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The orbital period of the SU Ursae Majoris star AK Cancri*

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Abstract. A time resolved spectroscopic study of the SU UMa subtype dwarf nova AK Cancri is presented. An orbital period of $P_{orb} = 0\overset{\mathrm{d}}{.}0651(2)$ and a radial velocity half-amplitude of K = 50(3) km s^{-1} were obtained from the radial velocity variations of the Hα emission line. Assuming various observational constraints, we deduced a most likely mass ratio of $q = 0.28 \pm 0.02$ and stellar masses of $M(1) = 0.41 \pm 0.03 M_\odot$ and $M(2) = 0.1135 \pm 0.0002 M_\odot$, for the primary and secondary star, respectively. In addition, the dynamical solution indicates a low orbital inclination ($i = 36^o \pm 3^o$).

Key words: accretion, accretion disks – stars: fundamental parameters – stars: individual: AK Cnc – stars: novae, cataclysmic variables

1. Introduction

Cataclysmic Variables (CVs) are close binary systems consisting of a white dwarf – the primary – accreting material via an accretion disk from a secondary star, thought to be a late-type, near-main-sequence star. The secondary fills its Roche-lobe and transfers material onto the accretion disk through the inner Lagrangian point. The SU Ursae Majoris (SU UMa) subtype dwarf novae belong to CVs, being characterized by normal outbursts, superoutbursts and "superhumps", which are $\sim 0\overset{\mathrm{m}}{.}2$ amplitude oscillations of the light curve during superoutburst repeating on a period very close to the orbital one. Theoretical reviews of SU UMa stars have been given by Smak (1984), Osaki (1989) and Cannizzo (1993); observations have been summarized by la Dous (1994), Warner (1995), Cordova (1995) and Osaki (1996).

AK Cancri (AK Cnc, hereafter) was classified as a SU UMa subtype dwarf novae after the discovery of superhumps by Kato (1994). The basic cycle length (between normal outbursts) seems to be 47 days, whereas the superhump period is $P_{sup} = 0\overset{\mathrm{d}}{.}06749(1)$ (Mennickent et al. 1996). These authors analyzed photometry obtained during the 1995 March superoutburst. In this paper we present spectroscopic observations obtained just after this photometric run, covering the stage of superoutburst decline.

Table 1. Spectroscopic observations of AK Cnc. N is the number of spectra taken per night. HJD$_{begin}$ is the initial Julian day (zero point HJD$_0$=244 9700). Estimates of the nightly mean V magnitude, along with their rms error are also given.

UT	N	HJD$_{begin}$	Exposure Time (s)	V
21/03/95	21	97.5319	300/600	17.3 ± 0.3
22/03/95	3	98.5230	600	18.3 ± 0.0
23/03/95	20	99.5139	600	18.4 ± 0.1
24/03/95	11	100.5063	600	18.6 ± 0.1

2. Observations and data reduction

Spectroscopic observations were carried out in the 250 cm-telescope at Las Campanas Observatory using the Modular Spectrograph. The TEK #2 chip (1024×1024 pixels), 21 μ/pix width, was used with grating 600 tilted by an angle of 37^o (at second order), yielding a spectral range of 4000-7000 Å and a spectral resolution of 4.5 Å. Details of the observations are given in Table 1.

All CCD images were reduced with standard IRAF routines, correcting them for bias level and detector response. One-dimensional spectra were extracted and calibrated in wavelength using comparison spectra with typically 25 He-Ar-Ne lines. The rms of the calibration function was typically lower than 0.05 Å (2 km s^{-1} at Hα). We tested the stability of the CCD to shifts in wavelength by cross-correlating comparison lamps taken the same night. In all cases, the mean shift was less than 0.1 pixel. In order to get rough flux calibrated spectra, we took wide-slit exposures of the standard star *LTT 2415* ($\alpha_{2000} = 05^h 56^m 24^s 30$, $\delta_{2000} = -27^o 51' 24''.30$, V =12.21, $B - V = +0.40$, Hamuy et al. 1992, 1994). Residuals of the flux-calibrated standard star with respect to published spectrophotometric magnitudes were of order of $0\overset{\mathrm{m}}{.}1$. On the other hand, the comparison star in the center of the field shown by Misselt (1996, V = $16\overset{\mathrm{m}}{.}36$) was also included in the slit. This enabled us to measure the nightly averaged differential (spectrophotometric) magnitudes listed in

