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On the cosmic ray cross field diffusion in the presence of oblique MHD waves

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Abstract. The process of particle cross-field diffusion in the presence of finite MHD amplitude wave fields is considered using the Monte Carlo particle simulations. We derive the cross-field diffusion coefficient $\kappa_\perp$ and the parallel diffusion coefficient $\kappa_\|$ for the flat and the Kolmogorov wave spectra, including the waves propagating oblique to the mean magnetic field $\boldsymbol{B}_0$. We note a substantial difference in the cross-field diffusion efficiency between the considered turbulent fields. Much larger values of $\kappa_\perp$ appear in the presence of fast-mode waves in comparison to the Alfvén waves and we reproduce the expected increase of $\kappa_\perp$ with the growing power of waves propagating perpendicular to $\boldsymbol{B}_0$. We interpret these results in terms of the particle drifts in non-uniform magnetic fields.

Key words: magnetohydrodynamics – turbulence – waves – ISM: cosmic rays

1. Introduction

Understanding particles motions in perturbed magnetic fields is essential for a wide range of problems in astrophysics including describing galactic cosmic ray transport and acceleration at shock waves. In spite of large progress achieved since the paper by Jokipii (1966) there are a number of issues still poorly understood. In the previous paper (Michalek & Ostrowski 1997 ($\equiv$ MO97)) we presented simulation of particle transport in the presence of 1-D, 2-D and 3-D finite amplitude turbulence patterns composed of Alfvén waves propagating parallel to the average magnetic field. Those simulations were limited to a somewhat unrealistic flat turbulence spectrum. In the present paper we extend these considerations to the Kolmogorov turbulence spectrum proposed to be a viable model for interplanetary space (e.g. Jokipii 1971). For these considerations we adopt a simple model of a turbulent magnetic field including waves propagating obliquely to the mean field (cf. Miller et al. 1996) to study the cross-field diffusion coefficient $\kappa_\perp$ and the parallel diffusion coefficient $\kappa_\|$ in the presence of finite amplitude ($\delta B/B \sim 1$) magnetosonic and Alfvén waves. In the next section (Sect. 2) we present a short review of quasi-linear results. As described in Sect. 3 the anisotropic wave distribution is modelled by selecting the wave vectors from a finite opening cone directed along the mean magnetic field $\boldsymbol{B}_o$. In our simulations we use the fast-mode and the Alfvén mode turbulence with the flat or the Kolmogorov wave spectrum selected from a finite wave-vector range. The results and a short discussion are presented in Sections 4 and 5, respectively. We note substantial differences in the cross-field diffusion efficiency at the same perturbation amplitude, depending on the form of the turbulent field considered. We prove the possibility of larger values of $\kappa_\perp$ occurring in the presence of the fast-mode waves in comparison to the Alfvén waves of the same amplitude.

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2. Quasi-linear cross-field diffusion coefficient

To date, the analytical derivations of $\kappa_\perp$ in turbulent magnetic fields have been limited to the quasi-linear approach, valid for small amplitude field perturbations, $\delta B \ll B_0$. The first considerations by Jokipii (1966, 1967, 1971) applied the Fokker-Planck equation to describe particle motion in terms of the magnetic-field perturbations' power spectrum. Jokipii showed that scattering at the small-scale magnetic inhomogeneities drives pitch-angle diffusion, allowing for transverse guiding-centre diffusion across the field lines. For a particle distribution close to isotropy the distribution function averaged over pitch-angle satisfies the diffusion equation, with the diffusion tensor expressible in terms of the correlation function of the irregular magnetic field. If the fluctuating field depends only on the z-coordinate (= direction of the mean magnetic field), the Fokker-Planck coefficients for pitch-angle diffusion and for cross-field diffusion derived by Jokipii take the form, respectively,

$$\frac{\langle \Delta\mu^2 \rangle}{\Delta t} = \frac{(1-\mu^2)}{|\mu| v} \frac{Z^2 e^2}{\gamma^2 m_o^2 c^2} P_{xx}(k = \Omega^{(o)}/\mu v) \quad , \qquad (2.1)$$

and

$$\frac{\langle \Delta x^2 \rangle}{\Delta t} = \frac{\langle \Delta y^2 \rangle}{\Delta t} =$$

$$\frac{\mu v}{B_o^2} P_{xx}(k=0) + \frac{(1-\mu^2)}{2|\mu| v} \frac{v}{B_o^2} P_{zz}(k = \Omega^{(o)}/\mu v) \quad . \qquad (2.2)$$

At the present time one knows that the cross-field diffusion coefficient (2.2) must vanish for the 1-D turbulence model (Giacalone & Jokipii 1994; see also discussion below). However, as these expressions were widely used for discussion of the cross-field diffusion process, and as the interpretation of the terms in (2.2) is mostly valid for the 3-D turbulence, we repeat this expression following the original paper. The coefficients (2.1,2) describe particle scattering by fluctuations that are resonant with the particle's gyromotion in the averaged magnetic field $\mathbf{B}_0$. In coefficients (2.2) describing cross-field diffusion one can note an additional non-resonant term $\propto P_{xx}(k=0)$. It represents the tendency of particles to follow the meandering or random walk of magnetic field lines.

