Astron. Astrophys. 338, 62–68 (1998)

ASTRONOMY
AND
ASTROPHYSICS

Two-photon annihilation in the pair formation cascades in pulsar polar caps

Bing Zhang[1,2] **and G.J. Qiao**[1,2,3]

[1] Department of Geophysics, Peking University, Beijing, 100871, P.R. China,
[2] Beijing Astrophysics Center (BAC)* Beijing, 100871, P.R. China,
[3] CCAST, P.O. Box 8730, Beijing 100080, P.R. China

Received 21 April 1998 / Accepted 4 June 1998

Abstract. The importance of the photon-photon pair production process ($\gamma + \gamma' \to e^+ + e^-$) to form pair production cascades in pulsar polar caps is investigated within the framework of the Ruderman-Sutherland vacuum gap model. It is found that this process is unimportant if the polar caps are not hot enough, but will play a non-negligible role in the pair formation cascades when the polar cap temperatures are in excess of the critical temperatures, T_{cri}, which are around $4 \times 10^6 K$ when $P = 0.1$s and will slowly increase with increasing periods. Compared with the $\gamma - B$ process, it is found that the two-photon annihilation process may ignite a central spark near the magnetic pole, where $\gamma - B$ sparks can not be formed due to the local weak curvatures. This central spark is large if the gap is dominated by the "resonant ICS mode". The possible connection of these central sparks with the observed pulsar "core" emission components is discussed.

Key words: pulsars: general – X-rays: stars

1. Introduction

Recent X-ray observations and data analyses show strong evidence of thermal emission components from the whole surfaces as well as the hot polar caps of some pulsars (e.g. Ögelman 1995; Greiveldinger et al. 1996; Wang & Halpern 1997; Becker & Trümper 1997). The presence of these thermal photons near the neutron star (especially near the polar caps) may bring various new physical effects which have not been fully investigated before. One important process which has stimulated interest is the inverse Compton scattering (ICS) processing of these photons by the high energy particles. Recent studies show that the ICS process is not only a significant energy loss (Xia et al. 1985; Daugherty & Harding 1989; Dermer 1990; Chang 1995; Sturner 1995) and radiation (Sturner & Dermer 1994) mechanism, but is also the dominant mechanism to ignite pair production cascades near the pulsar polar caps and hence, to limit the parameters of the pulsar inner accelerators (Zhang & Qiao 1996, hereafter ZQ96; Qiao & Zhang 1996; Luo 1996; Zhang et al. 1997a, hereafter ZQLH97a; Zhang et al. 1997b, hereafter ZQH97b; Harding & Muslimov 1998, hereafter HM98). Within the framework of the Ruderman-Sutherland (1975, hereafter RS75) vacuum gap model, ZQ96 found that the ICS-induced $\gamma - B$ process is the most important mechanism to cause an inner gap breakdown. The two characteristic frequencies in the up-scattered ICS spectra lead to two different modes of the inner gaps, which are dominant within different temperature regimes (ZQLH97a, ZQH97b).

The mean free path of pair production is the key parameter to study pulsar polar cap physics, since it is the relevant quantity to determine the length of the inner accelerator. It is necessary to compare the relative importance of the various pair formation processes in the pulsar polar cap region. *A process with shorter pair formation mean free path will be more likely to dominate pair formation cascades in the inner gaps.* For example, for the $\gamma - B$ process, the ICS-induced pair formation mean free path is shorter than the curvature radiation (CR)-induced one, so the CR mode is usually suppressed. For the ICS process, when the polar cap temperature is high enough so that the scatterings off the soft photons near the peak of the Planck spectrum become important, this so-called "thermal-peak ICS mode" will suppress the otherwise dominant "resonant ICS mode", which is mainly the contribution of the resonant ICS process. This is also because the mean free path of the "thermal" ICS mode is shorter than that of the "resonant" one (ZQLH97a, ZQH97b).

Another process which might also be important with the presence of the hot thermal photon fields in the neutron star vicinity is the photon-photon pair production or the two-photon annihilation. The two ingredients which take part in the process are: the soft thermal X-rays and their inverse Compton γ-rays. In this paper, we will explicitly study the possible importance of the $\gamma - \gamma$ process and compare its mean free path with that of the $\gamma - B$ process. The method to calculate the mean free path of the two-photon annihilation in the polar caps is described in Sect. 2. A comparison between the relative importance of the $\gamma - \gamma$ and the $\gamma - B$ processes in a dipolar magnetic field configuration is presented in Sect. 3. The possible connection of the $\gamma - \gamma$-induced central sparks with the pulsar "core" emission components is discussed in Sect. 4.

