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On the role of self-organised criticality in accretion systems

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Abstract. Self-organised criticality (SOC) has been suggested as a potentially powerful unifying paradigm for interpreting the structure of, and signals from, accretion systems. After reviewing the most promising sites where SOC might be observable, we consider the theoretical arguments for supposing that SOC can occur in accretion discs. Perhaps the most rigorous evidence is provided by numerical modelling of energy dissipation due to magnetohydrodynamic turbulence in accretion discs by G Geertsema & A Achterberg (*A&A* **255**, 427 (1992)); we investigate how "sandpile"-type dynamics arise in this model. It is concluded that the potential sites for SOC in accretion systems are numerous and observationally accessible, and that theoretical support for the possible occurrence of SOC can be derived from first principles.

Key words: accretion, accretion discs – stars: novae, cataclysmic variables – galaxies: active – X-rays: stars

1. Introduction

The question whether accretion discs can be in a state of self-organised criticality (SOC) (Bak et al. 1988, Kadanoff et al. 1989) was raised explicitly by Mineshige et al. (1994), and strongly implicitly by Bak et al. (1988). Young & Scargle (1996) have raised the related question of transient chaos in accretion systems. A distinctive feature of SOC is flickering energy transport with no characteristic lengthscale or time separation, displaying a $1/f$ power spectrum; in this context, $1/f$ is shorthand for inverse power law frequency dependence with unspecified index. In SOC systems, for which mathematical sandpiles (Bak et al. 1988, Kadanoff et al. 1989) provide a paradigm, global transport occurs as a result of self-organised avalanches which are triggered locally when the accretion of sand leads to a critical gradient being exceeded at a given point. This causes local redistribution of sand, which may lead to the critical gradient being exceeded at neighbouring points, resulting in further redistribution and, cumulatively, to a global avalanche. Following the avalanche, the system returns to a subcritical configuration; accretion then continues until it again creates a local excess gradient, triggering a further avalanche. In the SOC-sandpile paradigm, it is helpful to note that the word "critical" is being used in two senses: the sandpile has a critical gradient, redistribution being triggered wherever this gradient is exceeded; and the self-organised global avalanches which emerge from the integrated effects of local redistribution may be scale-free, in which case they are linked to the generic field of critical phenomena.

In mathematical models and also some experimental realisations (Nagel 1992, Feder 1995, Frette et al. 1996, Christensen et al. 1996), avalanche statistics display scale-free $1/f$ characteristics. SOC sandpile algorithms are extremely simple and possess the attraction of any successful reduced system: it becomes unnecessary to attempt to model the detailed, and perhaps insuperably complex, microphysics of transport in the real system. Such an approach has been applied to the statistics of solar X-ray bursts (Lu & Hamilton 1991; for a recent treatment, see MacKinnon & Macpherson 1997 and references therein) and to transport phenomena in magnetically confined plasmas (Newman et al. 1996, Carreras et al. 1996, Dendy & Helander 1997, 1998). It is clearly of interest to establish whether, in certain circumstances, the detailed modelling of astrophysical accretion flows could be substituted by a simple SOC sandpile paradigm. This is a highly cross-disciplinary question. In the present paper, we aim to carry forward the debate in two ways: by identifying in greater detail the classes of accretion flow where SOC might play a role; and by examining theoretical arguments that we believe point clearly towards SOC in some accretion flows.

2. Some sites of interest

It was noted by Bak et al. (1988) that flickering signals with $1/f$ power spectra have been observed for the X-ray variability of active galactic nuclei (AGNs), specifically: 0.05-2keV X-rays from the Seyfert galaxy NGC4051 (Lawrence et al 1987) and 2-7keV X-rays from the Seyfert galaxy NGC5506 (McHardy & Czerny 1987), and for 10-140 keV X-rays from the massive compact binary Cyg X-1 (Nolan et al. 1981). Similar $1/f$ spectra for X-ray variability from binary accreting systems were noted by Mineshige et al. (1994) (neutron stars, Makashima 1988) and by Geertsema & Achterberg (1992) (cataclysmic variables and dwarf novae, Wade & Ward 1985). While we believe this

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list can be extended, as we discuss below, it is also important to be more specific about where, in these diverse systems, SOC might be occurring.