Achatz et al. (1991) re-derived the Fokker-Planck equation for charge particle transport in a slab turbulence superimposed on a homogeneous magnetic field, involving all phase-space variables. In contrast to the previous papers they included dispersive effects of the waves by considering whistler-mode waves in addition to the Alfvén waves. They confirmed the previous results of Jokipii that the diffusion perpendicular to the magnetic field could be solely due to the wandering of field lines (waves with zero wave vectors). The same result was obtained in a different way by Achterberg & Ball (1994), who studied the requirements for efficient electron acceleration in young supernova remnants, where the shock is expanding into the progenitor's stellar wind with the magnetic field lines forming a tightly-wound spiral. Then the intersection point between the shock and the magnetic field line moves along the mean magnetic field at a speed exceeding c. To allow the shock wave to accelerate electrons to GeV energies required to account for the observed radio emission, efficient particle diffusion across the magnetic field is necessary. In the considered situation relativistic particles can be scattered by resonant low-frequency MHD waves. The waves with a wave-vector component perpendicular to the magnetic field $\boldsymbol{k}_\perp$, contribute to the s-th resonance with a weight $J^2_{s\mp1}(k_\perp v/\Omega^{(o)})$, where J_n is the n-th order Bessel function. For $k_\perp v_\perp/\Omega^{(o)} \leq 1$ the dominant contribution comes from the $s=\pm1$ resonances. These resonant waves produce a stochastic change in particle momentum as well. The net diffusion process in particle momentum is accompanied by guiding centre shift across the field. In the case of turbulence symmetry around $\mathbf{B}_0$, $< k_x^2 >=< k_y^2 >= k_\perp^2/2$, they estimated the diffusion tensor components as:

$$\kappa_\parallel \approx \frac{v^2}{3\nu_s} = \varepsilon^{-1}\kappa_B \quad , \tag{2.3}$$

$$\kappa_\perp \approx \frac{\varepsilon}{2}\left(\frac{k_\perp}{k_\parallel}\right)^2 \kappa_B \quad , \tag{2.4}$$

where $\varepsilon \approx \nu_s/\Omega^{(o)}$, ν_s is the effective pitch-angle scattering frequency and the Bohm diffusion coefficient $\kappa_B = \frac{1}{3}r_g^2\Omega^{(o)}$. One may note that the waves with $k_\perp = 0$ do not contribute to particle transport across the magnetic field.

Vanishing of the cross-field diffusion for turbulence models involving 1- or 2-dimensional perturbation fields was proved by Giacalone & Jokipii (1994). They demonstrated that if one or two ignorable co-ordinates appear in the magnetic field description, the ions are effectively tied to the magnetic lines of force, independent of the turbulent field's amplitude.

The process leading to particle diffusion perpendicular to the average magnetic field due to field line wandering (or braiding) has been considered in some forms since the first papers of Jokipii & Parker in 60th. Achterberg & Ball (1994) discuss the case with long-wavelength perturbations leading to stochastic excursions of magnetic field lines transverse to $\mathbf{B}_0$. Particles stay in a given patch of field lines for a time t_c, travelling a distance $L_c = s(t_c)$ along a field line. The distance L_c is necessary for particle to enter a neighbouring, statistically independent, patch of field lines. Application of this model yields the perpendicular diffusion coefficient

$$\kappa_\perp = D_m\left(\frac{L_c}{t_c}\right) \quad , \tag{2.5}$$

where D_m is the field lines' diffusion coefficient. Recently, a regime of sub-diffusive transport and of compound diffusion in the presence of 'braided' magnetic field was discussed by Duffy et al. (1995). For times less than t_c, the particles undergo sub-diffusion, which is a combination of diffusion along a fixed field line, which itself diffuses. The defining characteristic of sub-diffusion is that the mean square cross-field deviation of a particle is not proportional to t as in ordinary diffusion, but rather to the time with power lower than 1 ($\sqrt{t}$ for the most often discussed cases). Subsequently, the particle decorrelates and undergoes compound diffusion with the ordinary $\propto t$ behaviour. The compound diffusion combine wandering of field lines and diffusion of particles along and across the local field. This problem was discussed by Kirk et al. (1996) for the issue of cosmic ray acceleration at perpendicular shock waves (see also Giacalone & Jokipii (1996) for numerical modelling).

3. Description of simulations

Following the approach applied by Michalek & Ostrowski (1997) and Michalek et al. (1998), in the present paper we use numerical Monte Carlo particle simulations. The general procedure is quite simple: test particles are injected at random positions into a turbulent magnetized plasma and their trajectories are followed by integration of particle equations of motion. For each particle we have the individual set of randomly selected waves allowing particles to move diffusively in space and momentum. By averaging over a large number of trajectories one derives the diffusion coefficients for turbulent wave fields. In the simulations we consider relativistic particles with velocity $v \gg V_A$ and use dimensionless units (cf. Appendix A): $\delta B \equiv \delta B/B_o$ for magnetic field perturbations, $1/\Omega_o$ for time and k/k_{res} for wave vectors.

Below we describe the models of the turbulent MHD fields applied in the simulations and we also consider two topics used later in discussion of the particle cross-field diffusion: the diffusion of magnetic field lines in a turbulent medium and particle guiding centre drifts in the magnetic field perturbed by an individual wave.