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* Based on observations obtained at Las Campanas Observatory, Chile

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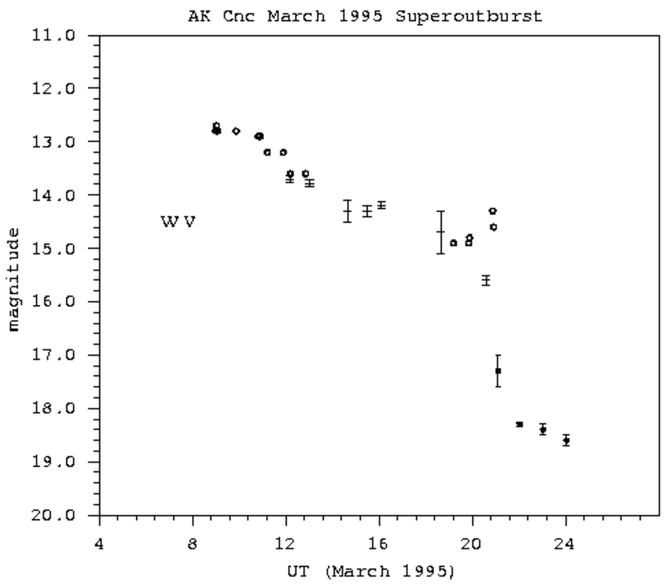

Fig. 1. Light Curve of AK Cnc during its superoutburst on 1995 March. The vertical axis shows the V magnitude and the horizontal axis shows Universal Time (UT) in days. CCD and visual data, given by Mennickent et al. (1996), are shown by (+) and (○) respectively. Our derived estimates of the CCD magnitude of AK Cnc are indicated by (●).

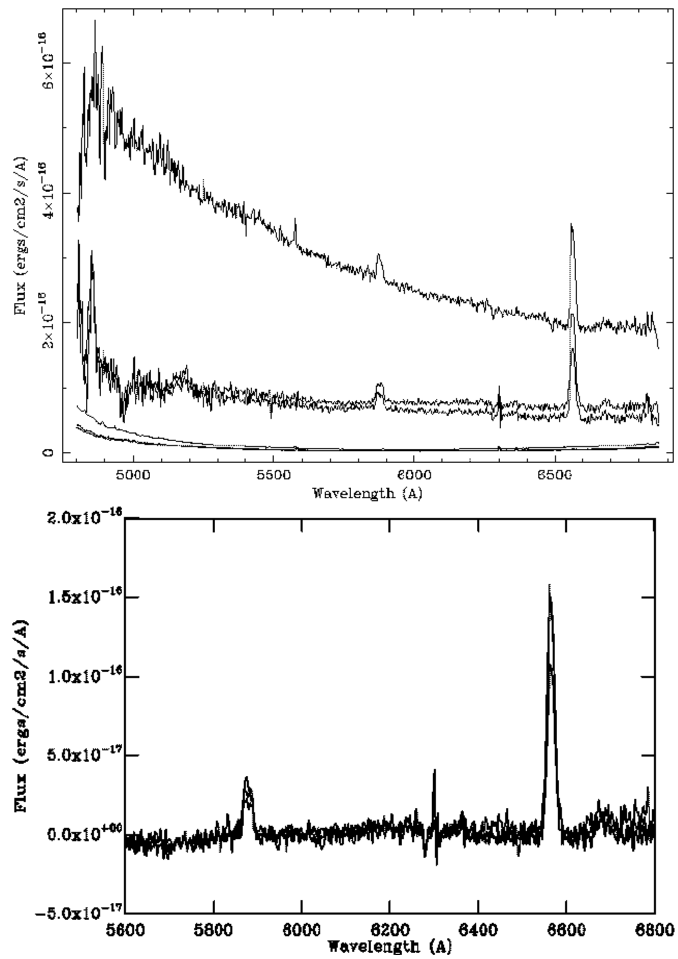

Fig. 2. *Upper panel:* flux-calibrated spectra of AK Cnc on March 21, 23 and 23. Flux is given in erg cm^{-2} s^{-1} Å^{-1}, the upper spectrum corresponds to March 21nd, the figure also show the rms associated to each average spectra (the lines at the bottom). *Lower panel:* continuum-subtracted spectra of AK Cnc corresponding to March 21, 23 and 24.