Send offprint requests to: B. Zhang

* BAC is jointly sponsored by the Chinese Academy of Sciences and Peking University

2. Two-photon annihilation and polar gap sparking

The threshold condition of two-photon annihilation for a high-energy photon (energy E) and a low-energy photon (energy ϵ) colliding with an incident angle of $\cos^{-1}\mu_c$ is

$$\epsilon E(1-\mu_c) \geq 2(m_0c^2)^2, \qquad (1)$$

where m_0c^2 is the static energy of the electron/positron. The high energy γ photons produced by the ICS process by the quasi-monoenergetic electrons/positrons over the semi-isotropic thermal photons are found to have two characteristic energies, the relative importance of which is determined by the polar cap temperature (ZQLH97a). The first characteristic energy is the contribution of the "resonant" scatterings, in which the scattering cross sections are tremendously enhanced due to the rapid transitions between the ground and the first Landau levels of the electrons in strong magnetic fields. Various resonant scatterings over photons with different energies coming from different angles all contribute to this sharp characteristic energy

$$E_{res} = 2\gamma\hbar\omega_B = 2\gamma\hbar\frac{eB}{m_0c} \simeq 2.3\times10^3\gamma_5 B_{12}\mathrm{MeV}, \qquad (2)$$

where $\gamma = 10^5\gamma_5$ is the Lorentz factor of the electrons/positrons, $\omega_B = eB/m_0c$ is the cyclotron frequency in a magnetic field $B = 10^{12}B_{12}\mathrm{G}$. The second broad peak in the ICS spectrum is the contribution of the scatterings over the soft thermal X-rays near the Planck peak, with the characteristic energy

$$E_{th} \sim \gamma^2\cdot 2.82kT \simeq 2.4\times10^4\gamma_4^2 T_6\mathrm{MeV}, \qquad (3)$$

where $T = 10^6T_6\mathrm{K}$ is the polar cap temperature. This second "thermal mode" is only important when the polar cap temperature is high enough (beyond the "critical temperature", see details in ZQLH97a). For a blackbody-like low-frequency photon field, we adopt

$$\epsilon \sim 2.82kT \simeq 240T_6\mathrm{eV}. \qquad (4)$$

From Eqs. [2-4] we find that the threshold condition (1) is easily satisfied, since the scatterings have a wide range distribution of μ_c.

To calculate the mean free path of two-photon annihilation, we follow the approach of Gould & Schréder (1967, hereafter GS67), but generalize it to be able to treat the case of anisotropic collisions. Assuming that the soft photon gas near the neutron star surface is blackbody-like (a discussion concerning this assumption is presented in the last section), then the absorption probability per unit path length is

$$\frac{d\tau}{dx}(x) = \frac{\alpha^2}{\pi\Lambda}\left(\frac{kT}{m_0c^2}\right)^3 f(\nu,x) \simeq 2.1\times10^{-6}T_6^3 f(\nu,x)\mathrm{cm}^{-1}, \qquad (5)$$

with $\alpha = e^2/\hbar c \simeq 1/137$, $\Lambda = \hbar/m_0c \simeq 3.86\times10^{-11}\mathrm{cm}$, and $\nu = (m_0c^2)^2/(EkT)$. The function $f(\nu,x)$ reads

$$f(\nu,x) = \nu^2\int_{\frac{2}{1-\mu_c(x)}\nu}^{\infty}(e^\epsilon-1)^{-1}\bar{\varphi}(\epsilon/\nu,x)d\epsilon \qquad (6)$$

(GS67, their Eqs. [17,18]). Our generalization of the GS67 formalism is to multiply the lower-limit of integration of GS67 to a factor of $2/(1-\mu_c)$ so as to be able to deal with the case of anisotropic collisions. Also the function $\bar{\varphi}(s_0,x)$ is re-defined as

$$\bar{\varphi}(s_0,x) = \int_1^{\frac{1-\mu_c(x)}{2}s_0(\epsilon)} s\bar{\sigma}(s)ds, \qquad (7)$$

with the upper limit of integration multiplied by a factor of $(1-\mu_c)/2$ with respect to the GS67's result (their Eq. [9]). Here $s_0 = \epsilon E/m_0^2c^4$, and the integrand of Eq. [7] can be deduced as

$$s\bar{\sigma}(s) = \left(2+\frac{2}{s}-\frac{1}{s^2}\right)\ln\frac{1+(1-\frac{1}{s})^{1/2}}{1-(1-\frac{1}{s})^{1/2}} - 2\left(1-\frac{1}{s}\right)^{1/2}\left(1+\frac{1}{s}\right) \qquad (8)$$

according to the GS67 formalism (see their Eqs. [1,3,4,9]).