3.1. The wave field models

In the modelling we consider a superposition of 384 plane MHD waves propagating obliquely to the average magnetic field $\boldsymbol{B}_o \equiv B_o \hat{\mathbf{e}}_z$. The wave propagation angle with respect to $\boldsymbol{B}_o$ is randomly chosen from a uniform distribution within a cone ('wave-cone') along the mean field. For a given simulation two symmetric cones are considered centered along $\boldsymbol{B}_o$, with the opening angle 2α, directed parallel and anti-parallel to the field direction. The same number of waves is selected from each cone in order to model the symmetric wave field. Related to the i-th wave, the magnetic field fluctuation vector $\delta\boldsymbol{B}^{(i)}$ is given in the form:

$$\delta\boldsymbol{B}^{(i)} = \delta\boldsymbol{B}_o^{(i)} \sin(\boldsymbol{k}^{(i)} \cdot \boldsymbol{r} - \omega^{(i)} t) \qquad . \tag{3.1}$$

The electric field fluctuation related to a particular wave is given as $\delta\boldsymbol{E}^{(i)} = -\boldsymbol{V}^{(i)} \wedge \delta\boldsymbol{B}^{(i)}$. For the Alfvén waves (A) we consider the dispersion relation

$$\omega_A^2 = k_\parallel^2 V_A^2 \qquad , \tag{3.2}$$

where $V_A = B_o/\sqrt{4\pi\rho}$ is the Alfvén velocity in the field B_o. The wave magnetic field polarization is defined by the formula

$$\delta\boldsymbol{B}_A = \delta B_A(\boldsymbol{k}, \omega_A)\,(\boldsymbol{k} \times \hat{e}_z)\,k_\perp^{-1} \qquad . \tag{3.3}$$

In considered here low-β plasma the fast-mode magnetosonic (M) waves propagate with the Alfvén velocity and the respective relations are:

$$\omega_M^2 = k^2 V_A^2 \tag{3.4}$$

$$\delta\boldsymbol{B}_M = \delta B_M(\mathbf{k}, \omega_M)\,(\boldsymbol{k} \times (\boldsymbol{k} \times \hat{e}_z))\,k^{-1}\,k_\perp^{-1} \qquad . \tag{3.5}$$

In the simulations we adopt $V_A = 10^{-3} c$. One should be aware of the fact that the considered turbulence model is unrealistic at large δB and the present results can not be considered as the exact ones. In particular, in the presence of a finite amplitude turbulence the magnetic field pressure is larger than the mean field pressure and the wave phase velocities can be greater than the $V_A(B_o)$ assumed here.

3.2. Spectrum of the turbulence

In the simulations we consider power-law turbulence spectra, where the irregular magnetic field in the wave-vector range (k_{min}, k_{max}) can be written as

$$\delta B(k) = \delta B(k_{min}) \Big(\frac{k}{k_{min}}\Big)^{-q/2} \tag{3.6}$$

where the wave vector $k_{min} = 0.08$ ($k_{max} = 8.0$) corresponds to the considered longest (shortest) wavelength and q is the wave spectral index. In the present simulations we consider the flat spectrum with $q = 1$ and the Kolmogorov spectrum with $q = 5/3$. Considering the flat spectrum we try to refer to our earlier simulations (MO97) where we used parallel Alfvén waves with the flat spectrum. On the other hand this kind of turbulence spectrum is very convenient for numerical simulations due to the presence of a large number of short waves.

For our flat spectrum case the wave vectors are drawn in a random way from the respective ranges: $2.0 \le k \le 8.0$ for 'short' waves, $0.4 \le k \le 2.0$ for 'medium' waves and $0.08 \le k \le 0.4$ for 'long' waves. The respective wave amplitudes are drawn in a random manner so as to keep constant

$$\Big[\sum_{i=1}^{384} (\delta B_o^{(i)})^2\Big]^{1/2} \equiv \delta B, \tag{3.7}$$

where δB is a model parameter, and in each separate wave-vector range analogous sums equal $\delta B/\sqrt{3}$.

In the case of the Kolmogorov turbulence spectrum. all wave vectors are drawn in a random manner from the whole range ($0.08 \le k \le 8.0$) but amplitudes ($\delta\boldsymbol{B}^{(i)}$) are fitted to the respective waves according to the Kolmogorov distribution as to satisfy the formula (3.7). With such turbulence spectrum most of the energy is carried by 'long' waves.

In the discussion below we will consider four cases for turbulence:
i. Alfvén waves with the flat spectrum - AF,
ii. Alfvén waves with the Kolmogorov spectrum - AK,
iii. Magnetosonic-M waves with the flat spectrum - MF,
iv. Magnetosonic-M waves with the Kolmogorov spectrum - MK.

Thus for a given simulation we use one of the above models characterized by parameters α and δB. Here we have to note that the assumption of a superposition of Alfvén waves when $\delta B \sim 1$ can be questionable due to non-linear effects. We use this simplification due to lack in the literature of the more realistic in a wide wave-vector range models.

