Astron. Astrophys. 337, 883–886 (1998)

ASTRONOMY
AND
ASTROPHYSICS

One possibility of dip formation in magnetic field lines associated with a filament

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Received 26 February 1998 / Accepted 10 June 1998

Abstract. The shape of a curve representing the axis of a filament of finite length in equilibrium in an external magnetic field is studied. The magnetic field of the filament is a twisted flux tube. The ends of the tube are anchored at the photosphere. The direction of the filament current corresponds to models of inverse polarity. When the angle γ between the magnetic field and the axis of the filament becomes greater than $\approx 30°$, a dip arises in the middle part of the equilibrium contour which represents the filament axis. This dip can be treated as necessary condition for cold dense material accumulation in the filament flux tube.

Key words: Sun: activity – Sun: corona – Sun: filaments – Sun: magnetic fields – Sun: prominences

1. Introduction

It is now widely accepted that the support of dense cold filament (prominence) material high in the corona is due to a magnetic field. The magnetic field lines near the prominence location must have some dips for the stability of plasma equilibrium. If all sources of magnetic field are below the photosphere (current-free approximation for coronal field) the only possibility to have a dip in the field lines is in the vicinity of saddle points (lines). Such peculiarities in the coronal magnetic field could appear if photospheric magnetic fields have at least a quadrupolar configuration. Whether these configurations are as widespread on the Sun as filaments are is a debatable question (Antiochos 1992; Uchida 1997; Cheng & Choe 1997).

In simple bipolar magnetic regions some deformations of magnetic field lead to the appearance of a “dipped” geometry if electrical currents are allowed in the corona. Wu et al. (1990), Choe & Lee (1992) proposed that the weight of the dense prominence material deforms the coronal arches sufficiently that dips form. However, this mechanism will not work if the plasma beta is low as estimated in the corona. Antiochos (1992) put forward a model in which a dip is a direct consequence of a strong shear on a flux tube near a polarity inversion line. The 3D structure of prominences is essential for their formation in this model. The footpoints of the flux tube are moved by the strong shear motions into regions of weak field, whereas the midpoint remains in the region of strong field. Consequently, the expansion of sheared force-free flux tubes will be greater near the footpoints.

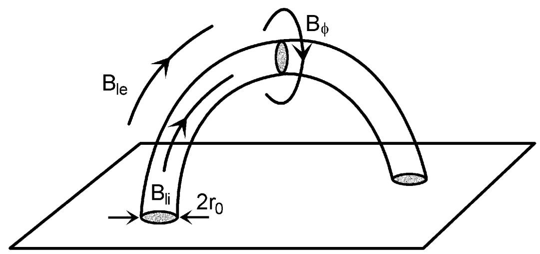

Fig. 1. Sketch showing the filament flux tube with the electric current and the magnetic field components

Twisted flux tube also have fragments of field lines with the “U” shape at the bottom of the helix. Helical field lines can be formed by non-uniform vortical photospheric motions (Browning 1991) or as result of reconnection of magnetic flux transported to a neutral line by converging motion (Van Ballegooijen & Martens 1989; 1990). Rust & Kumar (1994) suggested a twisted flux tube is lifted up from below the photosphere.

In this paper we intend to show the possibility of dip formation due to line-tying of the ends of a filament to the photosphere. A finite length of a filament is essential as in the model of Antiochos (1992) though greater height of the parts near the footpoints has another origin. The physical principles of the model are described and estimates are made in Sect. 2. In Sect. 3 the numerical model of a filament is presented which confirms the idea suggested in Sect. 2. Conclusions are drawn in Sect. 4.

2. Physical model

Two-dimensional equilibrium of a filament represented by a linear current has been studied by Kuperus & Raadu (1974), Van Tend & Kuperus (1978), Kaastra (1985), Molodenskii & Filippov (1987), Martens & Kuin (1989). Consider a situation differing from that analyzed in these works by a finite length of a filament. Nevertheless, the filament length is much greater than its height above the photosphere. The ends are anchored in the photosphere which has been assumed here to be a rigid con-