It is of interests to calculate the shortest mean free path, so it is necessary to get the maximum value of the function $f(\nu)$. For an isotropic soft photon field, the minimum incident angle cosine is $\mu_c = -1$, which means that head-on collisions are important. In this case, Eqs. (7,8) are reduced to the GS67 results (their Eqs. [18,9]), and $\mathrm{Max}[f(\nu)] \simeq 1$. For the case of pulsar polar cap region, however, the emission is directed radially outward, so that the photon gas is only semi-isotropic ($\mu_c = 0$). Moreover, at higher altitudes, μ_c is even larger due to the finite size of the polar cap. In these cases, $\mathrm{Max}[f(\nu)]$ cannot be that large. With variable μ_c, it is not easy to get an analytic expression and asymptotic forms of Eq. [6] similar to what GS67 did. We thus performed a numerical calculation to get $\mu_c - \mathrm{Max}[f(\nu)]$ relation. The results are shown in Fig. 1. A fairly good polynomial fit to the data gives

$$\mathrm{Max}[f(\nu)] = 0.270 - 0.507\mu_c + 0.237\mu_c^2, \qquad (9)$$

which we will use directly in the further calculations. We see that $\mathrm{Max}[f(\nu)] \simeq 1$ when $\mu_c = -1$ according to Eq. [9], which is in accordance to the GS67 result.

With the above mentioned preparation done, we can calculate the shortest mean free path of the two-photon annihilation process. This minimum mean free path (denoted by $l_{\gamma-\gamma,min}$) can be obtained through the relation

$$\int_0^{l_{\gamma-\gamma,min}}\left(\frac{d\tau}{dx}\right)_{max}dx$$

$$= \int_0^{l_{\gamma-\gamma,min}} 2.1\times10^{-6}T_6^3\mathrm{Max}[f(\nu,x)]dx = \tau = 1, \qquad (10)$$

where $\mathrm{Max}f(\nu,x)$ is a function of μ_c, and hence, of the height x from the surface.

The last relationship which is necessary to calculate $l_{\gamma-\gamma,min}$ is the x dependence of μ_c. First, it is worth noticing that there exists a thermal "corona" (Chang 1995) or "atmosphere" (e.g. Pavlov et al. 1995) near the neutron star surface, the scale height of which is determined by $m_0gh \sim kT$, where $g \sim 10^{14}-10^{15}\mathrm{cm\ s}^{-2}$ is the typical gravitational acceleration of a neutron star. Hence, this scale height is

$$h = x_0 \simeq 820T_6\mathrm{cm}. \qquad (11)$$

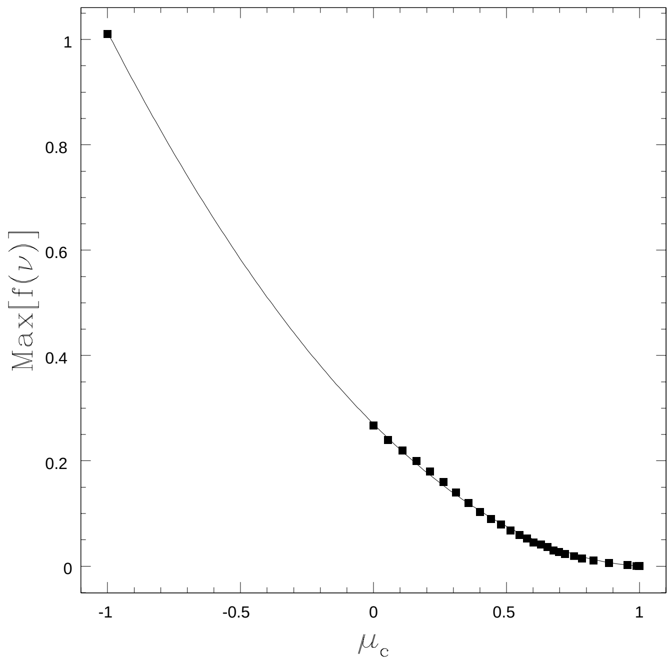

Fig. 1. The numerical results and a polynomial fit (Eq. [9]) of the $\mu_c - \mathrm{Max}[f(\nu)]$ graph.