3.3. Magnetic field line diffusion coefficient

Following MO97 we derived the magnetic field diffusion coefficients, D_m, for all considered field models. Examples of such derivations are presented in Fig. 1, where D_m versus δB is given for waves with the Kolmogorov spectrum, for three opening angles $\alpha = 45°$, $60°$, $90°$. As expected, D_m increases with δB, however, the particular behaviour of this relation depends on the field model considered. For Alfvén waves D_m grows uniformly with α, but this is not the case at larger δB when fast-mode waves are present. For these waves with $\delta B > 0.6$ $D_m^{\alpha=45°,60°}$ is about two times larger than $D_m^{\alpha=90°}$. Simulated values of D_m for the flat spectrum (not presented in the figure) show the same features, but for large amplitudes $\delta B \ge 0.6$ the magnetic field lines' diffusion coefficients are about two times smaller than with the respective models of the Kolmogorov turbulence. For all turbulence models D_m have to vanish when $\alpha \to 0°$. In fact

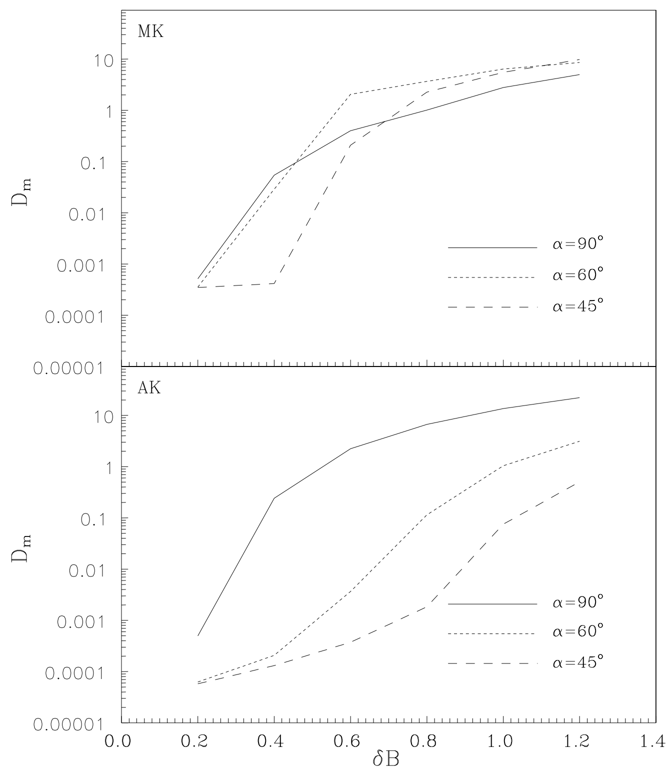

Fig. 1. Simulated values of magnetic field diffusion coefficient D_m versus δB for $\alpha = 45°, 60°$ (dotted line) and $90°$ (solid line) are presented at the upper panel variation of D_m for the magnetosonic turbulence and at the bottom panel for the Alfvén turbulence.

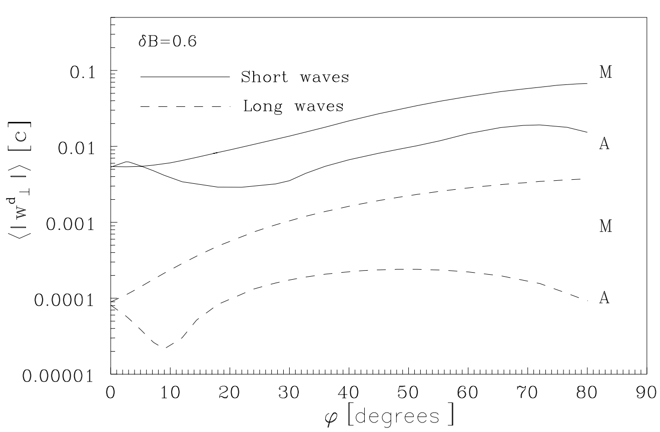

Fig. 2. Variation of the $< |w^d_\perp| >$ versus a wave propagation angle for the Alfvén (A) and the magnetosonic (M) wave. The results for a long wave ($k = 0.1$) are presented with dashed lines, and the ones for a short wave ($k = 10$) with solid lines.

our simulations for $\alpha = 0°$ yielding $D_m = 0$ provide a good accuracy check for the computations.

3.4. Particles drifts

Any force $\boldsymbol{F}_\perp$ acting on a gyrating particle, the one which is constant on the time and space scales large compared with the particle gyromotion, perpendicular to the magnetic field causes its drift directed perpendicular to both $\boldsymbol{F}_\perp$ and $\boldsymbol{B}$, with velocity

$$\boldsymbol{w}^d = \frac{c}{e} \frac{\boldsymbol{F}_\perp \times \boldsymbol{B}}{B^2} \quad . \tag{3.8}$$