Table 1. The zero point was obtained assuming that the comparison's V magnitude corresponds to the instrumental magnitude.

Radial velocities, referred to the Local Standard of Rest (V_{sun} = 20 km s^{-1}, $\alpha_{sun} = 18^h$ and $\delta_{sun} = +30^o$), were measured using the double gaussian method first proposed by Schneider & Young (1980) and refined by Shafter (1983) and Horne et al. (1986). This method, implemented in the *MOLLY* program, provides a robust diagnostic test for investigating the behaviour of different line profile sections during the orbital cycle giving also a reliable error estimate for every single measurement of radial velocity. The method consists of simultaneously shifting two gaussians of standard deviation σ_g (or alternatively full width at half maximum $FWHM_g$) and separation Δ along the emission profile until a velocity is found for which the convolved flux in both is the same. Changing Δ and $FWHM_g$ we can probe different velocity sections of every profile.

In addition, the full width at half maximum ($FWHM$) was measured by fitting a simple gaussian to the emission lines. The equivalent width (W_λ) was measured as the area of this gaussian over the continuum level. Internal errors of these parameters were of order of 10%.

3. Results

3.1. The spectrum and estimated magnitudes

Fig. 1 shows the reported visual and CCD visual magnitudes during the 1995 March superoutburst along with our estimated magnitudes. It is evident that our observations were obtained just at the end of superoutburst decline; the star drops by about $3\overset{m}{.}0$ the first night – with respect to previous observations – and shows a roughly constant flux level during the three following nights (see Fig. 2). Nightly mean spectroscopic parameters are given in Table 2.

Due to that the variable star was observed through a rather narrow 1" slit (matching the $fwhm$ of the point spread function every night) and the flux standard was observed using a wide 5" slit, a systematic flux loss of about $0\overset{m}{.}2$ is expected in our flux-calibrated spectra. The spectra, along with their associated rms scatter, are shown in Fig. 2. They show a bright blue continuum during March 21 indicating the inner disk was still hot and emitting mostly at short wavelengths. Fig. 2 also shows emission lines at Hβ, He 5875 and Hα.

3.2. The revised $P_o - P_s$ relationship and the orbital period

In order to find the orbital period of AK Cnc, we measured Hα radial velocities using the double gaussian method with $\Delta = 2 \rightarrow$ 50 Å and $FWHM_g$ =15 Å. The phase-dispersion-minimization program (Stellingwerf 1978) implemented in IRAF was then applied to every dataset. In general, the periodograms showed

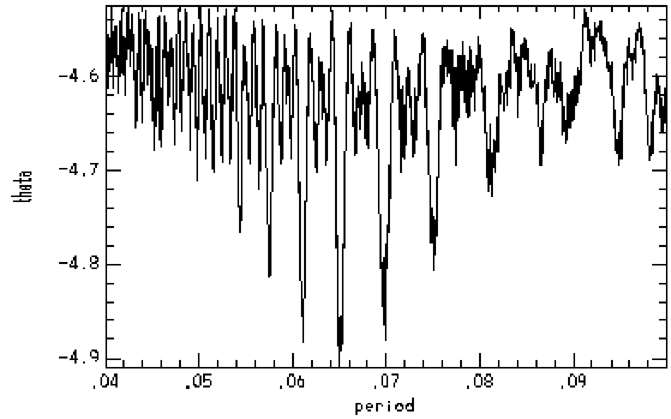

Fig. 3. Periodogram of the Hα radial velocities. The most probable period is P$_{orb} = 0\overset{d}{.}0651(1)$.

a main minimum at P$_{orb} = 0\overset{d}{.}0651(1)$ flanked by its ± 1 c/d aliases at $0\overset{d}{.}061$ and $0\overset{d}{.}071$ (Fig. 3).

