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Magnetic field evolution of accreting neutron stars

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Abstract. We study the evolution of the magnetic field of an accreting neutron star in the frozen field and incompressible fluid approximations. The plasma is accreted onto two polar caps and squeezes some of the surface material of the neutron star toward the equator. The frozen B-field is then pushed toward the equator and is eventually buried there. The magnetic field within the polar cap areas, which is defined by the Alfvèn radius, decreases due to the expansion of the polar cap areas resulting from the physical motion of the accreted material, which conserves the magnetic flux. But the decrease of the magnetic field also changes the Alfvèn radius which modifies the size of the polar cap and also affects the decrease of the magnetic flux within the polar caps. Therefore, the magnetic field enclosed by the polar caps appears to decay rapidly with a time scale of $\sim 10^5 \frac{m_B/10^{-3}M_\odot}{\dot{M}/(10^{18}gs^{-1})}$ years. As a consequence the magnetic field outside the polar cap is increasing because the total flux of the entire stellar surface is conserved in our approximations. The decrease of the polar cap magnetic field will stop and reach a minimum value $\sim 10^8$G when the magnetic field outside the polar cap reaches $B_{out} \sim 10^{15}$G, which is strong enough to stop the motion of the accretion material across the stellar surface. However, this strong B_{out} cannot be observed because the accreted matter stopped by this strong field cannot move toward the equator. Instead it moves inward and pulls this field inside the crust with a time scale $\sim 10^6 H_5 R_6^2 \rho_{14} \dot{M}_{18}^{-1}$ yr. Pulsars accreting similar masses but having very different magnetic field may result from different equations of state.

Key words: stars:neutron – stars: magnetic fields – X-rays: stars – stars: binaries: close

1. Introduction

The generation and evolution of magnetic field of neutron stars have been the subject of long-standing debate since the discovery of neutron stars (e.g. Bhattacharya and van den Heuvel, 1991; Ruderman, 1991 a,b,c; Bhattacharya et al. 1992; Chanmugam, 1992; Lewin et al. 1993; Phinney and Kulkarni, 1994). It was believed that the magnetic field of radio pulsars decay with a time scale about a few million years (e.g.Ostriker and Gunn 1969; Lyne and Smith, 1990; Narayan and Ostriker 1990). The recent analysis by Bhattacharya et al. (1992) concludes that the magnetic fields of isolated radio pulsars may not decay at a time scale of 100 Myr. However, a large decrease of magnetic field seems to occur in the binary mass transfer phase. The clear evidence is that the evolution of magnetic field is mostly correlated with the duration of the mass accretion in the binary phase, and that the decay of the field is inversely correlated with the total amount of matter accreted (Taam and van den Heuvel, 1986; van den Heuvel et al. 1986). A simple inverse correlation between field strength and the estimated transfer mass is also suggested by Shibazaki et al. (1989).

Theoretically, the first calculation of the evolution of the neutron star magnetic field is based on the ohmic dissipation which indicates that the magnetic field may not decay significantly in a Hubble time if the field fills initially the entire stellar crust (e.g.Flowers and Itoh, 1976; Flowers and Ruderman, 1977; Sang and Chanmugam, 1987). Romani (1990) suggested that the accretion-induced thermal effect can speed up the ohmic dissipation of the crustal magnetic field and that the advection of field lines in the equator can bury the field there. More detailed studies of the accretion-induced thermal effect (Geppert and Urpin 1994; Urpin and Geppert 1995) showed that the thermal history of the neutron star can be drastically changed if the accretion rate is high and lasts for a long period of time. The crust of neutron stars is heated up and the conductivity can decrease by a few orders of magnitude compared with that of cool neutron stars. Hence the field can decay to $10^{-3} - 10^{-4}$ of its original value if the high accretion rate lasts for $10^6 - 10^7$ years. This is because the conductivity depends sensitively on the temperature (Yakovlev and Urpin 1980). It is generally believed that the core of neutron stars is a quantum liquid consisting of quantized vortex lines and quantized magnetic fluxoids(cf. Pines 1980 for a review). The interactions between the vortex lines and fluxoids determine the evolution of the core magnetic field and hence affect the overall magnetic evolution (Srinivasan et al 1990; Ruderman 1991; Ding, Cheng & Chau 1993).