Such drifts move the particles across the magnetic field lines. In the presence of MHD waves the drifts can arise due to introduced curvature and perpendicular gradient of the magnetic field. These drifts induce local fluxes of particles moving perpendicular to the average magnetic field. Averaging over these fluxes in a turbulent magnetic field leads to particle diffusion across the average magnetic field. For Alfvén waves the gradient drift is caused by the variation of the magnetic field, being the second-order in δB. For magnetosonic waves containing compressive components such drifts are associated with the first-order in δB field variations. To evaluate the role of drifts in the simulations we derived the *formal* drift velocity perpendicular to the average magnetic field, $\boldsymbol{w}^d_\perp$, in the presence of a single long (longer than r_g) or short (shorter than r_g) wave propagating at some angle with respect to the mean magnetic field. We consider both drifts arising due to the curvature and the perpendicular gradient of the magnetic field. Then $\boldsymbol{w}^d_\perp = \boldsymbol{w}^G_\perp + \boldsymbol{w}^C_\perp$, where the component due to the field gradient is

$$\boldsymbol{w}^G_\perp = \frac{r_g^2 \Omega}{2B^2} (\boldsymbol{\nabla}_\perp B \times \boldsymbol{B}_\circ) \quad , \tag{3.10}$$

and the component due to the field curvature

$$\boldsymbol{w}^C_\perp = \frac{v_\parallel^2}{\Omega B^3} [\boldsymbol{B} \times (\boldsymbol{B}\boldsymbol{\nabla})\boldsymbol{B}] \quad . \tag{3.11}$$

In this discussion we consider the isotropic distribution of particle velocity vectors with $v^2 = v_\perp^2 + v_\parallel^2 = c^2$. The drift velocity fluctuates in the wave magnetic field but the average $< |\boldsymbol{w}^d_\perp| >$ of its absolute value provides information about efficiency of drifts in generating the cross-field particle transport. Variations of $< |\boldsymbol{w}^d_\perp| >$ versus the wave propagation angle ϕ for the Alfvén and the magnetosonic wave are shown at Fig. 2. In the upper panel the results are presented for the short wave ($k = 10k_{res}$) and in the bottom panel for the long wave ($k = 0.1k_{res}$). We observe a much more rapid increase of $< |\boldsymbol{w}^d_\perp| >$ with ϕ for the magnetosonic wave than for the Alfvén wave. Due to larger gradients, for a short wave $< |\boldsymbol{w}^d_\perp| >$ is about twenty times larger than in the presence of a long wave. If one considers separately the drifts due to the curvature and the magnetic field gradient (not presented in the figure) one finds that in the presented example $< |\boldsymbol{w}^G_\perp| >$ is about ten times larger than $< |\boldsymbol{w}^C_\perp| >$.

4. Derivation of the diffusion coefficients

The simulated cross-field diffusion coefficients $\kappa_\perp$ for different wave-cone opening angles and for different turbulence amplitudes are presented in Fig. 3. For the flat spectrum turbulence a systematic increase of $\kappa_\perp$ with amplitude occurs and the rate of this increase roughly scales as δB^2. The value of $\kappa_\perp$ at any given δB is a factor ~ 10 larger for the fast-mode waves in comparison to the Alfvén waves (cf. Section 5). It grows substantially with increasing wave cone opening α, i.e. with increasing power of waves perpendicular to the mean magnetic field. For the Kolmogorov spectrum a dependence of $\kappa_\perp$ on the perturbation amplitude is flatter, the values of the cross-field diffusion coefficient at small δB are larger and there is a smaller difference between the fast-mode and the Alfvén waves.

The simulated parallel diffusion coefficients $\kappa_\parallel$ for different wave cone opening angles and wave amplitudes are presented in Fig. 4. One may note that $\kappa_\parallel$ depends only weakly on the considered wave model if the turbulence spectrum is flat. In this case the main parameter influencing the value of $\kappa_\parallel$ is the wave amplitude. However, one may note a small (in the logarithmic scale) departure from the general trend for magnetosonic waves with intermediate opening angles (our case of $\alpha = 40°$) providing more effective scattering and smaller $\kappa_\parallel$. As in MO97, for the flat spectrum turbulence one can reasonably fit the data by the quasi-linear relation $\kappa_\parallel \propto (\delta B)^{-2}$ (in the 'worst' presented case for magnetosonic waves with $\alpha = 40°$ one obtains $\kappa_\parallel \propto (\delta B)^{-2.4}$).

The situation is more complicated for the Kolmogorov spectrum. Then $\kappa_\parallel$ depends both on the wave amplitude and the turbulence model. For Alfvén waves $\kappa_\parallel$ decreases in a monotonic way with both δB and α. For magnetosonic waves an exceptional behaviour occurs again for the intermediate α MK model (see $\alpha = 40°$). These data are not fitted well with the quasi-linear relation, as approximately $\kappa_\parallel \propto \delta B^{-2.9}$, except for magnetosonic waves with $\alpha = 40°$, where $\kappa_\parallel \propto \delta B^{-3.9}$. One should note that for $\alpha = 0°$ the parallel diffusion is about fourteen times larger for the Kolmogorov turbulence than for the flat turbulence spectrum.

