reemission is larger than that in the cyclotron case, for long-period pulsars it is still rather low.

Proceeding to short-period pulsars ($P \sim 0.1$s) we recall that in this case one can assume $r_c \sim r_L$, $\theta \sim 1$. Then the frequency of spontaneous emission (5.1) may be rewritten as

$$\nu_s = 5\cdot 10^{12}\nu_9\gamma_2^2 \quad \mathrm{Hz} \tag{5.5}$$

In accordance with Eq. (3.7), $\alpha_{max} < 1$ provided that $P \sim 0.1$s. So one can estimate $\beta_\perp$ as $\sim \theta\sqrt{\alpha_{max}} \sim \sqrt{\alpha_{max}}$. The luminosity (5.3) then takes the form

$$L_s = 2.5\cdot 10^{22}L_{28}P^{-5}\gamma_2 B_{\star 12}^2 \quad \mathrm{erg}\cdot\mathrm{s}^{-1}. \tag{5.6}$$

Hence, for short period pulsars the spontaneous synchrotron emission appears to be rather high.

6. Conclusions

We investigated absorption of radiation by an ultrarelativistic magnetized electron-positron plasma in pulsars. In the cyclotron limit the process appears to be significant at pulsar conditions. In case the energy absorbed becomes large enough to provide relativistic electron gyration in the guiding-centre frame, the customary theory of cyclotron absorption is expected to be no longer appropriate. This is really the case in pulsars because absorption of the pulsar radio emission cause the electrons to acquire relativistic gyration. Thus a more general consideration of synchrotron absorption and emission is necessary.

As the electron pitch-angle is much less than the angle θ, which radiation absorbed makes with the magnetic field, the resonance frequency and the synchrotron absorption coefficients for both modes turn out to coincide with the cyclotron ones. Therefore the estimates (2.8) and (2.9) for the optical depth are valid until the absorbed energy remains less than initial energy of absorbing plasma. For the long-period pulsars, $P \sim 1$s, absorption is significant if only the multiplicity factor values $\kappa_3 \gtrsim 1$. As for the short-period pulsars, $P \sim 0.1$s, synchrotron absorption is significant at any conceivable parameters. Absorption in the very fast pulsars ($P < 0.1$s for the normal pulsars and $P < 10$ms for the low-field recycled ones) is not considered here because in these pulsrs the cyclotron resonanse occurs beyond the light cylinder, where the magnetic field structure is badly known. Note that the estimated large optical depth does not argue against the models of pulsar emission in which the waves originate well within the magnetosphere. The problem of radio wave escape can be naturally avoided through the assumption of the nonuniform plasma distribution within the open field line tube (Blandford & Scharlemann 1976). Then the radiation passing through the regions of sufficiently dilute plasma is weakly absorbed, the most part of it reaching the observer.

The power of synchrotron radiation resulting from spontaneous reemission of the energy absorbed proved to exceed significantly the cyclotron one. Being strongly dependent on pulsar period, spontaneous emission of relativistically gyrating electrons appears to be rather intense for short-period pulsars, so that an essential fraction of the energy absorbed is reemitted in the far infrared band.

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Appendix A: treatment of synchrotron absorption in terms of plasma theory

Here we consider the synchrotron absorption process by plasma methods to find the condition on which ignoring the plasma influence is appropriate and, correspondingly, the above expressions for absorption coefficients, (4.7)–(4.8), are valid. The dispersion relation for normal waves in the uniformly magnetized plasma (see, *e.g.*, Krall & Trivelpiece 1973, Eqs. (8.10.10)—(8.10.11)) can be easily generalized for the relativistic plasma particle motion as follows:

$$\mathrm{Det}\ ||\ (1-\frac{k^2c^2}{\omega^2})\delta_{\alpha\beta} + \frac{k_\alpha k_\beta c^2}{\omega^2} - a_{\alpha\beta}\ ||= 0, \tag{A1}$$