The recent studies of magnetic field evolution in accreting neutron stars indicate that the evolution of the stellar magnetic field is affected by: the hydrodynamic flow in the melted ocean

Send offprint requests to: K.S. Cheng of Dept. of Phys., The University of Hongkong and the plasma instabilties(e.g Woosley & Wallace 1982)- the advection of the magnetic field-the crustal motions-the density and pressure profiles in the crust-the spatial distribution of the magnetic field- accretion-induced heating and dissipation(Romani 1990; Urpin & Konenkov 1997a,b; Urpin, Geppert & Konenkov 1998), Hall conductivity (Jones 1987), diamagnetic screen (Zhang et al 1994, Zhang 1998) etc.. Therefore it is logical to include all these effects in studying the evolution of the magnetic field. In other words, we need to solve a set of highly non-linear and coupled differential equations. Obviously various approximations are necessary. At the end, some important features may be lost due to various approximation schemes. On the other hand, there may be a few dominant mechanisms which govern the evolution of the magnetic field. In this paper, we intend to identify one possible key mechanism and to study how it affects the evolution of the magnetic field.

Recently, van den Heuvel and Bitzaraki (1995a, 1995b), from a statistical analysis of 24 binary radio pulsars with nearly circular orbits and low mass companions ($0.2-0.4M_{\odot}$ helium white dwarfs in most cases), discovered a clear correlation between spin period and orbital period, as well as between magnetic field and orbital period. These relations strongly suggest that an increased amount of accreted mass leads to the decay of the magnetic field. A 'bottom' field strength of several 10^8 G is also implied. Furthermore, van den Heuvel and Bitzaraki (1995a) proposed a plausible mechanism for field decay, which is similar to that proposed by Romani (1990). In their mechanism, the accretion material is initially channeled by the strong magnetic field into the two polar patches, corresponding to an Alfvèn radius of several hundreds of star radii, pushes the field lines aside and thus dilutes the polar field strength due to the flow of accreting material from pole to equator zone where the magnetic field will be buried. The bottom field should be reached when the accretion becomes isotropic, which corresponds to the Alfvèn radius matching the star radius with a magnetic field strength of $\sim 10^8$ G. In this paper, we propose an accretion induced field decay model, which is very similar to that proposed by van den Heuvel and Bitzaraki (1995a) and by Romani (1990).

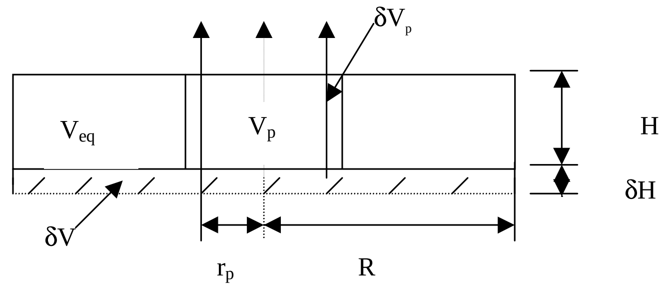

Fig. 1. Illustration of the cross section of plane magnetic slab. The thickness of the slab, H, represents the depth of the neutron star crust; r_p is the radius of the accretion polar patch; $V_p=\pi r_p^2 H$ and $V_{eq}=\pi H(R^2-r_p^2)$ are the volumes of polar zone and the equatorial zone respectively. The volume (δV) immersed in the liquid core of the neutron star is shown by the shaded area. δH is the immersed depth. δV_p is the expansion volume of the polar zone.