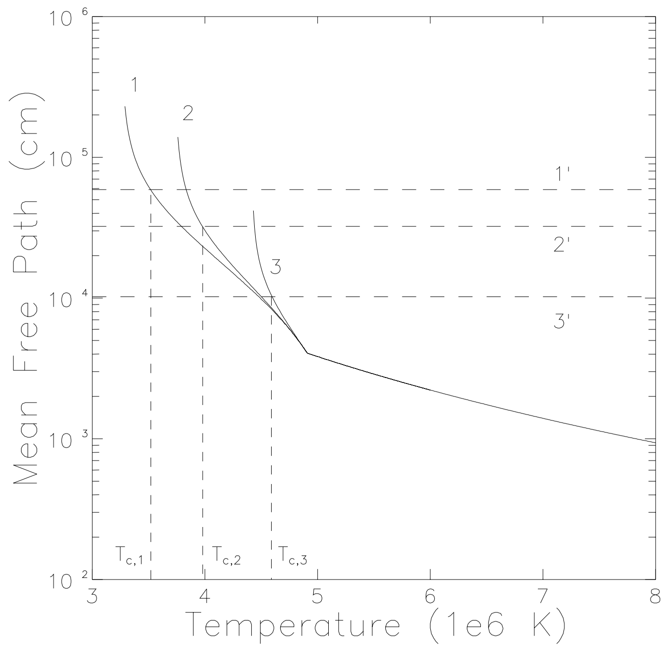

Fig. 2. The temperature dependence of the shortest mean free paths of two-photon annihilation. Solid lines indicate the temperature-dependent mean free path of two-photon annihilation, while the dashed lines denote the corresponding maximum gap height. The numbers 1, 2, 3 indicate pulsar periods $P = 0.03, 0.1, 1$s, respectively. We find that a critical temperature T_c clearly exists when the pulsar period is given.

Within this hot corona, the soft photon gas can be regarded as nearly isotropic, since thermal equilibrium might be set up there. Beyond this hot corona, the photon gas begins to show strong anisotropy, so that μ_c increases with x, while $\mathrm{Max}[f(\nu)]$ decreases rapidly with x. For the configuration of a hot spot with the dimension of the polar cap, the $\mu_c - x$ relation is

$$\mu_c = \begin{cases} -1, & x < x_0 \\ (1 - (\frac{R+x_0}{R+x})^2)^{1/2}, & x_0 \leq x < x_{cri} \\ \frac{x-x_0}{\sqrt{r_p^2+(x-x_0)^2}}. & x \geq x_{cri} \end{cases} \tag{12}$$

Here x_{cri}, which satisfies the relation of $\frac{(x_{cr}-x_0)^2}{r_p^2+(x_{cr}-x_0)^2} = \frac{(R+x_{cri})^2-(R+x_0)^2}{(R+x_0)^2}$, is the critical height at which the horizon is just the area of the polar cap, and R is the radius of the neutron star. Note that here we only consider the thermal emission from the "hot spot" at the polar caps, while neglecting the contribution of the whole surface. This is because only very high temperatures (see Figs. 2,3) can make the effect of two-photon annihilation become important. Such a high temperature is not likely for a cooling neutron star unless additional re-heating takes place in its polar caps.

Using Eqs. [10,12], we can calculate the temperature-dependent two-photon annihilation mean free path. In the RS75's vacuum gap model, the gap height cannot exceed

$$h_{max} = \frac{r_p}{\sqrt{2}} = 1.0 \times 10^4 P^{1/2}\mathrm{cm}. \tag{13}$$

Thus we regard

$$l_{\gamma-\gamma,min} \leq h_{max} \tag{14}$$

as the absolute criterion for occurrence of the two-photon annihilation process. Hence, we compare the temperature-dependent $l_{\gamma-\gamma,min}$ with h_{max} in Fig. 2. A critical temperature T_{cri} is thus defined, beyond which the two-photon process becomes important. From Fig. 2 we find that, $l_{\gamma-\gamma,min}$ is very sensitive to the polar cap temperatures. When the temperature increases slowly near T_{cri}, the mean free path may drop tremendously from infinity to less than h_{max}. For a pulsar period $P = 0.1$s, $T_{6,cri} \sim 4$. This means that the temperature necessary to switch this $\gamma-\gamma$ process on is very high. In Fig. 2, the three mean-free-path curves for different periods (curves 1,2,3) merge together eventually at some temperature, beyond which the mean free paths are less than the height of the "hot corona" x_0.

The critical temperature (T_{cri}) only depends on the pulsar period P, which determines the size of the polar cap, and hence, determines the importance of this effect. However, the T_{cri} variation is insensitive to P. Fig. 3 plots the critical temperatures for two-photon annihilation for different pulsar periods. We find that $T_{6,cri}$ increases with increasing period. The increasing rate is rapid for the fast-rotating pulsars, but becomes slower when the periods get longer. For most pulsars with period $P \sim 0.03 - 1.5s$, $T_{6,cri}$ ranges from 3.5-4.7.

Now we see that the importance of the two-photon annihilation for forming pair cascades in the pulsar polar cap depends completely on the temperatures of the polar caps. But can the polar caps be that hot?

