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On the accuracy of helioseismic determination of solar helium abundance

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Abstract. The Helium abundance in the solar envelope is one of the most important seismic observables. We investigate the accuracy of its determination taking into account uncertainties in the data and in the inversion procedure. Our best value for the helium abundance in the photosphere is $Y_\odot = 0.248$. The estimated uncertainty of 0.002 is dominated by uncertainties in the inversion. This does not account for possibly larger inaccuracies in the thermodynamical data.

Key words: Sun: oscillations – Sun: abundances

1. Introduction

A long time ago, Gough (1984) noted that the strong sensitivity of p-modes frequencies to the value of the adiabatic exponent, Γ_1, in HeII ionization zone should allow a seismic determination of the helium abundance in outer layers of the Sun, $Y_\odot$. It took few years before the first suitable data became available (Libbrecht et al., 1990). Shortly afterwards, the first seismic values of $Y_\odot$ were published (Christensen-Dalsgaard and Pérez Hernández, 1991; Dziembowski, et al., 1991 Vorontsov et al., 1991; Kosovichev et al., 1992). The reported values were in the range 0.23 - 0.25 which was significantly less than the initial abundance Y_0 inferred by means of the Standard Solar Model construction. Dziembowski et al. (1991) pointed out that the difference was in rough agreement with that expected from gravitational settling as calculated by Cox et al. (1989).

The importance of a seismic determination of $Y_\odot$ with properly assessed errors follows from the fact that we cannot use spectroscopy to accurately measure the photospheric He abundance. There are abundance data from the study of the solar wind, the corona, and flares. These values, however, are discrepant and subject to large errors. Their accuracy may eventually improve. Still, in these latter environments a complicated element stratification takes place, they will never yield true photospheric value of the helium abundance.

The most valuable application of $Y_\odot$ is in testing models of the Sun's internal structure. For the radiative interior, helioseismology yields the value of squared sound-speed, which to a very good approximation, is proportional to the ratio of the temperature to the mean molecular weight, T/μ. Unfortunately, in this way, we still cannot obtain separate information on T and μ. Consequently, we cannot disentangle from the difference between the true solar and model sound speeds which effects are due to opacity errors and which are due to an inadequate treatment of the chemical evolution. Information about $Y_\odot$ does not solve this problem but it does provide an important constraint.

Our aim here is to asses the overall uncertainty in the seismic value of $Y_\odot$ arising from various sources. This is not a new problem. Partial discussions of it were given in the papers presenting the first determination of $Y_\odot$. More recently, the accuracy of the determination was discussed by Dell'Innocenti et al. (1997) and by Basu (1998). We believe that the discussion here is more comprehensive. Most of the results presented here is based on frequency data from the SOHO/MDI instrument (Rhodes et al., 1998)

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2. Inverse problem with adjustable radius

Recently, it has been discovered (Schou et al., 1997; Antia, 1998) that fitting the f-mode frequencies requires a re-scaling of the solar radius to a value significantly below that implied by the uncertainty in photometric measurements. Basu (1998) showed that the radius change has important consequences for helioseismic inferences from the p-mode frequencies.

Dziembowski et al.(1990) formulated the inverse problem for structure in which the value of $Y_\odot$ is a directly inferred quantity. Here we follow that method with a modification consisting of allowing an adjustment in the radius, R. Our basic structural variable is now a dimensionless quantity $u = \frac{P}{\rho}\frac{R}{GM}$ and the inverse problem equation is posed by

$$\left(\frac{\Delta\nu}{\nu}\right)_i = \int_0^1 \mathcal{K}_{u,i}\frac{\Delta u}{u}dx + \mathcal{J}_i\Delta Y_{\rm cz} - 1.5\frac{\Delta R}{R} + \frac{F(\nu)}{I_i}, \quad (1)$$

where $i \equiv (\ell, n)$ identifies the mode, $x = r/R$, and $\Delta Y_{\rm cz}$ denote the difference between solar and model helium abundance in the convective zone. The equation is equivalent to Eq. (7) of Dziembowski et al.(1990) if $\Delta R \equiv 0$. The explicit forms of