Stolz & Schoembs (1984) discovered a possible linear relationship between the orbital and the superhump period. Later, Molnar & Kobulnicky (1992) and Howell & Hurst (1994) recalibrated an empirical relationship between them for SU UMa star, the relationship established by Howell & Hurst is:

$$P_{orb} = 0.036(\pm 0.352) + 0.94(\pm 0.1)\, P_{sup} \quad (1)$$

where the period is given in hours. This equation is of great utility to find orbital periods of SU UMa stars, because most periods found in literature correspond to superhump periods, which are easier to determine than orbital periods. Howell & Hurst (1994) used data of 19 dwarf novae given by Molnar & Kobulnicky (1992). We extended this dataset using data published by Warner (1995) for 39 SU UMa stars, finding the following linear relationship (Fig. 4):

$$P_{orb} = 0.1353(86) + 0.888(5) P_{sup} \quad (2)$$

Some orbital periods given by Warner were estimated from the superhump period using empirical relationships from the $\Delta P_{sup} (= P_{sup} - P_{orb})$ or P_{sup}/P_{orb} versus P_{orb} diagrams, so Eq. (2) is based on well determined as well as estimated periods. Subsequent iterations changing P_o of the aforementioned stars accordingly to Eq. (2) and then re-fitting the data did not improve the fit accuracy. Eq. (2) fits all known data to 1% very well. For example, by using the AK Cnc superhump period given by Mennickent et al. (1996) – P$_{sup} = 1^{\mathrm{h}}620$ – an expected orbital period of P$_{orb} = 1^{\mathrm{h}}574$ ($0\overset{d}{.}0656$) is derived. Remarkably, the orbital period found, P$_{orb} = 0\overset{d}{.}0651(1)$, differs only by 0.07% from that predicted by Eq. (2).

3.3. Ephemeris and radial velocity half-amplitude

Subsequently, we found the time of the superior conjunction of the white dwarf (HJD$_0$) by analyzing the radial velocity curve characterized by a Δ = 16 Å and $FWHM_g$ = 15 Å. The corresponding ephemeris, assuming the emission lines follow the white dwarf motion, is:

$$HJD = 244\,9796.9694(5) + 0\overset{d}{.}0651(1)E. \quad (3)$$

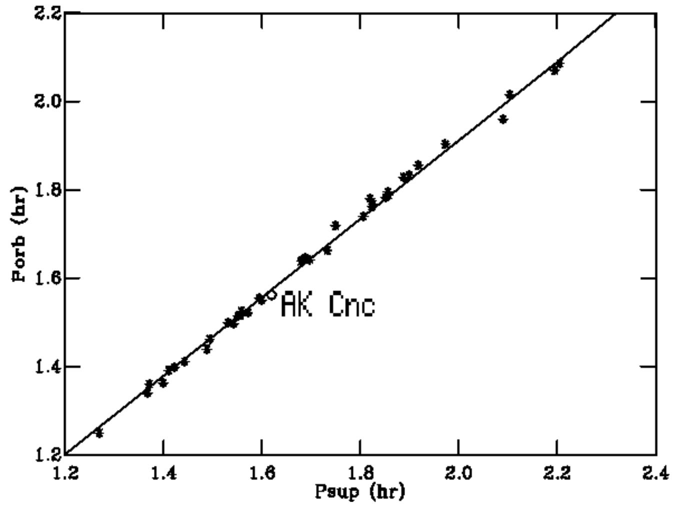

Fig. 4. The linear relationship between the superhump and the orbital period for 39 SU UMa-type stars. The values are from Warner (1995), except that of AK Cnc, which is from this paper.

Table 2. Nightly averages and standard deviation of the equivalent widths and full widths at half maximum, in Å, of the Hα and He 5875 emission lines.

Date	-W_{5875}	$FWHM_{5875}$	-W_{α}	$FWHM_{\alpha}$
21/03/95	3 ± 1	16 ± 5	16 ± 3	17 ± 3
22/03/95	8 ± 0	24 ± 4	40 ± 1	21 ± 1
23/03/95	14 ± 3	28 ± 7	52 ± 5	22 ± 2
24/03/95	9 ± 2	24 ± 7	43 ± 6	20 ± 2

In order to calculate the radial velocity half-amplitude K, we binned the spectra in intervals of 0.1 phase units, calculating the radial velocities for different gaussian separations and making sinusoid fits:

$$RV = \gamma + C_2 sin(2\pi\phi) + C_3 cos(2\pi\phi) \quad (4)$$

here K is given by $\sqrt{C_2^2 + C_3^2}$ while γ represents the velocity of the system's mass center.