Theoretical analysis and recent X-ray observations do show that pulsars are likely to have a hot polar cap with temperatures of the order of 10^6K. In the inner gap model, the maximum

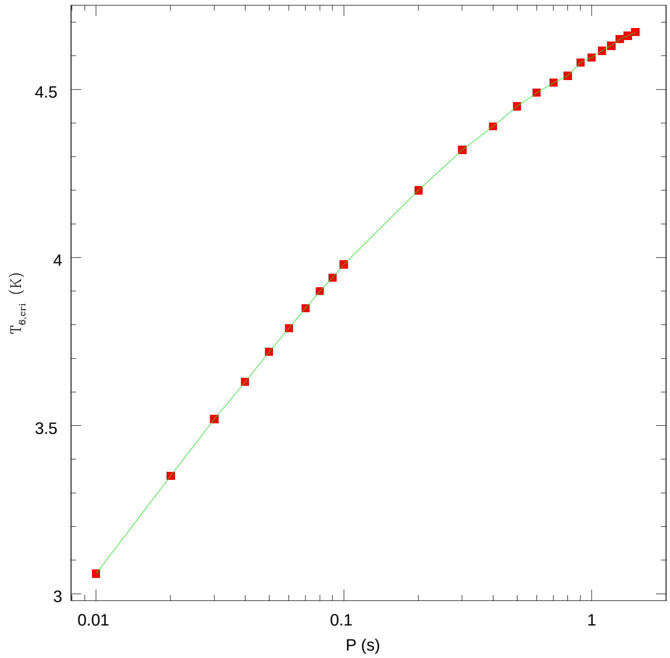

Fig. 3. Period-dependence of the critical temperatures.

polar cap temperature can be achieved by assuming the rotation energy loss deposited on the conventional polar cap area $A = \pi r_p^2 = 6.6 \times 10^8 / P \mathrm{cm}^2$. This gives

$$T_{max} = (\frac{\dot{E}_{rot}}{\sigma \pi r_p^2})^{1/4} \simeq 5.7 \times 10^6 \dot{P}_{-15}^{1/4} P^{-1/2} \mathrm{K}, \qquad (15)$$

where $\sigma = 5.67 \times 10^{-5} \mathrm{ergcm^2 K^4 s}$. Such high temperatures are indeed observed, e.g. $T_6 \sim 1.5$ for PSR 0656+14 and $T_6 \sim 3.7$ for 1055-52 (Greiveldinger et al. 1996), $T_6 \sim 5.14$ for PSR 1929+10 and $T_6 \sim 5.70$ for PSR 0950+08 (Wang & Halpern 1997). In view of this, we see that the two-photon process should play an non-negligible role in the pulsar polar cap physics.

3. Comparing the $\gamma - \gamma$ and the $\gamma - B$ processes

The final pair production cascades are the consequence of competition among various processes. To judge the relative importance of these processes, we again use the "shortest mean free path principle" discussed in Sect. 1. So it is necessary to compare the above-discussed $\gamma - \gamma$ process with the conventional $\gamma - B$ process, which has been long regarded as the only important mechanism to form polar cap pair cascades.

The basic difference between the two processes is that, the $\gamma - \gamma$ mean free path strongly depends on polar cap temperatures, while the $\gamma - B$ mean free path is quite sensitive to the field line curvatures. Hence, a direct picture is: $\gamma - B$ cascades can develop easily near the rim of the polar cap, but not easily near the magnetic pole where the curvature is weak. The $\gamma - \gamma$ process, however, has the almost same probability to be developed all over the polar cap region. In the RS75 inner gap model, it is reasonable to assume that the gap potential only breaks down with a local spark where the pair-production avalanche takes place, but keeps unbroken in regions where pair cascade condition does not hold. This is because the pair flows cannot cross the field lines to fill the vacuum region other than the local spark, as the transverse energies of the secondary pairs are lost rapidly via synchrotron radiation. Thus a picture of gap sparking is: the gap grows at the speed of light, then sparks develop at the rim of the gap edge via the $\gamma - B$ process, but the gap height keeps growing near the pole until more $\gamma - B$ sparks take place. Finally, a $\gamma - \gamma$ induced central spark might be formed if the surface temperature is in excess of T_{cri}. While this picture is not presented quantitatively, we believe that it is at least qualitatively sound though the cascade near the rim might compensate for the lack of cascades in the central region.