Send offprint requests to: B.P. Filippov ductive surface. The condition of the middle part of the filament is not far from the equilibrium conditions of a straight current. In the external horizontal uniform magnetic field B_0, directed orthogonal to the filament axis, the equilibrium equation of the filament is

$$\frac{B_0 I_l}{c} = \frac{I_l^2}{c^2 h} , \quad (1)$$

where I_l is the current component directed long the filament axis, h is equilibrium height. From (1) it follows that

$$h = \frac{I_l}{cB_0} . \quad (2)$$

In addition to axial component of the current, which exerts a dominant influence on the filament equilibrium, the azimuthal current I_φ should also be taken into consideration at least partially. The azimuthal current creates the axial magnetic field component B_l in the filament (Fig. 1). The tension of longitudinal field lines ultimately helps to hold and stabilize the filament when the ends are anchored. If we consider the filament to be a cylindrical plasma pinch with radius r_0, the condition of radial equilibrium takes the form (Shafranov 1963)

$$\langle p \rangle + \frac{\langle B_{li}^2 \rangle}{8\pi} = \frac{B_\varphi^2}{8\pi} + \frac{B_{le}^2}{8\pi} , \quad (3)$$

where $\langle p \rangle$ is the plasma pressure averaged over the cylinder cross section, $\langle B_{li}^2 \rangle$ is the mean internal axial field, and B_{le} is the external longitudinal field. This relationship shows the importance of the longitudinal field for the internal equilibrium of the filament. If the external field B_{le} is absent, i.e. B_{li} is created solely by the azimuthal current of the filament, then $B_{li} \leq B_\varphi$. However,

$$B_\varphi / B_{li} = \tan\gamma , \quad (4)$$

where γ is the angle between the field line and the filament axis. From field measurements in quiescent prominences, this angle is usually $20° - 30°$ (Leroy et al. 1984). It is also clearly visible in the fine structure of filaments, and increases to $40° - 50°$ shortly before a flare and filament eruption occur (Kulikova et al. 1989). For our purpose of the analysis of the external equilibrium of a flux tube we can assume that axial component of the magnetic field only creates the tension

$$f_T = \frac{\langle B_{li}^2 \rangle}{8\pi} (\pi r_0^2) . \quad (5)$$

If the curvature of the flux tube is low, as can be expected near the midpoint for a long enough filament, the tension does not influence the equilibrium height h. Another matter is in the vicinity of the footpoints. First, the repulsive force between an element of the current $I d\mathbf{l}$ and the mirror image $-I d\mathbf{l}^*$ is proportional to $1/r$ and $(d\mathbf{l} \cdot \mathbf{r})/(dl \cdot r)$, where $\mathbf{r}$ is the radius-vector from the mirror image element and $d\mathbf{l}$. The inset in Fig. 2 shows the vectors which enter into the expression for the repulsive force. Near the footpoints h and hence $\mathbf{r}$ is very small so $(d\mathbf{l} \cdot \mathbf{r})/(dl \cdot r)$ must be small too to have limited repulsive force. Therefore, the flux tube must approach the photosphere at

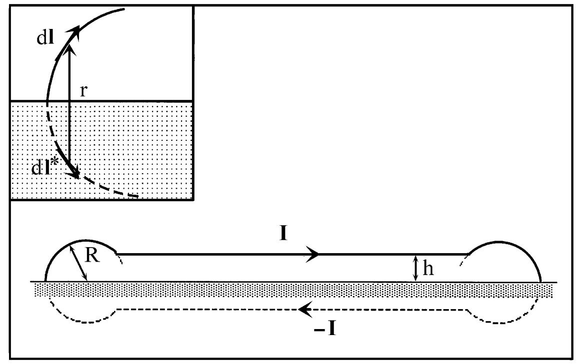

Fig. 2. Schematic shape of the filament flux tube

right angle. Then it goes into horizontal position along a smooth curve, which can be assumed approximately as a fraction of a circle.

Stretching of a ring by the current flowing along it is given by (Landau et al. 1984)

$$f_I = \frac{I^2}{c^2} \left(\ln \frac{8R}{r_0} - \frac{3}{4} \right) , \quad (6)$$

where R is the radius of the circle. For contours having small to moderate noncircularity, it is shown by Garren & Chen (1994) that the Lorenz self-force on a segment with local major R and minor r_0 radii is approximately that for an axisymmetric torus having uniform R and r_0.

The external field B_0 strives to shrink a half of a circle with the force

$$f_B = \pi R \frac{B_0 I}{c} \quad (7)$$

in order to decrease the curvature radius.

The curved end segments of flux tube will be in equilibrium if

$$f_I = f_B + f_T . \quad (8)$$

The substitution of Eqs. (4)–(7) in (8) yields

$$R = h \left(\ln \frac{8R}{r_0} - \frac{3}{4} - \frac{1}{2\tan^2\gamma} \right) . \quad (9)$$

A dip in the middle part of the tube means that $R > h$, therefore

$$\ln \frac{8R}{r_0} - \frac{3}{4} - \frac{1}{2\tan^2\gamma} > 1 , \quad (10)$$

and

$$\tan^2\gamma > \frac{1}{2\left(\ln \frac{8R}{r_0} - \frac{7}{4} \right)} . \quad (11)$$

Assuming $R/r_0 \approx 10$ one can find

$$\gamma > 25° . \quad (12)$$

This value is not very sensitive to ratio R/r_0 because the latter enters into Eq. (10) as the argument to a logarithm.