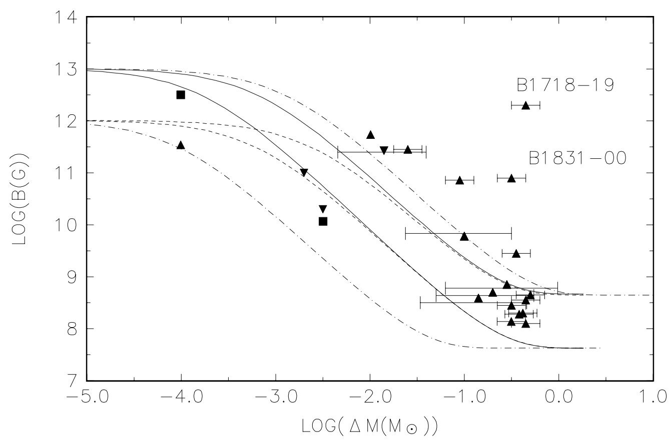

Fig. 2. Magnetic field versus accreted mass with various initial magnetic fields, accretion rates and crustal masses. The solid curves are intial magnetic field $B_o=10^{13}$ G and $\dot{M}=10^{18}gs^{-1}$(upper solid curve) or $10^{16}gs^{-1}$(lower solid curve). The dashed curves are the same as those in solid curves except the initial field $B_o=10^{12}$G. The upper (lower) dot-dashed curve is for the initial field $B_o=10^{13}$ G ($B_o=10^{12}$ G) and M$_{cr}=0.4M_{\odot}$ (M$_{cr}=0.03M_{\odot}$).

2. Evolution of the magnetic field

We consider a simplified accreted neutron star structure as depicted in Fig. 1. We take the spherical crust of the neutron star as a plane slab for simplification. We assume that the magnetic field fills the entire crust of the neutron star. It may be generated at the birth of the neutron star by either the frozen-in primeval magnetic field of the parent star in its presupernova stage (e.g. Woltjer, 1964; Pacini, 1967) or by a self-excited hydromagnetic dynamo mechanism (e.g. Parker, 1979; Zel'dovich, Ruzmaikin and Sokoloff, 1983) or by thermomagnetic effects (Urpin and Yakovlev, 1980; Blandford, Applegate and Hernquist, 1983; Blondin and Freese, 1986; Urpin, Levshakov and Yakovlev, 1986). The large conductivity implies that the field lines are frozen in the crust. Initially, if the field is enough strong, 10^{12} G in general, then the accreted matter will be channeled into two polar caps by the field lines. The piled accreted matter in the polar patches will be compressed and become part of the crustal material. Here, we neglect the details of the surface motion and the instability of the plasma, which is not clear yet (Woosley and Wallace 1982), and suppose that most of the accreted matter under the accretion polar patches arises from the expansion of the polar zone. Otherwise, the excess accreted matter would form the 'mountain' over the polar and result in a mass quadrupole, which is in contradiction with the observation (for a review, see Baym and Pethick, 1975; Shapiro and Teukolsky, 1983, chap. 9). The expansion of the polar zone should not change the equilibrium distribution of matter between the polar zone and equatorial zone. As a result, a part of the material in the equatorial zone will be squeezed into the core of the star such that the volume occupied by the crust remains unchanged. For a realistic equation of state (e.g. Wiringa, Fiks and Fabrocini, 1988), we can use the formula proposed by Lorentz, Ravenhall and Pethick (1993) to estimate the changes of the crustal density and the crustal volume for a neutron star from $1.4M_\odot$ to $1.6M_\odot$. That are 4% and 15% respectively. Under the incompressible fluid approximation together with the assumption of a constant crustal volume, the volume of the polar zone (V_p) will be expanded by (cf. Fig. 1)

$$\delta V_p = \frac{\dot{M}\delta t}{\rho} - A_p \delta H \quad , \tag{1}$$

where A_p is the surface area of the polar cap, ρ is the average density of the crust, $\dot{M}$ is accretion rate, δt is a particular accretion duration, H is the thickness of the crust and $\delta H/H$ is the fraction of the crust thickness dissolving into the core which is given by

$$\frac{\delta H}{H} = \frac{\dot{M}\delta t}{M_{cr}} \quad , \tag{2}$$

where $M_{cr} = \rho \pi R^2 H$ is the crust mass.