Take a similar absolute criterion

$$l_{\gamma - B, min} \leq h_{max} \qquad (16)$$

as Eq. [14], we examine how large is the area near the pole where the $\gamma - B$ process fails. In the pure dipolar configuration, consider a field line whose "root" is at a colatitude of $\xi \theta_p$, where $\theta_p = r_p / R$ is the size of the polar cap and $0 < \xi < 1$ indicates the distance to the pole. Then the local field line curvature reads

$$\rho = \frac{4}{3} (\frac{\xi^{-2} \Omega R}{c})^{1/2} \simeq 9.21 \times 10^7 \xi^{-1} P^{1/2} \mathrm{cm}. \qquad (17)$$

The characteristic parameter

$$\chi = \frac{E_\gamma}{2 m_0 c^2} \frac{B_\perp}{B_q} = \frac{E}{2 m_0 c^2} \frac{B}{B_q} \frac{l_{\gamma - B}}{\rho} \qquad (18)$$

to calculate the $\gamma - B$ mean free path (Erber 1966) is around 1/15 when $l_{\gamma - B}$ is of the order of h_{max} (RS75). Thus the area where $\gamma - B$ fails is

$$\xi \leq 2.7 \times 10^4 (\frac{\chi}{15^{-1}}) P^{1/2} \dot{P}_{-15}^{-1/2} E_\gamma^{-1} (\mathrm{Mev}). \qquad (19)$$

Using Eqs. [2,3], and adopt

$$\gamma = \gamma_{max} = \frac{e}{m_0 c^2} \frac{\Omega B}{c} h_{max}^2 \simeq 1.3 \times 10^7 P^{-3/2} \dot{P}_{-15}^{1/2}, \qquad (20)$$

we finally get

$$\xi_{res} \leq 0.09 (\frac{\chi}{15^{-1}}) P^{3/2} \dot{P}_{-15}^{-3/2} \qquad (21)$$

and

$$\xi_{th} \leq 6.6 \times 10^{-7} (\frac{\chi}{15^{-1}}) P^{7/2} \dot{P}_{-15}^{-3/2} T_6^{-1}. \qquad (22)$$

If the surface temperatures are higher than T_{cri}, Eqs. [21,22] just indicate the smallest area of the $\gamma - \gamma$ -induced central spark for the "resonant ICS mode" and the "thermal ICS mode", respectively (ZQLH97a, ZQH97b).

A conventional polar gap spark usually occupy a region characterized by the gap height (RS75; Gil et al. 1997; Gil & Cheng 1998, hereafter GC98). Using the ICS-induced gap parameters (see ZQH97b, their Eqs. [12,20]), we find a conventional $\gamma - B$ spark usually occupy a proportion

$$\frac{h_{res}}{r_p} \simeq 0.075 P^{1/3} \dot{P}_{-15}^{-1/2} \qquad (23)$$

or

$$\frac{h_{th}}{r_p} \simeq 1.9 \times 10^{-2} P^{3/5} \dot{P}_{-15}^{-3/10} T_6^{-1/5} \quad (24)$$

of the polar cap. Comparing Eqs. [23,24] with Eqs. [21,22], we find that, if the $\gamma - B$ process is in the resonant mode, the central $\gamma-\gamma$-induced central spark is large enough to be compared with the conventional sparks, and hence, have a non-negligible effect in forming polar cap pair cascades. If the $\gamma - B$ process is in the thermal mode, however, the central $\gamma - \gamma$ spark is too small compared with the conventional ones, and hence, negligible.

4. Possible connection with core emission

It is known that the radio emission beams of pulsars usually have two kinds of components, namely the "core" and the "conical" ones (two cones, Rankin 1983; or one cone, Lyne & Manchester, 1988). The "conical" components are a direct consequence of the hollow cone model, while the "core" components, which cannot interpreted directly by the conventional curvature radiation model (e.g. Sturrock 1971; RS75), have stimulated many imagination (e.g. Beskin, Gurevich & Istomin 1988; Wang, Wu & Chen 1989; Weatherfall & Eilek 1997; Qiao 1988, Qiao & Lin 1998, hereafter QL98; Gil & Snakowski 1990). Among these models, the latter two require the inner gap sparking process, e.g. the pair production avalanches continuously developed in a vacuum gap in the polar cap region supposing that positive ions are bound in the neutron star surface (RS75). The inverse Compton scattering model of Qiao et al. (Qiao 1988; QL98; Xu & Qiao 1998) discusses the pulsar radio observation properties (e.g. emission beams, polarization properties) as a consequence of the ICS of the secondary particles with the low-frequency wave produced by the inner gap sparking. This model can naturally results in the "core" and "conical" emission components of the pulsars, with their properties being interpreted successfully. The sparks of this model are supposed to be located at a rim in the polar cap near the last field lines, which is a natural consequence of the $\gamma - B$ pair production process in a standard dipolar configuration. In this model, the "core" emission does not require to a spark located in the central region of the polar cap (though it is better if there IS one), but is a consequence of the geometrical effect of the coherent ICS processes at lower altitudes. The model by Gil et al., however, requires a strong central spark near the magnetic axis of the pulsar to account for the "core" emission, with some other weak ones drifting around the central one to account for the "conical" emissions (e.g. Gil & Kijak 1992). Recently, they (Gil et al. 1997; GC98) found that it is reasonable to assume that the sparks are located all over the polar cap region, with characteristic dimension as well as the typical distance between neighbours approximately equal to the height of the gap. This picture is supported by observations, since those pulsars showing complex profiles (e.g. cT, Q and M profiles, Rankin 1983) all lie in the region in the $P - \dot{P}$ diagram where the parameter $a = r_p/h \simeq 4$.