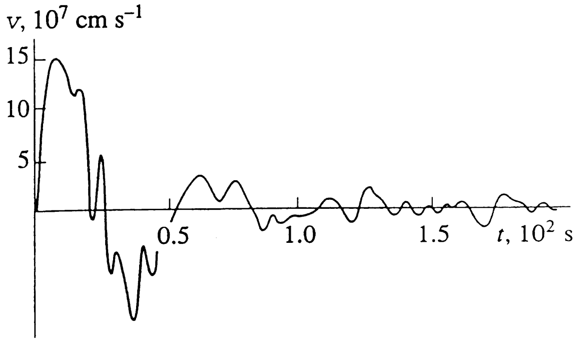

Fig. 3. Time variation of the velocity of a single filament element for damped oscillations in a uniform external field

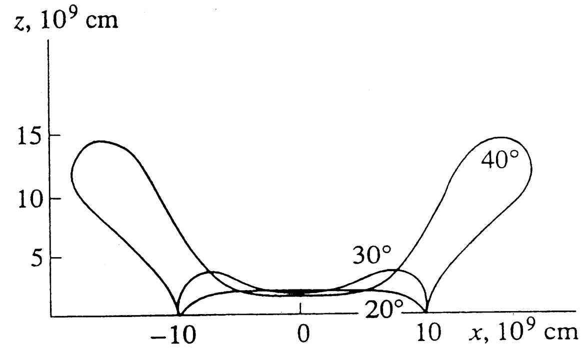

Fig. 4. Equilibrium shape of a contour in a uniform field for different angles γ, $I_l = 2 \times 10^{12}$ A, $B_0 = 100$ G

3. Numerical calculations

The position of the stable equilibrium of the flux tube filled with plasma can be determined by finding a stationary solution of the equation of motion. One way in which this can be done is to add an artificial dissipation to the equation of motion which gives damped oscillations of a filament toward equilibrium.

An element of the current $I d\mathbf{l}$ generates a magnetic field

$$d\mathbf{B} = \frac{1}{c}\frac{I}{r^3}\,(d\mathbf{l} \times \mathbf{r}) \ . \tag{13}$$

The fulfillment of the boundary condition dB_z/dz at the photospheric surface $z = 0$ is equivalent to introducing the mirror image of the element $-I d\mathbf{l}^*$.

Turning to finite differences, we write for the ith current element:

$$\Delta\mathbf{l}_i = \{x_{i+1} - x_i, 0, z_{i+1} - z_i\} \ , \tag{14}$$

$$\mathbf{r} = \{x - (x_{i+1} + x_i)/2, 0, z - (z_{i+1} + z_i)/2\} \ , \tag{15}$$

$$\Delta\mathbf{l}_i^* = \{x_{i+1} - x_i, 0, z_i - z_{i+1}\} \ , \tag{16}$$

$$\mathbf{r}^* = \{x - (x_{i+1} + x_i)/2, 0, z + (z_{i+1} + z_i)/2\} \ . \tag{17}$$

The y component of the field near the jth current element can then be represented as the sum

$$B_{yi} = \sum_{i=1, i\neq j}^{N} \Delta B_{yi} + \sum_{i=1}^{N} \Delta B_{yi}^* + B_y^e \ , \tag{18}$$

where ΔB_i and ΔB_i^* are the fields generated by elements of the current $I\Delta\mathbf{l}_i$ and of the mirror current $-I\Delta\mathbf{l}_i^*$, and B^e is the external field. This field acts upon each element of the current with the Lorenz force

$$\mathbf{F}_j = \frac{1}{c}\,(\Delta\mathbf{l}_j \times \mathbf{B}_j) \ . \tag{19}$$

The continuity condition for the current I links the coordinates of adjacent elements: the end of one element coincides with the beginning of the next one, forming nodes. It is easiest to write the equations of motion of the current elements for the nodes, assigning them a mass equal to the average of the masses of the neighboring elements $(m_j + m_{j-1})/2$. The tension of the axial field lines acts upon a node from the adjacent element along its direction $\Delta\mathbf{l}_j$. It is proportional to the relative change in length of the element $\Delta l_j/\Delta l$, where Δl is the initial length of the current element. The equation of motion of the jth node is

$$\begin{aligned}\frac{(m_j + m_{j-1})}{2}\frac{d\mathbf{v}_j}{dt} &= \frac{1}{2}\,(\mathbf{F}_j + \mathbf{F}_{j-1}) + f_T(\Delta\mathbf{l}_j - \Delta\mathbf{l}_{j-1})/\Delta l \\ &\quad + \frac{1}{2}\,(m_j + m_{j-1})\,\mathbf{g}_\odot - k\mathbf{v}_j \ ,\end{aligned} \tag{20}$$

where

$$\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_j \ , \tag{21}$$

$\mathbf{r}_j = \{x_j, 0, z_j\}$, $g_\odot$ is the free fall acceleration on the Sun, and k is the coefficient chosen so as to bring the filament to a steady state after 2–3 periods of damped oscillations.