The area of the accretion polar patch can be expressed as (Shapiro and Teukolsky, 1983)

$$A_p = \pi r_p^2 = \pi R^3/R_o \quad , \tag{3}$$

where r_p is the radius of polar patch, R$_o = \alpha$ R$_A$ is the inner radius of the accretion disk, and R_A is the Alfvèn radius given by

$$R_A = 3.2 \times 10^8 \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} \left(\frac{M}{M_\odot}\right)^{1/7} \quad \mathrm{cm} \quad , \tag{4}$$

α is shown to be ~ 0.5 (Ghosh and Lamb 1979). Here M is the mass of neutron star, $\dot{M}_{17}$ is the accretion rate in units of 10^{17} gs^{-1}, and μ_{30} is the magnetic moment in units of 10^{30} Gcm^3. Therefore, using the relation of $\delta V_p = H\delta A_p$ and Eqs. (3) and (4), the variation of polar zone volume can be related to the variation of magnetic field in the polar zone δB_p by

$$\delta V_p = -V_p \frac{4\delta B_p}{7B_p} \quad . \tag{5}$$

Substituting Eqs. (2) and (5) into Eq. (1), we obtain the variation equation of the field evolution:

$$\frac{1}{(B_p/B_f)^{4/7} - 1} \frac{\delta B_p}{B_p} = -\frac{7\dot{M}}{4M_{cr}} \delta t \quad , \tag{6}$$

where B_f is defined by the inner radius of the accretion disk matching the radius of neutron star, i.e. $R_o(B_f) = R$, or

$$B_f = 4.3 \times 10^8 \left(\frac{\dot{M}}{\dot{M}_{Ed}}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/4} R_6^{-5/4} \quad \mathrm{G} \quad , \tag{7}$$

where $\dot{M}_{Ed}$ is the Eddington accretion rate and R_6 is the radius of the neutron star in units of 10^6cm. Solving Eq. (6) with the initial condition $B_p(t=0) = B_\circ$, the analytic solution is

$$B_p(t) = \frac{B_f}{\left[1 - C\exp\left(-\frac{\Delta M}{M_{cr}}\right)\right]^{7/4}} \quad , \tag{8}$$

where $C = 1 - \left(\frac{B_f}{B_o}\right)^{4/7}$ and $\Delta M = \dot{M}t$.

Fig. 2 indicates that the B-field seems to decay by one order of magnitude for an accretion mass $\Delta M = 10^{-2} M_\odot$ for $\dot{M} = \dot{M}_{Ed}$ and $B_o = 10^{12}$ G. Such decay rate appears unreasonable. However, we want to point out that, unlike the Ohmic decay process(Urpin and Geppert 1995), the decrease of the polar cap magnetic field (N.B. not the real decay of the magnetic field) results from the expansion of the polar cap and the field is frozen. Furthermore even if the field lines are completely frozen, the magnetic flux enclosed by the polar cap defined in Eq. (3) is not conserved. This is because the "real" expanded polar area $A_p^{'}(t+\delta t)$ due to the accreted material during δt is given by

$$A_p^{'}(t+\delta t) = A_p(t)\left(\frac{B_p(t)}{B_p(t+\delta t)}\right) \tag{9}$$

if the field lines are frozen. However, the polar cap defined by Eq. (3) is given by

$$A_p(t+\delta t) = A_p(t)\left(\frac{B_p(t)}{B_p(t+\delta t)}\right)^{4/7} \tag{10}$$

Since $\frac{B_p(t)}{B_p(t+\delta t)} > 1$, so $A_p^{'}(t+\delta t) > A_p(t+\delta t)$, which means that the new constructed polar cap area at time $t+\delta t$ is smaller than that of the real expanded polar cap area. In other words, the decrease of the magnetic field within the polar cap are determined by two mutually related factors, i.e. the expansion of the polar cap area due to the frozen field approximation and the rearrangement of the polar cap boundary due to the change of the Alfvèn radius which also depends on the magnetic field.

Again, we want to emphasize that the magnetic field of the entire star in the frozen field approximation is conserved, in other words,

$$B_p \pi r_p^2 + B_{out}\pi(R^2 - r_p^2) = B_o \pi R^2 \tag{11}$$

where B_{out} is the magnetic field outside the polar cap. Hence, the magnetic field outside the polar cap is given by

$$B_{out}(t) = B_o \frac{1 - \left(\frac{B_f}{B_p(t)}\right)^{4/7} \frac{B_p(t)}{B_o}}{1 - \left(\frac{B_f}{B_p(t)}\right)^{4/7}}, \tag{12}$$