The characteristic features seen in Figs. 3 and 4 can be qualitatively explained with the use of simple physical arguments involving results of Sects. 3.3,4. Comparison of Figs. 1 and 3 shows much larger increases of respective D_m than $\kappa_\perp$. It proves that in the range of δB considered here $\kappa_\perp$ is in a substantial degree controlled by the cross-field drifts and the resonance cyclotron scattering, and not by the field line diffusion. Let us stress that the substantial cross-field shifts accompany wave particle interaction involving the so called 'transit time damping resonance', where for the effective cross-field drift the particle velocity $v_\parallel$ and the wave phase velocity $V_\parallel$ along the mean field should be approximately equal:

$$v_\parallel \approx V_\parallel \qquad , \tag{4.1}$$

where $V_\parallel = V_A$ for the Alfvén waves and $V_\parallel = V_A(k/k_\parallel)$ for the magnetosonic fast-mode ones. For $V_A = 10^{-3}$ and $v = 0.99$ considered in our simulations a noted difference between $\kappa_\perp$ for the Alfvén and the fast-mode waves is expected to occur as a result of satisfying the resonance condition (4.1) in a wider range of $v_\parallel$, when the oblique fast-mode waves are present. Another difference arises from the fact that the linear compressive terms occur only in the fast-mode waves. It enables the gradient drifts to be revealed by these waves at smaller perturbation amplitudes. Also, the 'effective' particle gyroradius $r_g(\boldsymbol{B}_0 + \delta\boldsymbol{B}) < r_g(\boldsymbol{B}_0)$ and particles can interact resonantly with shorter waves. In the case of the Kolmogorov spectrum, the power of such waves is smaller and the importance of resonance interactions decreases. At the same time, the long waves enable uncorrelated long distance drifts leading to a net grow of $\kappa_\perp$ with turbulence amplitude (decorrelation length for drifts at short waves is much shorter and makes the net effect smaller even with the formally larger drift speeds). Such effects are expected to be responsible for the slightly flatter curves for the fast-mode waves in Fig. 3.

In Fig. 5 variations of the products $\kappa_\parallel \cdot \kappa_\perp$ versus the perturbation amplitude δB for various wave cone openings are presented. The results for the flat and the Kolmogorov turbulence spectra are significantly different. In the former case $\kappa_\parallel \cdot \kappa_\perp$ is slowly varying within the considered waves' amplitudes. At the same time, in the presence of the Kolmogorov spectrum $\kappa_\parallel \cdot \kappa_\perp$ decreases significantly with increasing δB for all models. We fitted these data with power law relations with respect to δB. Then $\kappa_\perp \cdot \kappa_\parallel$ scales from $\propto \delta B^{-3.9}$ for the steepest curve (magnetosonic waves with $\alpha = 40°$) to $\propto \delta B^{-1.9}$ for the flattest one (Alfvén waves with $\alpha = 90°$). Thus the sometimes applied in the literature scalling $\kappa_\perp \kappa_\parallel \approx \kappa_B^2$ (eg. Drury 1983) may be wrong even at the quantitative level.

5. Summary and discussion

We modelled particle transport in the presence of oblique Alfvén and magnetosonic waves with finite amplitudes. The diffusion transport of energetic particles is mediated by resonant scattering of energetic particles, drifts in turbulent magnetic fields and diffusion of magnetic field lines. Under the considered conditions, the main factors causing the cross-field diffusion are particle drifts requiring the 'n=0' resonance and the cyclotron ('n=1') resonant scattering. We suspect that the magnetic field line diffusion will become important at smaller δB. In accord with the analytical derivations of Giacalone & Jokipii (1994) our simulations confirm vanishing of $\kappa_\perp$ in the 1-dimensional turbulent fields, the case occurring in the simulations for $\alpha = 0$). As expected, $\kappa_\perp$ increases with the turbulence amplitude but for the Kolmogorov turbulence this effect is less pronounced as compared to the flat spectrum. We also note the possibility of larger values of $\kappa_\perp$ occurring – up to ten times larger with the flat spectrum and up to five times with the Kolmogorov spectrum – in the presence of the compressive fast-mode waves in comparison to the Alfvén waves of the same amplitude. This difference can be partly explained as a result of more effective drifts with magnetosonic waves. For the Kolmogorov spectrum, which prefers long waves, this increase is somewhat less significant. From Eq. 2.4 one obtains an analytic approximation $\kappa_\perp \propto {<k_\perp>}^2/{<k_\parallel>}^2$. In our simulations the last factor ≈ 0.13 for models with $\alpha = 40°$ and ≈ 1 for $\alpha = 90°$ what gives $\kappa_{\perp\alpha=90°}/\kappa_{\perp\alpha=40°} \approx 7$. It is consistent with models presented in Fig. 4 for our 'small' wave amplitudes. As velocities of the considered waves are very small ($u = 0.001c$) any induced electric fields do not influence the derived cross-diffusion in a noticeable way.