Table 1. Parameter of solar models

Model	R 6.96Mm	$Y_{\rm cz}$	Diffusion	Convection	Opacity OPAL	EOS
JCD	1.00068	0.24467	Yes	MLT(std)	Older	OPAL
RS0	1.00016	0.28266	No	Canuto	Oldest	MHD
RS1	1.00009	0.24281	Yes	MLT(std)	Newer	OPAL
RS2	0.99987	0.24319	Yes	MLT(extreme)	Newer	OPAL
RS3	1.00119	0.24280	Yes	CM	Newer	OPAL
RS4	0.99995	0.25022	Yes	CM	Older	OPAL
RS5	1.00002	0.26391	No	CM	Newer	OPAL

$\mathcal{K}_{u,i}$ and $\mathcal{J}_i$ were given in the same paper. Also, the $F(\nu)/I_i$ term, where I_i denotes mode inertia, has the same meaning. The free $F(\nu)$ functions allows one to eliminate various *the near surface uncertainties*, mostly due to the effects of vigorous nonadiabatic convection which cannot be reliably calculated.

Let us recall that the integrand of $\mathcal{J}_i$ contains the $\frac{\partial\Gamma_1}{\partial Y}$ factor. It is the most important thermodynamic parameter in our problem. Note that its evaluation requires the second derivatives of the gas pressure and energy . Thus, there is a high requirement on the precision of the equation of state calculation, as well as a source of uncertainty that is difficult to estimate.

The coefficient -1.5 at $\Delta R/R$ is only an approximation. A small mode-dependent contribution arising from the derivatives of Γ_1 with respect to P and ρ was ignored. We also omitted a small term involving $\Delta M/M$, which arises in the same way. In fact, it is regrettable that the coefficient the $\Delta M/M$ term is very small because this means that helioseismology cannot contribute to the determination of the universal gravitational constant G.

$\Delta Y_{\rm cz}$ is determined simultaneously with a large number of other parameters from inverting Eq. (1). In the standard inversion, we use 42 cubic splines to represent $\Delta u/u$ and 20 Legendre polynomials to represent $F(\nu)$. There is a coupling between the parameters we determine. In our present application, the most important one is that among the parameters determining $\Delta u/u$ in the HeII ionization zone. There is also a role for coupling to the $F(\nu)$ parameters as well as ΔR. In fact we will see that we cannot determine this latter quantity from the p-mode frequencies we have. We will use here results with and without the ΔR term to assess the consequences of the uncertainty in radius.

3. Reference models

In order to investigate cross-talk between the determination of $Y_\odot$ and Δu, we constructed a number of standard solar models (RS1-RS5) which were subsequently used as the reference models in the inversions described in previous section. Some characteristics of the models are given in Table 1.

The radius, R, is understood there as the distance between the center and the temperature minimum, and it is measured in units of 6.96 Mm and $Y_{\rm cz}$ denotes present helium abundance in the convective envelope. The main difference between the models is in the treatment of the convection. In addition to the standard mixing-length theory, MLT(std), we use a new theory proposed by Canuto and Mazzitelli (1991), CM, as well as our own modification of MLT, MLT(extreme). Of the two formulations of CM, we chose the one with the standard expression for the mixing length. Our modification of the standard MLT was aimed at reproducing CM results for the superadiabatic temperature gradient in the subphotospheric layers by an *ad hoc* change of the numerical coefficients. The model selected by us is extreme in the sense that the maximum value of the gradient is the highest (3.78). By comparison, standard MLT yields 0.95 and CM used here 1.76. There is also a model (RS4) calculated with an older version of OPAL opacities and a model (RS5) calculated ignoring elemental settling. More data and information about these models may be obtained on request from R. Sienkiewicz (rs@camk.edu.pl). In Table 1, we also provide data on the model (JCD) of Christensen-Dalsgaard et al.(1996) which has been used as a reference model in most recent frequency inversions, as well as a model named RS0 (Dziembowski et al., 1994) used in our earlier works. In the latter model, an approximate treatment of convective transport developed by Canuto (1990) was used.

In Fig. 1 we show the relative differences in $u(x)$ between the models. The large differences in the outermost layers between the model calculated with a different treatment of convection are forgotten in the lower convective zone. For determination of $Y_\odot$ the HeII ionization zone is essential. Its localization is marked in Fig. 1 with two vertical lines. We can see that the differences in u among the models with different treatments of convection remain large. However, the adiabatic approximation applies in the HeII ionization zone and therefore the values of u are determined by a single parameter e.g. the specific entropy in the adiabatic part of the convective zone. The HeI and H ionization zones are located in layers where seismic probing is unreliable. The reason is that the effect of differences in u and Y in the outermost layers between the Sun and models of it on the frequency difference has the same $F(\nu)/I$ dependence as other near surface effects. The extent of these layers depends on the modes used for probing. For the data sets used in this work, it is less than $0.01R$ from the outer boundary.