with

$$a_{xx} = \frac{\omega_p^2 m^2}{\omega}\sum_{l=-\infty}^{\infty}\left[\left[\frac{l^2\omega_H^2}{k_\perp^2}J_l^2\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\chi\right]\right], \tag{A2}$$

$$a_{xy} = -a_{yx} = \frac{i\omega_p^2 m}{\omega} \tag{A3}$$

$$\times\sum_{l=-\infty}^{\infty}\left[\left[\frac{l\omega_H p_\perp}{k_\perp}J_l\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)J_l'\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\chi\right]\right],$$

$$a_{xz} = \frac{\omega_p^2 m}{\omega}\sum_{l=-\infty}^{\infty}\left[\left[\frac{l\omega_H p_\parallel}{k_\perp}J_l^2\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\Lambda\right]\right], \tag{A4}$$

$$a_{yy} = \frac{\omega_p^2}{\omega}\sum_{l=-\infty}^{\infty}\left[\left[p_\perp^2 J_l'^2\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\chi\right]\right], \tag{A5}$$

$$a_{yz} = \frac{-i\omega_p^2}{\omega}\sum_{l=-\infty}^{\infty}\left[\left[p_\perp p_\parallel J_l\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)J_l'\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\Lambda\right]\right], \tag{A6}$$

$$a_{zx} = \frac{\omega_p^2 m}{\omega}\sum_{l=-\infty}^{\infty}\left[\left[\frac{l\omega_H p_\parallel}{k_\perp}J_l^2\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\chi\right]\right], \tag{A7}$$

$$a_{zy} = \frac{i\omega_p^2}{\omega} \sum_{l=-\infty}^{\infty} \left[\left[p_\perp p_\parallel J_l \left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right) J_l' \left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right) \chi \right]\right], \tag{A8}$$

$$a_{zz} = \frac{\omega_p^2}{\omega} \sum_{l=-\infty}^{\infty} \left[\left[p_\parallel^2 J_l^2 \left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right) \Lambda \right]\right], \tag{A9}$$

Here the magnetic field is assumed to be directed along the z-axis and the y-component of the wave vector is taken to be zero; $k_\parallel$, $k_\perp$ are the wave vector components k_z, k_x parallel and perpendicular to the magnetic field, respectively, $\omega_p \equiv \sqrt{\frac{4\pi Ne^2}{m}}$ the plasma frequency, i the imaginary unity,

$$\chi \equiv \frac{\partial f}{\partial p_\perp^2}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) + \frac{k_\parallel v_\parallel}{\omega}\frac{\partial f}{\partial p_\parallel^2}, \tag{A10}$$

$$\Lambda \equiv \frac{\partial f}{\partial p_\parallel^2} - \frac{l\omega_H}{\gamma\omega}\left(\frac{\partial f}{\partial p_\parallel^2} - \frac{\partial f}{\partial p_\perp^2}\right). \tag{A11}$$

Double square brackets in Eqs. (A2)–(A9) denote the integral operator

$$[[F(p_\perp, p_\parallel)]] \equiv \int \frac{4\pi F(p_\perp, p_\parallel) p_\perp dp_\perp dp_\parallel}{\gamma(k_\parallel v_\parallel + l\omega_H/\gamma - \omega)}$$

$$= \mathcal{P} \int \frac{4\pi F(p_\perp, p_\parallel) p_\perp dp_\perp dp_\parallel}{\gamma(k_\parallel v_\parallel + l\omega_H/\gamma - \omega)} \tag{A12}$$

$$+ i\pi \int 4\pi F(p_\perp, p_\parallel)\delta(k_\parallel v_\parallel + l\omega_H/\gamma - \omega)\frac{p_\perp dp_\perp dp_\parallel}{\gamma}.$$