Next we studied K, γ, σ/K (where σ is the fit dispersion) and $\Delta\phi$ (the phase where the RV changes from positive to negative values) as a function of different gaussian separations.

The best K value corresponds to the best gaussian separation used just before the fit starts to degrade. From Fig. 5, this occurs with gaussian separations 14 $\rightarrow$ 18 Å (5 velocity datasets). For illustration, we plotted the RV sample for all nights as a function of the orbital phase in Fig. 6. In order to get K and γ, we fitted the 5 RV datasets altogether. The best value for the Hα radial velocity half-amplitude is $K = 50 \pm 3$ km s^{-1} and $\gamma = 125 \pm 2$ km s^{-1}.

From Fig. 5, $\Delta\phi$ is roughly constant whereas γ steeply increases (by $\approx$ 120 km s^{-1}) with gaussian separation.

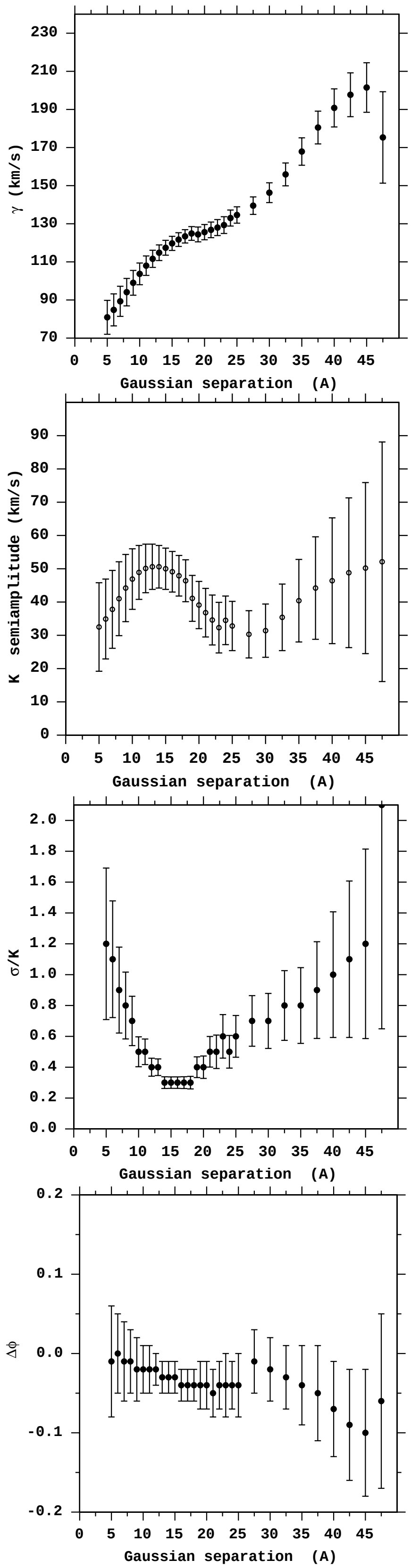

Fig. 5. The diagnostic diagrams for the Hα emission lines of AK Cnc. The γ-velocity, the radial velocity semi-amplitude K, the σ/K ratio and the phase-shift $\Delta\phi$ are plotted as a function of the gaussian separation.

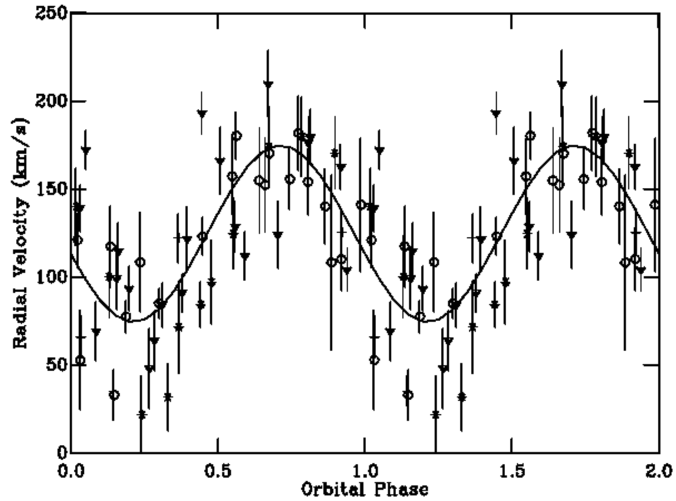

Fig. 6. The Hα radial velocities of all spectra obtained with Δ = 16 Å and $FWHM_g$= 15Å. The best sinusoid fit is also shown. First, 2nd, 3th and 4th night data are indicated by $\circ$, $+$, $\triangle$ and $*$ respectively.