Of course, $B_{out}(t)$ cannot increase unlimitedly. Physically, when the magnetic field energy density, $B_{out}^2/8\pi$ is comparable to the crustal stress, instead of pushing the magnetic field further toward the equator, the accreted matter will move toward to the core the star. We can estimate the maximum B_{out} by

$$\frac{(B_{out})_{max}^2}{8\pi} = \mu\theta, \tag{13}$$

where $\mu = 2 \times 10^{29}(z/40)(\rho/5 \times 10^{13}\mathrm{gcm}^{-3})^{4/3}\mathrm{ergcm}^{-3}$ is the shear modulus, z is the atomic number of nuclei, ρ is the crust density and θ is the shear angle. Taking the characteristic values z = 40, $\rho = 10^{14}\mathrm{gcm}^{-3}$ and $\theta = 10^{-2}\mathrm{rad}$, we get

$(B_{out})_{max} \sim 5 \times 10^{14}$ Gauss. This value corresponds to a minimum value of B_p obtained by solving Eq. (10):

$$(B_p)_{min} \sim B_f \left[1 + \frac{7B_o}{4(B_{out})_{max}}\right] \sim B_f \; if \; (B_{out})_{max} >> B_o \quad (14)$$

3. Applications

The evolutionary equation of the magnetic field of the neutron star is derived in our simplified model. From this model, four conclusions related to observations can be obtained: (i) In the case of a high mass X-ray binary (HMXB) or of the beginning stage of a low mass X-ray binary (LMXB), a little mass is accreted, $\Delta M \sim 10^{-4} - 10^{-5} M_{\odot}$, and the field evolutionary equation can be approximately written as

$$B_p = \frac{B_{\circ}}{1 + \frac{\Delta M}{m_B}} \; . \quad (15)$$

$m_B = (B_f/B_{\circ})^{4/7} M_{cr} \approx 10^{-3} M_{\odot}$ for $\dot{M} = 10^{18}\mathrm{gs}^{-1}$, which is the same form as the empirical formula of accretion-induced field decay, proposed by Shibazaki et al. (1989), with a well fitted mass constant $m_B = 10^{-3} - 10^{-5} M_{\odot}$. (ii) The apparent decay time scale of the polar cap magnetic field is given by

$$\tau_p \sim 10^5 \frac{(m_B/10^{-3} M_{\odot})}{\dot{M}/10^{18} gs^{-1}} \; \mathrm{yr}, \quad (16)$$

which seems an extremely short time scale. However, this time scale does not represent the real field decay time scale, because the decrease of the polar cap magnetic field results from the combined effects of polar cap expansion and the migration of magnetic flux lines out of the polar cap. Actually, the magnetic flux of the entire star is conserved. (iii) A minimum field exists in our model, namely,$B_{min} = B_f = 4 \times 10^8$ G which corresponds to $R_o = R$ and $\dot{M} = 10^{18}\mathrm{gs}^{-1}$ from Eq. (7). Hence the bottom field is reached when the accretion becomes isotropic. This was first predicted by van den Heuvel and Bitzaraki (1995a) from the analysis of the millisecond pulsar magnetic field. (iv) There exists a characteristic accretion mass, namely, $\Delta M = \frac{7}{4} M_{cr} \approx 0.2 M_{\odot}$, after which the bottom field is reached. This conclusion is consistent with the observed results summarized by van den Heuvel and Bitzaraki (1995a) with $\Delta M \approx 0.18 M_{\odot}$.