It should be noted that, in contrast to vacuum case, in the magnetized plasma the resonance may occur not only at positive harmonics of gyro-frequency but also at zeroth and negative ones. The case $l = 0$ corresponds to Cherenkov resonance; at $l < 0$ the electron transitions into higher Landau orbitals result, due to the anomalous Doppler effect, in emitting the waves. Corresponding resonance conditions may be met only in case plasma allows slow enough, perhaps $\omega/k_\parallel < c$, waves. Provided that initially the particles populate the ground Landau orbital, such waves should be amplified (Tsytovich & Kaplan 1972; Kawamura & Suzuki 1977, Machabeli & Usov 1979, 1989; Lominadze *et. al.* 1983, Kazbegi *et. al.* 1991). However, this instability is possible only at very high plasma densities, $\kappa \gg 10^4$, for waves propagating along the magnetic field. We do not consider such a case here. Note that an instability at $l > 0$ is also possible (synchrotron maser) but only in case higher Landau orbitals are overpopulated, $\frac{\partial f}{\partial p_\perp} > 0$. One can easily see that in this case the absorption coefficients (4.7) (see also below Eqs. (A15), (A18)) become negative. However such a distribution function cannot be formed in the course of absorption process, therefore we consider here only the case $\frac{\partial f}{\partial p_\perp} \le 0$, when the absorption coefficients are positive.

Obviously, plasma contribution to wave dispersion is represented by the terms $a_{\alpha\beta}$. In the case of interest $a_{\alpha\beta} \ll 1$, so that their products can be neglected. In this approximation, the dispersion equation (A1) becomes factorized, the wave modes being independent:

$$1 - \frac{k^2c^2}{\omega^2} - \frac{4\pi\omega_p^2}{\omega}\sum_{l=-\infty}^{\infty}\int J_l'^2\left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right) R_l p_\perp^3 dp_\perp dp_\parallel = 0, \tag{A13}$$

$$1 - \frac{k^2c^2}{\omega^2} - \frac{4\pi\omega_p^2}{\omega}\sum_{l=-\infty}^{\infty}\int \left[1 - \beta_\parallel^2 - \left(\frac{\omega - k_\parallel v_\parallel}{k_\perp c}\right)^2\right]$$

$$\times J_l^2\left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right) R_l \gamma^2 m^2 c^2 p_\perp dp_\perp dp_\parallel = 0, \tag{A14}$$

where

$$R_l \equiv \frac{\frac{\partial f}{\partial p_\perp^2}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) + \frac{k_\parallel v_\parallel}{\omega}\frac{\partial f}{\partial p_\parallel^2}}{\gamma(k_\parallel v_\parallel + l\omega_H/\gamma - \omega)}.$$

Eqs. (A13, A14) describe the extraordinary and ordinary modes, correspondingly.

Given $a_{yy} \ll 1$ one can assume that $n \equiv \frac{ck}{\omega} \approx 1$. Then the damping decrement for the extraordinary mode takes the form:

$$\mathrm{Im}k = \frac{2\pi^2\omega_p^2}{c}\int\left[\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\left(-\frac{\partial f}{\partial p_\perp}\right)\right.$$

$$\left. + \frac{k_\parallel v_\parallel}{\omega}\frac{p_\perp}{p_\parallel}\left(-\frac{\partial f}{\partial p_\parallel}\right)\right] \tag{A15}$$

$$\times \sum_{l=-\infty}^{\infty} J_l'^2\left(\frac{k_\perp v_\perp \gamma}{\omega_H}\right)\delta[l\omega_H/\gamma - \omega + k_\parallel v_\parallel]\frac{p_\perp^2 dp_\perp dp_\parallel}{\gamma}.$$

The latter equation coincides with the absorption coefficient μ_2 given by Eqs. (4.7)–(4.8) provided that one substitutes $\beta_\parallel \cos\theta$ for $k_\parallel v_\parallel/\omega$ and $\sin\theta$ for $k_\perp c/\omega$, with θ being the angle between the wave vector and the magnetic field. In particularly, then the terms with $l \le 0$ do not contribute to the sum in Eq. (A15), since the argument of delta-function does not turn into zero. Such a substitution is possible in case

$$|1 - n| \ll \theta^2, \tag{A16}$$

Indeed making use of $n \approx 1$, $\theta \ll 1$, $\beta_\perp \ll 1$, one can write

$$\omega - k_\parallel v_\parallel = \omega(1 - n + \theta^2/2 + \beta_\perp^2/2 + 1/2\gamma^2).$$