The three models with significant different u in the radiative interior are those calculated with different opacities or the one ignoring gravitational settling (RS5) of elements. However, we do not know enough of the technical details in the JCD model to comment on all the differences between it and the models of RS series.

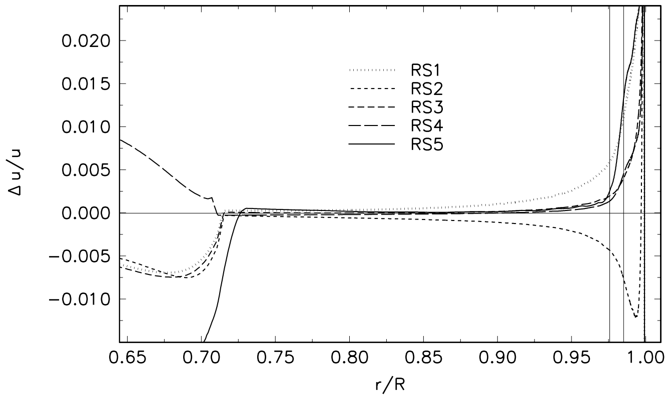

Fig. 1. Relative differences in u between various models and JCD model in outer layers. The vertical lines marked the position of the HeII ionization zone, where the value of $\frac{\partial \Gamma_1}{\partial Y}$ is greater than 0.1.

4. Choice of the parameters for the inversion

We use here the least square regularization method of the inversion. There is some freedom in choosing parameters for the method which leads to uncertainty in the results. This problem has been already discussed in some detail by Dell'Innocenti et al. (1997). Here, we just want to provide an assessment of the uncertainty of $\Delta Y_{\rm cz}$ resulting from this freedom.

Our choice is second derivative smoothing, that is we determine spline amplitudes in the representation of $\Delta u/u$, $\Delta Y_{\rm cz}$, and other parameters by means of minimizing

$$\chi^2_{\rm mod} = \chi^2 + \lambda \int_0^1 \left(\frac{d^2}{dx^2} \frac{\Delta u}{u} \right)^2 dx, \qquad (2)$$

where

$$\chi^2 = \sum_i \left(\frac{\Delta \nu_i}{\sigma_i} \right)^2$$

and σ_i are errors in the frequencies. Other options for regularization exist (see e.g. Dziembowski et al., 1994). There is also freedom in the discretization of $\Delta u/u$. However, all of it reduces, in fact, to a freedom in the degree of smoothing; therefore studying the effect in terms of a single parameter λ is sufficient. Naturally, an increase in λ implies a deterioration of the fit i.e. higher χ^2, but as long as the effect is small we should regard the resulting values of $\Delta Y_{\rm cz}$ as acceptable.

Next, there is a freedom in representation of the $F(\nu)$ function. An adequate representation is in terms of a series of N_F consecutive Legendre polynomials. An initial increase of N_F leads to a dramatic improvement of the fit, but at certain point the improvement ceases to be of consequence.

In Fig. 2, the effects of λ and N_F on the values of χ^2 and $\Delta Y_{\rm cz}$ are shown. The results were obtained by inverting the frequency data obtained with the SOHO/MDI instrument (Rhodes et al., 1998). The set contains frequencies for p-modes with ℓ from 0 to 194. It is the basic data set in this work and it is denoted S194. Alternative sets will be considered in Sect. 7. The reference model used in these inversions is JCD. We see in this figure that with decrease of λ and increase of N_F the values of $\Delta Y_{\rm cz}$ and χ^2 stabilize simultaneously. Our subjective estimate of the uncertainty based on this figure is 2×10^{-4}.