However, in the conventional gap sparking model with standard dipolar magnetic field configuration, it is quite difficult to get a central spark near the pole due to the "weak curvature problem". Although this problem might be solved by invoking multipolar magnetic field components (see, e.g. GC98), it is still uncertain since we don't know the real field configurations in the neutron star vicinity. A possible way out is to invoke the induced electric field as the dominant mechanism for the γ-photon absorption as suggested by Daugherty & Lerche (1975), but Zheng, Zhang & Qiao (1998) recently argued that this electric field absorption picture is unfortunately wrong and should be abandoned since the incident angles of the γ-photons are misused.

The $\gamma - \gamma$ process discussed in this paper seems to present another possible mechanism to account for the central sparks in the pulsar polar caps. Furthermore, it is interesting that the $\gamma-\gamma$-induced central sparks seem to have some properties relevant to the pulsar "core" emission. These evidence include:

1. "Core" emission components are usually not observed in the pulsars near the death line (Rankin 1990). This is understandable in our picture, since the region near the death line is just where the thermal ICS mode dominates the resonant ICS mode (ZQLH97a). The two hot-polar-cap old pulsars PSR 1929+10 and PSR 0950+08 (Wang & Halpern) should be in the thermal ICS mode according to our analysis. Actually, they do not show "core" components in their pulse profiles (Rankin 1990, 1993; GC98).

2. Pulsars with "core" emission components have relative shorter periods on average (Rankin 1990). It is natural in our picture, since shorter periods result in a lower threshold for the two-photon process (see Fig. 3). In this picture, millisecond pulsars should have the tendency to show the "core" feature. Though polarization data of millisecond pulsars are not sufficient to draw this conclusion, some millisecond pulsars do show central components in their pulse profiles (Han 1998, private communication). We hope that this prediction can test as the polarization data accumulate.

3. For the $\gamma - \gamma$ process, the secondary pairs are more energetic than those from the $\gamma - B$ one. This is because the $\gamma - \gamma$ secondaries inherit the energy of the high energy photons ($\gamma_\pm \sim E/(2m_0c^2)$), while the $\gamma - B$ secondaries rapidly lose their perpendicular energies via synchrotron radiation, so that only the parallel energy components remains (see ZQH97b). This is just the requirement of the "core emission" models, since higher energies are required for the particles along the central nearly-straight field lines both in the ICS (QL98) or CR (e.g. RS75) models in order to account for the emission with same frequencies.

4. The two photon annihilation cross section is greatly reduced with respect to $\epsilon E(1 - \mu_c) = 2(m_0c^2)^2$. This will results in steep spectra of the secondary particles, and hence, the steeper spectra observed in pulsar "core" emissions (Rankin 1983, 1990).

However, although we have mentioned evidences for the possible connection of the $\gamma - \gamma$-induced central spark with pulsar core emission, it is difficult to realize a large central spark in practice. This is due to the high critical temperature ($T_{6,cri} \sim 4$) for the two-photon annihilation. As discussed in Sect. 2, it is not difficult to realize a hot polar cap with $T_6 \sim 5$. The difficulty is that it is hard to find a pulsar with polar cap temperature $T_6 \sim 4$ but is still in the "resonant ICS mode". As discussed in ZQLH97a, there is still another critical temperature T'_{cri} to distinguish the two ICS $\gamma - B$-induced gap modes, which is also around $(3-4) \times 10^6 K$. So it is quite likely that, when the polar cap is hot enough to switch $\gamma - \gamma$ sparks, the $\gamma - B$ sparks are dominated by the thermal ICS mode, which produces much more energetic γ-rays. In this case, the $\gamma - \gamma$-induced central spark will be too small to play an important role (see Eq. [22]). But in contrast with T_{cri}, which is a strong constraint since $\gamma - \gamma$ mean free path is quite sensitive around it, T'_{cri} is rather uncertain since we do not know how violent the pair avalanche is. Different assumptions can change T'_{cri} by a factor of 2-3 (see detailed discussion in ZQLH97a). So there is still some room for the existence of a large-enough $\gamma - \gamma$-induced central spark when the $\gamma - B$ process is dominated by the resonant ICS mode. Furthermore, Eqs. [21,22] just show the smallest area of the central sparks. The central sparks can be even larger for a higher temperature since the $\gamma - \gamma$ mean free path will be much smaller. With all these in mind, we conclude that the question whether the $\gamma - \gamma$ process can form an important core-emission-connected central spark is still open.