Taking into account that we are interested in the case $\theta\gamma \gg 1$, $\theta \gtrsim \beta_\perp$, one can see that above substitution is indeed possible only on the condition (A16) but not on the weaker condition $n \approx 1$. Now we are to estimate the integral in the principal value sense in Eq. (A13) to find $|1 - n|$. Making use of the resonance condition, the Bessel function argument may be written as

$$\frac{k_\perp v_\perp \gamma}{\omega_H} \approx \frac{2l\beta_\perp\theta}{\beta_\perp^2 + \theta^2 + \gamma^{-2}} \sim l\frac{\beta_\perp}{\theta}.$$

Note that since the Bessel function steeply decreases with the order, only low harmonics mainly contribute to the integral. For these harmonics at the above argument one can take $J_l' \lesssim 1$. Taking into account the standard estimate

$$\mathcal{P}\frac{1}{k_\parallel v_\parallel + l\omega_H/\gamma - \omega} \sim \frac{1}{\omega},$$

one can finally find $|1-n| \sim \frac{\omega_p^2\theta^2}{\omega^2\gamma}$. Substituting this estimate into Eq. (A16), one can see that plasma influence on the absorption of the extraordinary wave is negligible on condition that

$$\omega_p^2 \ll \gamma\omega^2. \tag{A17}$$

Then the damping decrement (A15) reduces to the absorption coefficient μ_2 given by Eqs. (4.7)–(4.8).

The ordinary mode can be treated similarly. The damping decrement turns out to be:

$$\begin{aligned}\mathrm{Im}k = \frac{2\pi^2\omega_p^2}{c}\int\frac{p_\parallel^2 dp_\perp dp_\parallel}{\gamma}\left[1-\beta_\parallel^2-\left(\frac{\omega-k_\parallel v_\parallel}{k_\perp c}\right)^2\right] \\ \times\left[\left(1-\frac{k_\parallel v_\parallel}{\omega}\right)\left(-\frac{\partial f}{\partial p_\perp}\right)+\frac{k_\parallel v_\parallel}{\omega}\frac{p_\perp}{p_\parallel}\left(-\frac{\partial f}{\partial p_\parallel}\right)\right] \qquad \text{(A18)} \\ \times\sum_{l=-\infty}^{\infty}J_l^2\left(\frac{k_\perp v_\perp\gamma}{\omega_H}\right)\delta[l\omega_H/\gamma-\omega+k_\parallel v_\parallel],\end{aligned}$$

which certainly reduces to the absorption coefficient μ_1 if the condition (A16) is satisfied. Estimating the integral in the principal value sense in the dispersion relation for the ordinary mode, one can come to the same condition (A17).

Above we considered the one-component plasma. In general case one should take the sum over charge species in the right-hand sides of Eqs. (A2–A9). This implies the similar summing in the right-hand sides of Eqs. (A15), (A18). So the components of the pair plasma absorb radiation independently.

Now let us check whether the condition (A17) holds in cyclotron resonance zones of real pulsars. Substituting the parameters from Sect. 2 in Eq. (A17) yields

$$1.4\cdot 10^{-11}\frac{\kappa_3 B_{12}^{1.6}}{\nu_9^{2.6}P^{4.6}\gamma_2^{0.6}} \ll 1, \quad \text{for} \quad P\sim 1\text{s},$$

$$5.1\cdot 10^{-10}\frac{\kappa_3 B_{12}}{\nu_9^{2}P^{4}\gamma_2} \ll 1, \quad \text{for} \quad P\sim 0.1\text{s}.$$

These inequalities hold for any reasonable parameters therefore our neglect of the plasma influence on the absorption process is justified.

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