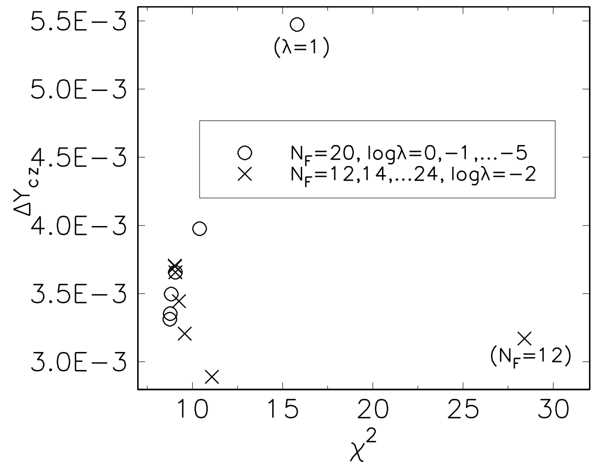

Fig. 2. Dependence of $\Delta Y_{\rm cz}$ and χ^2 on the regularization parameter, λ, and number of terms in representation of the $F(\nu)$ function, N_F. Decreasing value of λ from unity causes a large initial decrease of χ^2, but beginning with $\lambda = 0.01$ the effect ceases to be significant. There is also very little difference in $\Delta Y_{\rm cz}$ obtained with any $\lambda \leq 0.01$. A qualitatively similar effect is caused by an increase in N_F. There is a significant improvement in the fit when N_F is increased from 12 to 20. Further, increases beyond 20 has only a small effect both on χ^2 and $\Delta Y_{\rm cz}$.

5. Dependence of seismic $Y_\odot$ on the reference model

In Fig. 3, we summarize the results of the inversions made with different reference models.

The plot shows that the nominal observational errors contribute to very little to the uncertainty in the seismic $Y_\odot$ −− much less than the inversion procedure. This uncertainty is best revealed in the relatively large sensitivity of the result to the choice of reference model. We also see that the uncertainty in the radius contributes to the uncertainty in $Y_\odot$.

The mean value of those shown in Fig. 3 is $Y_\odot = 0.2475$. As the estimate of the uncertainty due to inversion at this point, we use the spread around the mean, which is 15×10^{-4}. The contributions from the freedom in λ and in N_F, which we discussed in the previous section, may be neglected if they are treated as being independent of those considered here, that is if we add the squares of the uncertainties.

6. How reliable is the EOS

It is most difficult to assess the consequences of errors and inadequacies in the thermodynamical parameters. The value obtained with RS0 as the references model, which uses the MHD EOS (Däppen et al., 1988) is $Y_\odot = 0.242$ which is well below the lower limit according to our previous estimate. The difference, which is about 0.006, cannot be regarded as the measure of the uncertainty. The approach adopted in the OPAL opacity calculations (Rogers et al., 1996) is certainly more sound. Furthermore, there is also a seismic evidence that it is more accurate.

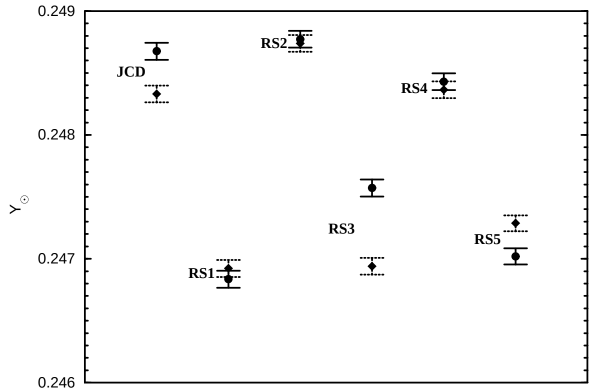

Fig. 3. Influence of the reference model and radius uncertainty on the seismic Y from SOHO/MDI data. The error bars shown represent the dispersion of the results obtained by means random numbers simulation of the observational errors. The two values shown for each model correspond to width to the inversions with the $\Delta R/R$ term (solid circle, solid error bar caps) and without it.

Helioseismology provides a clean test of the equation of state in the adiabatic part of the convective zone (Dziembowski et al., 1992; Christensen-Dalsgaard and Däppen 1992). Indeed, assuming an adiabatic stratification, we may evaluate $(\Delta\Gamma_1)_{\rm ad.eq.}(x)$ in terms of $\Delta u(x)$ and $\Delta P(x)$ which we determine from helioseismic inversion. The explicit integral expression for ΔP in terms Δu was given by Dziembowski et al. (1990). From a linearized condition of mechanical equilibrium, we get

$$\left(\frac{\Delta\Gamma_1}{\Gamma_1}\right)_{\rm ad.eq.} = \Gamma_1 \frac{d\ln x}{d\ln P}\left[\frac{d}{d\ln x}\left(\frac{\Delta u}{u}\right) - \left(1-\frac{d\ln\rho}{d\ln P}\right)\frac{d}{d\ln x}\left(\frac{\Delta P}{P}\right)\right], \quad (3)$$

On the other hand, from tables with the EOS data, we have $\Gamma_1(\rho, P, Y)$, hence we may evaluate $(\Delta\Gamma_1)_{\rm EOS}$ using again the seismic determinations of Δu, ΔP and, in addition, $\Delta Y_{\rm cz}$.