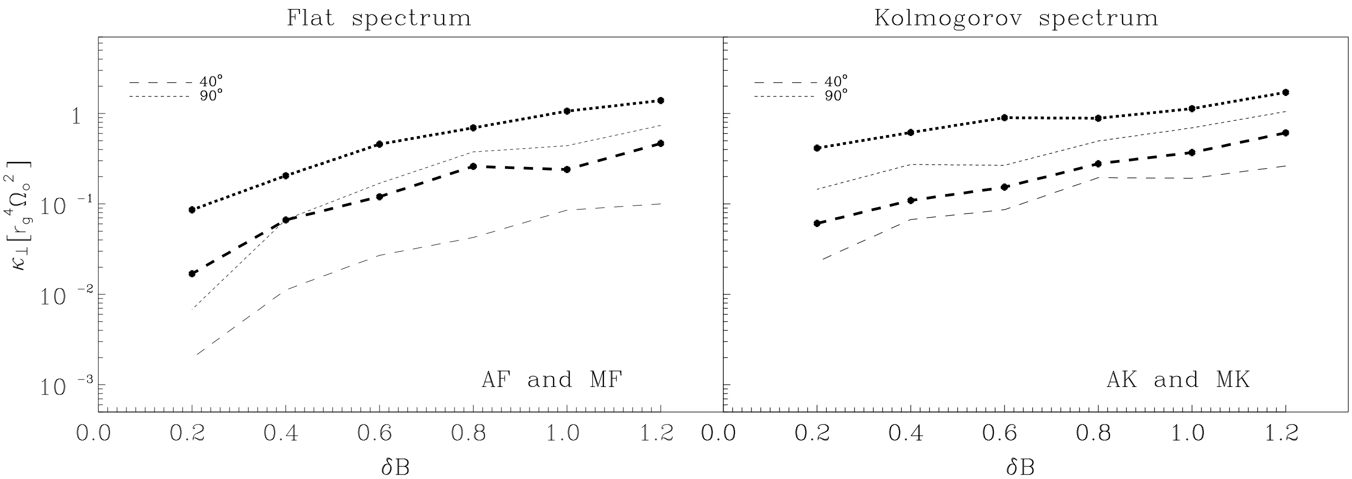

Fig. 3. Variation of the cross-field diffusion coefficient $\kappa_\perp$ versus the perturbation amplitude δB and the wave propagation anisotropy (angle α) for the flat spectrum and the Kolmogorov spectrum. Results for the Alfvén turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed at the same panels.

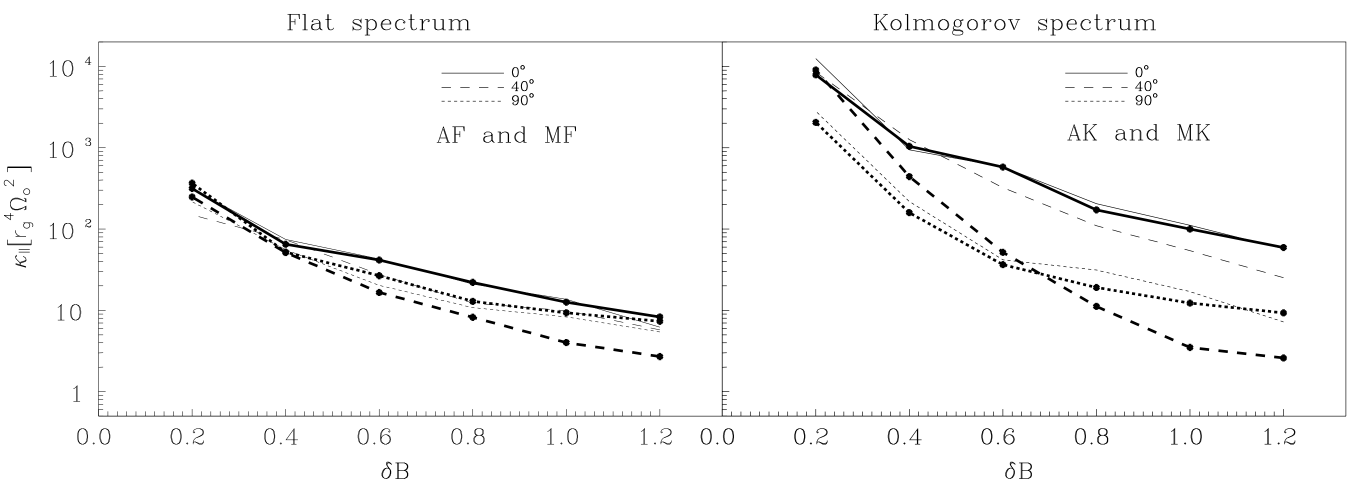

Fig. 4. Variation of the parallel diffusion coefficient $\kappa_\parallel$ versus the perturbation amplitude δB and the wave propagation anisotropy (angle α) for the flat spectrum and the Kolmogorov spectrum. Results for the Alfvén turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed at the same panels.