Perhaps the simplest case is presented by a paradigmatic AGN. Let us take for this the standard picture of a massive black hole fed by a cascade of structures: the accretion disc, the molecular torus and on a larger scale the galactic disc and its barred or spiral structure, which together generate a clumpy and irregular transfer of gas and stars. This irregular mass transfer could be analogous to the random and discrete feeding of a sandpile with grains of sand near its apex. It is believed that, at least in thin discs, the residence time τ_{res} of matter in the disc is much longer than the free–fall time, $\tau_{ff} \approx (R^3/GM)^{1/2}$ at radius R:

$$\frac{\tau_{res}}{\tau_{ff}} \sim \frac{R^2}{\alpha H^2}$$

where $H \ll R$ is the thickness of the disc, and $\alpha \ll 1$ is the well-known viscosity parameter. The condition $\tau_{res} \gg \tau_{ff}$, which is implicit in most accretion disc models, appears to be necessary, at least in principle, for the validity of a sandpile–type approach to mass transfer within the disc.

Thus if the accretion disc, like the sandpile, is in a state of SOC, there would be no clear link between the pattern of accretion from the torus and the pattern of avalanches leading to mass transfer across the disc, over its inner edge, and onto the black hole, giving rise to the X-ray signal. The latter would automatically display $1/f$ flicker. On larger space and timescales relating to our own Galaxy, even steady gas inflow from the Galactic bar could also result in avalanches to the inner regions, and these avalanches could give rise to episodes of AGN activity during an otherwise quiescent phase, with a duty cycle of a few percent, as inferred on statistical grounds for other AGNs (Mezger et al., 1996). The sandpile-SOC paradigm thus provides a candidate framework for summing up the consequences of the complex and intricate physics that relates the largescale dynamics of the Galactic disc to the activity of its central black hole.

Cataclysmic variables, and in particular dwarf novae, present further opportunities for SOC. First, the flow of matter from the secondary across the inner Lagrangian point could itself be an avalanching process governed by SOC. In this case, if the radiation source was a hot spot where the accreting mass flow reached the outer edge of an accretion disc, it would flicker with $1/f$ statistics. Steady mass flow from the secondary would also be compatible with a SOC signal, however, provided the latter originated from SOC mass transfers from the accretion disc (or accretion column in the strongly magnetised regime) to the white dwarf. Either regime appears possible in principle, both for dwarf novae and in the wider context of binary accreting systems. In dwarf novae, it is pointed out by van Amerongen et al. (1990) and Lasota et al. (1995) that outbursts are due to suddenly increased accretion towards the white dwarf, but that it is unclear whether the instability resides in mass transfer from the secondary to the accretion disc, or across the accretion disc and onto the white dwarf. We note that, observationally, flickering is more often strong during the quiescent phase of dwarf novae than during major eruptions (Wade & Ward 1985). Questions relating to the location of unstable flows in soft X-ray transients (Lasota et al. 1996) are similar to those in dwarf novae. For the wind-fed X-ray binary pulsar GX301-2, a model has been proposed (Orlandini & Morfill 1992) involving "noisy" accretion of blobs of matter formed by magnetohydrodynamical (MHD) instability at the magnetospheric radius, and not caused by inhomogeneities present in the stellar wind from the optical companion. This approach is somewhat reminiscent of that of Baan (1977), where accreted matter accumulates at the magnetopause of a rotating neutron star until an interchange instability is triggered, after which the released matter generates an X-ray burst. Baan (1977) suggested that, since most of the time between bursts is a refilling time, an approximately linear relation should exist between burst energy and the subsequent quiescent interval. However, in a SOC model where randomly arising local instability is sufficient to trigger a global avalanche, there would be no such correlation. This appears to be a key observational discriminant for the possible presence of SOC in a given accreting system. We also note that, from the theoretical point of view, Frank *et al.* 1992 have pointed out that a local instability in a given annulus of the disc can only trigger large–scale instability accross the disc if parameters in neighbouring annuli are such that the effect of this instability in these annuli can in turn trigger local instability there. This amounts to a prescription for a sandpile–type approach, and hence for the possibility of SOC.

It seems clear from the foregoing that there is good observational and interpretative motivation for testing for SOC in a broad range of accreting astrophysical systems, encompassing flickering AGNs and certain distinct locations within a variety of binary objects. Firm identification of SOC would yield information on the global consequences of the smaller-scale physics of the accretion process, while short-circuiting the need for detailed modelling. Let us now turn to theoretical arguments for expecting SOC.