In Fig. 2, we compare our model curves given by Eq. (8) with the observational data which are listed in Table 1. Two solid (dashed) curves represent $B_o = 10^{13}$ (10^{12}) Gauss and $\dot{M} = 10^{18}\mathrm{gs}^{-1}$ and $\dot{M} = 10^{16}\mathrm{gs}^{-1}$ respectively. The filled upward and downward triangles and squares represent LMXB and HMXB respectively. The companion mass of L/H is intermediate. The upper (lower) dot-dashed curve is for a neutron star with initial mass 1.4$M_{\odot}$ and M_{cr} = 0.4$M_{\odot}$ (0.03$M_{\odot}$) and B_o = 10^{13}G (10^{12}G). The former corresponds to an intermediate stiff equation of state (EOS) called WFF1 which has a very small crustal component (Wiringa, Fiks and Fabrocini, 1988) and the latter corresponds to a very stiff equation of state called PS which has almost the largest normal crustal mass (Pandharipande and Smith 1975). The data in Fig. 2 indicates that 10 out of 23 pulsars have magnetic fields between 10^8 to 10^9 G. Such a large fraction of pulsars concentrating in this region seems quite consistent with our model prediction in Eq. (7) in which the "bottom" fields are independent of the initial magnetic field and insensitive to the EOS (of course the stellar radius depends on EOS but it is about 10^6 cm for a very soft EOS, e.g. Friedman and Pandharipande 1981, and 1.5×10^6cm for a very stiff EOS, e.g. PS). We can see that most of the observed data fall within or close to our model curves except for PSR1718-19 and PSR1831-00, which are quite far away from our curves. The former is argued to be formed by accretion induced collapse of white dwarf (van den Heuvel and Bitzaraki 1995b) and the latter is located in a globular cluster and may be formed by capture (Cheng and Dai 1997). According to standard binary evolution models(for a review, c.f. Savonije 1983; van den Heuvel 1983), HMXBs accrete with the Eddington rate but the accretion time scale is just the thermal time scale,

$$\tau_{th} \sim 3 \times 10^5 (10 M_{\odot}/M)^2 \mathrm{yr} \quad (17)$$

Therefore, the total accreted mass $\Delta M \sim \dot{M}_{Ed}\tau_{th} \simeq 0.01 M_{\odot}$. However, Urpin (1998) points out that the time scale estimated in Eq. 17 is only the maximum possible time scale and can only be achieved in very tight binaries, e.g. orbital period of order a day. A more typical time scale of 10^4 yr for near-Eddington mass tranfer gives

$$B(\Delta M) = B_o/(1 + \frac{10^{-4} M_{\odot}}{10^{-3} M_{\odot}}) \approx B_o. \quad (18)$$

In the extreme case where $\Delta M = 0.01 M_{\odot}$, B may reduce to 10^{11} Gauss. On the other hand, the accretion rate of LMXBs, whose companion mass $\sim 1 M_{\odot}$, is still close to Eddington accretion rate but the accretion time is about $\sim 3 \times 10^7$ yr (cf. Eq. 17). Therefore the total mass transfer can be as high as $\sim 1 M_{\odot}$ and so we obtain,

$$B(\sim 1 M_{\odot}) \sim B_f \sim 4 \times 10^8 \mathrm{Gauss} \quad (19)$$

4. Discussion

We have proposed a simple model to explain the magnetic field decay due to accretion in the frozen field, in the incompressible fluid approximation. The magnetic field in the polar cap decreases in an extremely short time scale of $10^5 \dot{M}_{18}^{-1} (m_B/10^{-3} M_{\odot})$ years or

$$\tau_p \propto M_{cr} B_o^{-4/7} \dot{M}^{-5/7} \quad (20)$$

due to the expansion of polar cap resulting from accreting material and the migration of magnetic flux out of the polar cap region. However, the magnetic flux of the entire star remains constant. It is interesting to point out that the magnetic field of the pulsars witha stronger initial field, a larger accretion rate and a softer EOS decrease faster.

It appears that a strong magnetic field outside the polar cap $(B_{out})_{max} \sim 10^{15}$G can be observed. To be more

Table 1. Parameters of binary neutron stars

Source	$B_s(G)$	Period(s)	$\Delta M(M_\odot)$	Type	Ref.
J0034-05	1.1×10^8	0.00188	0.7	L	a
J0437-47	4.9×10^8	0.00576	0.77	L	a
B0655+64	1.2×10^{10}	0.196	0.003	L/H	b
B0820+02	3×10^{11}	0.8649	0.005/0.04	H	b
J1045-45	3.8×10^8	0.00745	0.7	L	a
B1620-26	3×10^9	0.0111	0.45	L	a
J1713+07	2×10^8	0.00457	0.65	L	a
B1718-19	1.28×10^{12}	1.004	0.7	L?	a
B1800-27	7.7×10^{10}	0.3344	0.18	L?	a
B1831-00	8.7×10^{10}	0.5209	0.8	L?	a
B1855+09	3.1×10^8	0.00536	0.75	L	a
B1913+16	2×10^{10}	0.059	0.003	H	b
B1953+29	4.3×10^8	0.00613	0.03/0.59	L	a,b
J2019+24	1.82×10^8	0.00393	0.65	L	a
J2317+24	1.27×10^8	0.00345	0.79	L	a
A0538-66	$> 10^{11}$	0.069	0.002	H	b
Her X-1	3.07×10^{12}	1.24	0.0001	L/H	b
1E2259+59	5×10^{11}	7.7	0.01	L	b
GX1+4	$> 3 \times 10^{11}$	120	0.0001	L	b
GX5-1	6×10^9	0.01	0.02/0.5	L	b
CygX-2	6×10^9	0.01	0.02/0.5	L	b
ScoX-1	5×10^8	0.002	0.05/0.8	L	b
MXB1730-335	7×10^8	?	0.1/0.8	L	b