5. Summary and discussions

We have discussed the possible importance of the two-photon annihilation in the pulsar polar gap sparking process. We found that this process may not be negligible when the polar cap temperature $T \geq T_{cri} \sim (3.5 - 4.7) \times 10^6$K. Compared with the $\gamma - B$ process, although this two-photon process is not important when the $\gamma - B$ process is dominated by the thermal ICS mode, it can be an important mechanism to form a central spark near the pole if the resonant ICS mode dominates. The uncertainty to distinguish the two ICS modes does not allow or rule out this possibility. If so, then this mechanism could be a possible interpretation to GC98's spark model.

There are some factors which might more or less influence the importance of the two-photon effect. First, we have used Eq. [10] to calculate $l_{\gamma-\gamma,min}$, in which Max$f[\nu]$ is used generally. This might have enhanced the importance of this effect a bit since the value ν at which Max$f[\nu]$ is achieved is not exactly aligned for different μ_c. But this variation is not large, so it does not severely influence the value of T_{cri}. Second, within the neutron star magnetosphere, the temperature is higher near the surface (Pavlov et al. 1995). This effect will enhance the importance of the two-photon process, since we only observe a "cooler" emission from infinity. Third, we have adopted a blackbody-like thermal photon field to do the calculation. This should be modified since neutron stars are not perfect blackbodies due to strong magnetic fields in the star vicinity (Pavlov et al. 1995). But the modification of the spectral shape will not change much the characteristic energy and the number density of the soft photon fields where they are the key parameters to determine the two-photon annihilation mean free path (see Eq. [5]). So the results for the mean free paths and the critical temperatures will not change much. Fourth, if strong multipole magnetic components with stronger curvatures do exist at the polar caps, the $\gamma - \gamma$ process will be further suppressed, so that the $\gamma - \gamma$-induced central spark would be smaller. Finally, we did not consider the influence of the strong magnetic fields on the two-photon annihilation cross section, which might yield some non-negligible modifications (see e.g. Burns & Harding 1984; Harding 1998, private communication). Both the cross sections of the two photon annihilation and its inverse process pair annihilation ($e^+ + e^- \to \gamma + \gamma'$) are reduced with the presence of strong magnetic fields (Daugherty & Bussard 1980; Baring & Harding 1992; Kozlenkov & Mitrofanov 1987), so that the critical temperature T_{cri} may be higher. Further analysis is required to present a quantitative analysis.

Our discussion throughout this paper is within the framework of the RS75 vacuum gap model. There are some observational evidences for unsteady flows of particles from the pulsar inner magnetospheres, such as microstructures, drifting subpulses, etc. Though a binding energy problem has been long recognized, there are still some theoretical approaches which support the RS-type gap (see a detailed discussion in GC98, Xu & Qiao 1998b). Furthermore, the vacuum gap model (RS75, ZQH97b) gives at least order-of-magnitude parameters for the inner accelerators, such as the heights, parallel electric fields, the potential drops, etc. It is natural to notice that the conclusion of this paper can also be extended to the steady space-charge-limited-flow model (e.g. Arons & Scharlemann 1979; Arons 1983; Muslimov & Tsygan 1992; Muslimov & Harding 1997; HM98). By taking the two-photon process into account, the "slot gap" (Arons 1983) will not be that "slot" near the pole if the polar cap temperature is hot enough ($T > T_{cri}$), since a pair formation front (PFF) will be formed there by the $\gamma - \gamma$ process.

We did not consider the anisotropy between the outgoing and back-flow particles. A qualitative discussion can be performed. As pointed out by HM98, the backflow ICS will produce more energetic photons, so that the $\gamma - B$ process is enhanced. The $\gamma - \gamma$ process, however, will be also enhanced since head-on collisions are dominant. Thus, the relative importance of the two processes might not be much changed.

Acknowledgements. The authors thank the referee C. D. Dermer for his important comments and suggestions and careful correction of the language problems. We are also grateful to Alice Harding for her careful reading the manuscript and insightful comments and suggestions, to Janusz Gil and J.L. Han for their useful comments, to R.X. Xu, J.F. Liu, B.H. Hong, Z. Zheng for helpful discussions, and to Z. Zheng and B.H. Hong for technique assistence. This work is supported by the NNSF of China, the Climbing Project of China, and the Project Supported by Doctoral Program Foundation of Institution of Higher Education in China. BZ acknowledges supports from China Postdoctoral Science Foundation.

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