3. Theoretical considerations

The identification of mechanisms responsible for the dissipation of shear flow energy within an accretion disc is an active field of research, because of its importance in determining transport across the disc and onto the compact object. If, as has been suggested (Bak et al 1988, Mineshige et al. 1994), the combined global effects of local transport physics result in SOC, observational signatures of the type described above will emerge. It is widely accepted (see, for example, Longair (1994) and Narayan (1997)) that anomalous viscosity caused by MHD turbulence probably plays an important role in the flux of angular momentum within accretion discs. MHD turbulence can arise naturally in accretion discs, see for example the instability mechanisms proposed by Tagger et al. (1990), Vishniac et al. (1990), and Balbus & Hawley (1991). It has also been pointed out (Chen et al. 1995) that there exist ranges of accretion rate $\dot{M}$ and disc radius R for which no stable steady state solution of the basic equilibrium equations is possible. In this case, it is suggested

(Chen et al. 1995, Narayan 1997) that the flow is forced into a time-dependent variable mode, satisfying the required $\dot{M}$ in the mean. This provides further motivation to test the applicability of sandpile-type models. In this section, we concentrate on the model for MHD turbulent energy dissipation in accretion discs presented by Geertsema & Achterberg (1992), because it appears to give rise to SOC. While the link to cataclysmic variable and dwarf nova observations (but not, explicitly, SOC) was made by Geertsema & Achterberg (1992) themselves, we believe that this work has much wider implications: for the general question of SOC in accretion flows; for the connection between SOC and turbulence in general; for the role of SOC in plasma physics, since it represents the first instance where SOC has been observed in a mathematical model derivable from the fundamental equations of MHD; and for the role of SOC in terrestial experimental systems - real sandpiles and ricepiles, as distinct from mathematical idealisations thereof - where uncertainties about its scope remain (Nagel 1992, Feder 1995, Frette et al. 1996, Christensen et al. 1996).

Before considering the model of Geertsema & Achterberg (1992) in greater detail, let us turn to its results. Fig. 12 of Geertsema & Achterberg (1992) shows the calculated times series of energy dissipation events within the disc. We note that this is qualitatively very similar to the observed time series of energy dissipation measured in an experimental ricepile displaying SOC, Fig. 2c of Frette et al. (1996), and in a related mathematical model of Dendy & Helander (1997, Fig. 3, and 1998, Fig.5). More quantitatively, Fig. 13 of Geertsema & Achterberg (1992) shows the power spectrum of energy dissipated by MHD disc turbulence, which displays the $1/f$ dependence characteristic of SOC; compare, for example, Fig. 3 of Frette et al. (1996) and Fig. 3 of Christensen et al. (1996) which show measured spectra of energy dissipation and particle transit times, respectively, in SOC ricepiles.

Given the clear indications of SOC emerging from the MHD turbulence model of Geertsema & Achterberg (1992), it will be of interest to establish how it has arisen. A full explanation must await diagnostic analysis of the code runs generated by this model. Pending this, we conclude the present section by seeking to identify some of the relevant salient features. In outline, the model of Geertsema & Achterberg (1992) is constructed as follows.

The accretion disc is regarded as a differentially rotating turbulent MHD fluid, and is modelled by a reduced system of equations reflecting the most important features of three-dimensional MHD turbulence. The disc is assumed to be thin in comparison with its diameter, and the flow is taken to be subsonic and hence incompressible. The possible existence of a large-scale magnetic field is neglected, and the only turbulent structures considered have a length scale shorter than the height of the disc, making the turbulence essentially three-dimensional. In a coordinate system rotating with the disc angular velocity $\mathbf{\Omega} = \boldsymbol{e}_z \Omega_K(R)$ at radius $r = R$, the simplified MHD equations for the flow velocity $\mathbf{u}$ and the normalised magnetic field vector $\mathbf{b} = \mathbf{B}/(4\pi\rho)^{1/2}$ are

$$\dot{\mathbf{u}} = -\nabla\Pi - 2\mathbf{\Omega} \times \mathbf{u} - \left(r\frac{d\Omega_K}{dt}\right)_R u_x \mathbf{e}_y + \nu\nabla^2\mathbf{u} + \mathbf{b}\cdot\nabla\mathbf{b} - \mathbf{u}\cdot\nabla\mathbf{u},$$

$$\dot{\mathbf{b}} = \left(r\frac{d\Omega_K}{dt}\right)_R b_x \mathbf{e}_y - \eta\nabla^2\mathbf{b} + \mathbf{b}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{b},$$

where $x = r - R$ and $y = R\phi$ are the local radial and azimuthal coordinates respectively, η is the resistivity, and ν is the viscosity coefficient. $\Pi = P/\rho + b^2/2$ is the total scalar pressure, while the $\mathbf{b}\cdot\nabla\mathbf{b}$ term contains the off–diagonal terms of the MHD stress tensor.