a: Van den Heuvel and Bitzaraki 1995
b: Taam and van den Heuvel 1986

precise, the magnetic moment outside the polar cap $\mu_{out} \sim B_{out} R(R^2 - r_p^2) \sim B_o R^3$ seems to be the dominant component. After the accretion phase, we should be able to observe this dipole component instead of the much smaller polar cap component. However, we have argued in Sect. 2 that such a strong field can stop the matter flow so that the accreted material moves toward the core of the star instead of being pushed toward the equator. Hence the field lines of B_{out} should be pulled and be buried in the crust. The decay time scale can be estimated as

$$\tau_d \sim \frac{H}{v_r} \tag{21}$$

where H is the depth of the inner crust, $v_r \sim \dot{M}/(\pi R^2 \rho)$ is the radial velocity of the accreting material at density ρ . Using $H \sim 10^5 \ \ \mathrm{cm}$, $\dot{M} \sim 10^{18} \ \ \mathrm{gs^{-1}}$, $\rho \sim 10^{14} \ \ \mathrm{gcm^{-3}}$ and $R \sim 10^6 \ \ \mathrm{cm}$, we obtain $\tau_d \sim 10^6$ years. After a time scale of $\frac{\Delta M}{\dot{M}}$, the μ_{out} will decay by a factor of $e^{-\frac{\Delta M}{\pi H R^2 \rho}} \sim 10^{-6\frac{\Delta M}{0.1 M_\odot}}$.

It should be noted that the flow kinetic energy density of the accreted matter is about $\sim \rho v^2 \sim 10^{-3} \ \ \mathrm{erg/cm^3}$, which is little compared with the magnetic energy density after the large amount of matter is accreted $B_{out}^2/8\pi \sim (10^{15})^2/8\pi \sim 10^{29} \ \ \mathrm{erg/cm^3}$. This increase of magnetic energy results from the internal transfer of the gravitational energy when the nuclear matter is compressed. The total pressure of the nuclear matter in the process of accretion-induced redistribution includes four parts, gravitational pressure P_g, matter pressure P_m, magnetic pressure and the flow of the kinetic energy pressure, i.e. $P_{total} = P_g + P_m + B^2/8\pi + \rho v^2$, where $P_g = \rho g H \sim 10^{33} \ \ \mathrm{erg/cm^3}$ and $g \sim 10^{14} \ \ \mathrm{cm/s^2}$ is the gravitational acceleration of neutron star surface. Since the gravitational pressure and matter pressure are much larger than that of the magnetic pressure, the variation of magnetic energy density induced by the deviation from the equilibrium of pressure distribution can be readjusted by the matter pressure and gravitational pressure during accreting compression. ρv^2 is produced by the residual force between gravitational pressure, matter pressure and the magnetic pressure.

In general, part of the matter accreted onto the polar cap will also sink to the core of the star and reduce the polar cap magnetic field. However, the magnetic field lines of the polar cap are interacting with the accretion disk. It is not easy to bury this type of ”open” field lines; on the other hand, the ”closed” field lines (B_{out}) are much less resistant against burying under the crust.

In our model, we have ignored the plasma instabilities, the variation of the crust during accretion, the melted surface flow, the details of how the magnetic field is buried in the equatorial zone, the Ohmic dissipation, the real spatial distribution of the magnetic field, etc. All these effects should be included in the future detail analysis of the problem of the accretion induced magnetic field decay.

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