4. System parameters

We cannot firmly establish the stellar masses in a single lined, non-eclipsing spectroscopic binary. Moreover, emission line radial velocities observed in dwarf novae may not represent well the orbital motion of the white dwarf. However, under some reasonable assumptions, we can obtain some useful estimates.

The mass ratio ($q = M(2)/M(1)$) was estimated from Jurcevic et al. (1994) relationship $FWHM/\alpha = K/f(q)$ where $\alpha = 2.00 \pm 0.05$ and $f(q) = q(0.5 - 0.227 \log q)^2$. Using the mean $FWHM$, measured from our Hα profiles, viz. 924 $\pm$ 137 km s^{-1} and our best $K = 50 \pm 3$ km s^{-1}, we obtained $q = 0.28 \pm 0.02$. This mass ratio is just in the upper limit of mass ratios of binaries where the superhump phenomenon is expected to appear (e.g. Whitehurst 1994).

Using the empirical $M_2 - P_{orb}$ relationship by Warner (1995, Eq. 2.100):

$$\frac{M(2)}{M_\odot} = 0.065\, P_{orb}^{5/4}(hr) \quad 1.3hr \leq P_{orb} \leq 9hr \tag{5}$$

we found $M(2) = 0.1135 \pm 0.0002 M_\odot$. The primary star mass following from q is $M(1) = 0.41 \pm 0.03 M_\odot$.

In order to estimate the radius of the primary star, we used the WD mass-radius relationship given by Hamada & Salpeter (1961)

$$\frac{R(1)}{R_\odot} = 0.0072 \left(\frac{M(1)}{M_\odot} \right)^{-0.8} \tag{6}$$

obtaining $R(1) = 0.015 \pm 0.001 R_\odot$. Since this is not an eclipsing binary, its inclination angle must be less than 65^o, but we found a lower limit by assuming the velocity in the inner disk edge is less than or equal to the Keplerian velocity at the star's surface:

$$v_{kep} = \sqrt{\frac{GM(1)}{R(1)}} sin(i) \geq v_{obs} = \frac{1}{2} FWZI \tag{7}$$

Table 3. Estimated stellar and orbital parameters for AK Cnc.

Parameter	Value
Orbital Period (days)	0.0651 ± 0.0002
K_1 (km s^{-1})	50 ± 3
Primary mass ($M_\odot$)	0.41 ± 0.03
Secondary mass ($M_\odot$)	0.1135 ± 0.0002
Inclination (^{o})	36 ± 3
Mass ratio ($q = M_2/M_1$)	0.28 ± 0.02

where M(1) and R(1) are the white dwarf mass and radius, i is the inclination angle and $FWZI$ is the full width at zero intensity. Using the mean $FWZI$ values from the phase binned spectra, (viz. $FWZI_\alpha = 2332 \pm 182$ km s^{-1} and $FWZI_{He\,5875} = 2343 \pm 485$ km s^{-1}), and both the primary mass and radius in Eq. 7, we found that the inclination angle must be greater than or equal to 33^o. To find the best value to this angle, we used the binary mass-function:

$$M(2)sin^3(i) = \frac{P_{orb}K^3}{2\pi G}\left(\frac{1+q}{q}\right)^2 \quad (8)$$

obtaining an inclination angle of $i = 36 \pm 3$. The most likely parameters for AK Cnc are given in Table 3. The low inclination found is compatible with the observed single-peak emission profiles.

4.1. Conclusions

- We reported time resolved spectroscopy obtained during late stages of the 1995 March superoutburst of AK Cancri.
- From the Hα radial velocity variations, an orbital period of $0.^{d}0651(2)$ was found.
- The radial velocity half amplitude, stellar masses and system inclination were estimated (Table 3) indicating a low inclination and small primary mass.

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