As these equations are still too complex for a numerical treatment over a sufficiently large dynamic range in scales, they were simplified further following a suggestion by Desnyanski & Novikov (1974) in the context of hydrodynamics, and later applied to MHD by Gloaguen et al. (1985). In this approximation, the space of wave vectors $\mathbf{k}$ is discretized into a finite set of k_n, and the non-linear interaction between different components of the Fourier transforms of $\mathbf{u}$ and $\mathbf{b}$ is described by the set of equations

$$\frac{du_n}{dt} + \nu k_n^2 u_n = \alpha k_n \left(u_{n-1}^2 - 2u_n u_{n+1}\right) + \beta k_n \left(u_{n-1}u_n - 2u_{n+1}^2\right) - (u \leftrightarrow b),$$

$$\frac{db_n}{dt} + \eta k_n^2 b_n = \alpha k_n \left(u_{n-1}b_{n-1} - 2u_n b_{n+1}\right) + \beta k_n \left(u_{n-1}b_n - 2u_{n+1}b_{n+1}\right) - (b \leftrightarrow u).$$

In the three-dimensional generalisation of this approximation scheme, the discretisation is made by dividing the $\mathbf{k}$-space into into spherical shells, thus discarding the information regarding the direction of $\mathbf{k}$. This allows a greatly simplified system of nonlinear equations to be written down, analogous to that of Gloaguen et al. (1985), which are supplemented with additional terms to account for the effects of differential rotation.

These equations, which are taken to model the turbulent cascade of MHD, were solved numerically by Geertsema & Achterberg (1992). They found that the turbulent shear stress can be very large, and has large, chaotic fluctuations on time scales of a few rotation periods. Perhaps the most striking feature of the simulations is, however, that the dissipation of energy at the smallest scales of the turbulent cascade is very intermittent. Energy is released in avalanches with a wide range of sizes, and the power spectrum of the dissipation rate obeys a $1/f$ power law over nearly two orders of magnitude in intensity, see their Fig. 13. This behaviour is, as already stated, similar to that of simple mathematical sandpile models. While a power law is to be expected from any scale-free model, in particular a Kolmogorov-type one in the inertial range, we note that the similarities between this system and mathematical sandpiles apparently extend further. Both are fundamentally governed by nearest-neighbour interactions between a discrete number of nodes. In the MHD accretion model, these reside in $\mathbf{k}$-space, so the interaction is between wave modes with similar wavelengths rather than between adjacent regions in real space. The dissipation is provided by viscosity and resistivity at large k, resembling the removal of material from the edge of a sandpile. The MHD accretion model is, of course, much more complex than the simple sandpile algorithms considered so far in the literature. Two fields, $\mathbf{u}$ and $\mathbf{b}$, each with three components, are involved rather than the single height parameter of conventional sandpile algorithms, and the time evolution is governed by differential equations rather than difference equations. Nevertheless, both the MHD accretion disc and simple sandpile models appear to exhibit similar self-organised, critical behaviour, supporting the claim often made in the sandpile literature that SOC is universal phenomenon shared by large classes of cellular automata.

4. Discussion

We have shown in this paper that the numerical simulations by Geertsema & Achterberg (1992) of viscous resistive MHD turbulence in an accretion disc give rise to behaviour characteristic of self-organised criticality in a sandpile. This similarity may help to explain certain observed properties of a range of accreting astrophysical systems, which we have reviewed. Furthermore, the result is of intrinsic scientific interest as an ab initio demonstration of the emergence of SOC from a system of MHD-based equations. Further analysis of the numerical results, to establish how the SOC-sandpile phenomenology arises, would be of great interest both for accretion disc astrophysics and for fundamental plasma physics. At the present stage of development of realistic MHD simulations (for example of the link between smallscale dissipation and the time behaviour of the flux of matter onto the central object), direct numerical proof of the occurence of SOC-sandpile phenomenology is still a remote objective. However, the capacity of the SOC-sandpile paradigm to circumvent complex “full mathematical descriptions” is the basis for its present attraction in many fields of physics. If future work confirms the importance of SOC-sandpile phenomenology in accreting systems that is suggested in the present paper, it would permit a dissociation (at least at zeroth order) of the detailed physics of turbulence in the disc from the global modelling of this class of astrophysical object.

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