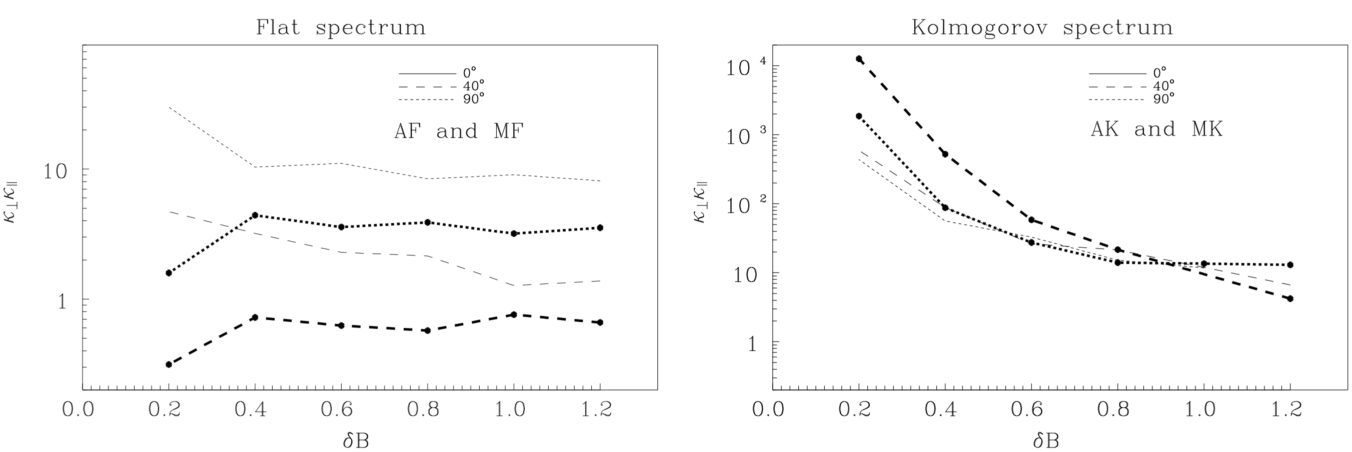

Fig. 5. Variation of $\kappa_\parallel \cdot \kappa_\perp$ versus the perturbation amplitude δB and the wave propagation anisotropy for the flat spectrum and the Kolmogorov spectrum. For comparison the results for both the Alfvén turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed at the same panels.

The main parameter influencing the value of $\kappa_{\|}$ is the form of the spectrum. At small wave amplitudes in the Kolmogorov spectrum a decreased number of waves resonantly scattering particles in pitch angle leads to substantially – a factor of 10 – larger $\kappa_{\|}$ (less effective scattering) in comparison to the flat spectrum turbulence, but the presence of oblique waves can decrease this difference at larger amplitudes. For comparison let us refer to the analytic evaluation by Schlickeiser (1989), who provided a relation $\kappa_{\|}^{\alpha=0^\circ} \propto (2/q^2 - 6q + 8)(\lambda_{max}/r_g)^{q-1}$ depending on the wave spectral index q. Using this formula for our simulations we can show that for $q = 5/3$ $\kappa_{\|}^{\alpha=0}$ should be about thirteen times larger in comparison to the $q = 1$. At large amplitudes $\delta B \sim 1$ the observed difference is preserved for the Alfvén waves, but for the fast-mode waves with intermediate α the respective scattering efficiency may increase substantially thus decreasing the difference. We also note that the flat spectrum data can be reasonably fitted with the quasi-linear relation $\kappa_{\|} \approx (\delta B)^{-2}$ (cf. MO97). At Fig. 4 we see that for the Kolmogorov turbulence spectrum $\kappa_{\|}$ is smaller about ten times for isotropic wave distribution ($\alpha = 90^\circ$) than for $\alpha = 0^\circ$.

Consideration of MHD waves propagation oblique to the mean magnetic field (e.g. Tademaru 1969, Lee & Völk 1975) shows that such waves are subject to effective processes dissipating their energy. Therefore the effect considered here of the cross-field diffusion enhancement due to fast-mode waves can occur only in a volume with the turbulence generation force acting. For example, in the vicinity of the strong shock, or in a region of magnetic field reconnection, the required fast-mode oblique waves could be effectively created. Also, the damping processes are less effective in the conditions of the low plasma-β (cf. Michalek et al. 1998).

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Appendix A: summary of notation

$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$ – a magnetic induction vector
$\mathbf{B}_0$ – a regular component of the background magnetic field
$\delta\mathbf{B}$ – a turbulent component of the magnetic field
c – the light velocity
$D_{\mu\mu}$– a pitch angle diffusion coefficient
D_m – a magnetic field lines' diffusion coefficient
$\mathbf{E}$ – an electric field vector
e – a particle charge
$g^j(k)$ – a magnetic field energy density for given waves 'j'
k – a wave vector
$k_{res} \equiv 2\pi/r_g$
$k_{\|}$ – a wave vector component along B_0
$k_{\perp}$ – a wave vector component perpendicular to B_0
L_c – a turbulence correlation distance
m – a particle mass
$\mathbf{p}$ – a particle momentum vector
$P_{ij}(k)$ – a power spectrum of $\delta\mathbf{B}$
q – a spectral index for waves
r_g – a particle gyro-radius
t_c – a turbulence correlation time
$\mathbf{v} \equiv c^2\mathbf{p}/\varepsilon$ – a particle velocity vector
V_A – the Alfvén velocity in the field B_0
$v_{\|}$ – a velocity along B_0
α– a wave-cone opening angle
$\gamma \equiv (1 - v^2/c^2)^{-1/2}$ – a particle Lorentz factor
λ_{max} – a maximum considered wave length
$\kappa_{\perp}$ – a transverse (cross-field) diffusion coefficient
$\kappa_{\|}$ – a parallel diffusion coefficient
ω – a wave frequency
$\Omega \equiv eB/\gamma mc$
$\Omega^{(o)} \equiv eB_\circ/\gamma mc$
$\Omega_\circ \equiv eB_\circ/mc$
ρ – a mass density of particles

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