We numerically solved the set of ordinary differential Eqs. (20) and (21) using a fourth-order Runge-Kutta method. Initial shape of the flux tube was a smooth loop and external field was $\mathbf{B} = \{0, B, 0\}$. In this field, a stable equilibrium of the straight current is possible for any current value. We took the number of elements in the calculations to be $N = 40$. This is sufficient to reproduce the complicated shape of the filament without requiring excessive computer time to integrate the equations.

The velocity of an individual node is shown in Fig. 3. The parameters are chosen corresponding to the following quantities on the Sun. Separation between the anchored feet of the filament loop is 2×10^{10} cm, $B_0 = 100$ G, $I_l = 2\times10^{12}$ A, filament mass, $m = 10^{16}$ g. With these values, the fundamental oscillation period of an element around its equilibrium position is about 50 s. There are, in addition, shorter period oscillations associated with individual, rather than collective, motions of an element.

Fig. 4 shows equilibrium shapes for various angle γ. For $\gamma < 30°$, the contour has the shape of a flattened arch. For $\gamma > 30°$ the ends of the contour have the greater height than the midpoint. So, the dip is formed in the middle part of the flux tube. The value of γ at which the dip starts to form is consistent with the estimate (12) made in the Sect. 2.

4. Conclusions

We have considered a model in which a filament is a stretchable contour with an electric current in an external magnetic field. The direction of the current corresponds to models of inverse polarity. The ends of filament flux tube are anchored to the photosphere, which is assumed to be a flat, rigid, ideally conductive surface.

When the angle γ between the magnetic field and the axis of the filament becomes greater than $\approx 30^\circ$, the dip arises in the middle part of the equilibrium contour. This dip can be treated as necessary condition for cold dense material accumulation in the filament flux tube. The matter is in the equilibrium conditions of end segments of the tube near the surface of the photosphere. The tube must enter the photosphere at right angle and this requirement leads to the curve of the contour which cannot be held by magnetic tension at small values of γ.

Acknowledgements. I wish to thank the referee Dr. P.Kuin for giving his valuable suggestions and correction of the language of this paper. This work was supported in part by Russian Foundation for Basic Research (grant 96-02-16285) and Russian State Astronomical Program.

References

Antiochos S.K., 1992, in: Coronal Streamers, Coronal Loops, and Coronal and Solar Wind Composition, ESA, Proc. of the First SOHO Workshop, p. 201
Browning P.K., 1991, Plasma Phys. and Contr. Fusion 33, 571
Cheng C., Choe G., 1997, 167 Coll. IAU (in press)
Choe G.S., Lee L.C., 1992, Sol. Phys. 138, 291
Garren D.A., Chen J., 1994, Phys. Plasmas 1, 3425
Kaastra J.S., 1985, Solar Flares: An Electrodynamical Model, Thesis, Utrecht Astronomical Institute
Kulikova G.N., Molodensky M.M., Starkova L.I., Filippov B.P., 1989, in: Teplitskaya R.B. (ed.) Solar Magnetic Fields and Corona, Proc. XIII Consultation Meeting on Solar Phys., Odessa, 1988, Nauka, Novosibirsk, V. 2, p. 156
Kuperus M., Raadu M.A., 1974, A&A 31, 189
Landau L.D., Lifshitz E.M., Pitaevskii L.P., 1984, Electrodynamics of Continuous Media, 2nd ed., Pergamon, Oxford, p. 124
Leroy J.L., Bommier V., Sahal-Brechot S., 1984, A&A 131, 33
Martens P.C.H., Kuin N.P.M., 1989, Sol. Phys. 122, 263
Molodenskii M.M., Filippov B.P., 1987, SvA 31, 564
Rust D.M., Kumar A., 1994, Sol. Phys. 155, 69
Shafranov V.D., 1963, in: Leontovich M.A. (ed.) The Problems of Plasma Theory, Gosatomizdat, Moscow, V. 2, p. 92
Uchida Y., 1997, 167 Coll. IAU (in press)
Van Ballegooijen A.A., Martens P.C.H., 1989, ApJ 343, 971
Van Ballegooijen A.A., Martens P.C.H., 1990, ApJ 361, 283
Van Tend W., Kuperus M., 1978, Sol. Phys. 59, 115
Wu S.T. et al., 1990, Sol. Phys. 125, 277