The difference between the values of $(\Delta\Gamma_1)$ tests the EOS. In Fig. 4, we show the results for the models RS0 (MHD EOS) and JCD (OPAL EOS). One may see that difference for JCD model is on average about one half of that for RS0. Thus, the OPAL EOS passed the test better but still not perfectly. We are reluctant to assign a specific number for the uncertainty in $Y_\odot$ that could be attributed to the EOS. It well may be greater that all the remaining uncertainties be combined.

7. Use of different data sets

In Fig. 3, we have seen that the quoted measurement errors contribute relatively little to the uncertainty of the inferred helium abundance. However, the χ^2 values shown in Fig. 2 suggest this contribution may be underestimated. To see what maybe the real uncertainty that could be attributed to the data, we studied the effect of using different data sets. The results are summarized in Table 2. For sake of comparison, we also included in the table some other seismicly inferred quantities.

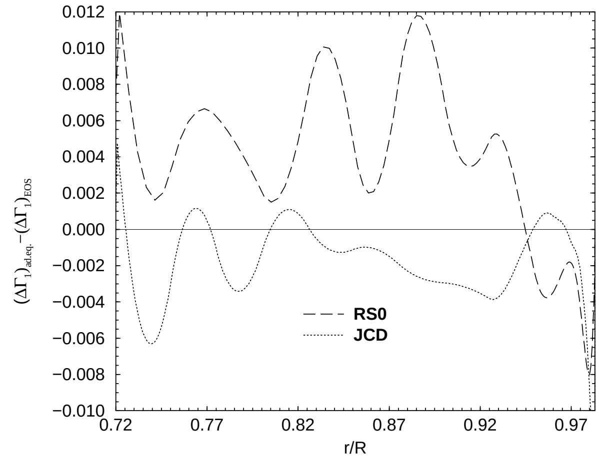

Fig. 4. The difference $(\Delta\Gamma_1)_{\rm ad.eq.} - (\Delta\Gamma_1)_{\rm EOS}$ evaluated on the basis of the inversion of the SOHO/MDI frequency data with the use of the two reference models employing different thermodynamical data.

The differences among the $\Delta Y_{\rm cz}$ inferred from different sets are significantly larger than the formal errors. We use the spread in these, ± 0.001, as the measure of the uncertainty attributed to data sets. However, it seems likely that most the spread in the results obtained with different sets may not reflect real observational errors, but rather limitation of the inversion method. In particular, for high degree, low-frequency modes, the approximation, by the $F(\nu)/I_i$, of the difference in the near surface layers between the Sun and models of it term may be inadequate at the precision of 10^{-3}. The changes in $\Delta Y_{\rm cz}$ with $\ell_{\rm max}$ support this. Since the mean value determined in the previous section was obtained for our basic data set (S194) and we have more trust in the value for the truncated sets, we adopt a value which is less by 0.005.

Decreasing l_{max} below 120 leads again to discrepant values of $\Delta Y_{\rm cz}$. This is caused by the cross-talk between the $\Delta Y_{\rm cz}$ and $F(\nu)$ terms. The stability of the 5.E-3 against variations of l_{max} is in fact the main argument for our choice of $\Delta Y_{\rm cz}$.

Including the uncertainties discussed here and in Sect. 3, we give as a final value,

$$Y_\odot = 0.248 \pm 0.002,$$

where the error does not include effects from the EOS.

Let us note the large variations in the inferred values of ΔR. Only for a complete set of p-mode data from SOHO/MDI is the value similar to that inferred from the f-mode frequencies which was $(-4.7\pm0.5)\times10^{-4}$. We cannot determine the radius of the Sun at the 10^{-4} accuracy level from p-mode frequencies. We actually are not sure that the radius is well defined at such a level. A consequence of the uncertainty in radius is the uncertainty in Δu. It should be noted, however that the differences between Δu at various points are nearly constant which means that they are, along with the $\Delta Y_{\rm cz}$, robust observables.

8. Conclusions

Our best value for the photospheric helium abundance, is $Y_\odot = 0.248$. It is very close to the value of 0.2488 inferred by Basu

Table 2. Results of inversions of different frequency sets

SET	NUMBER	$\Delta Y_{\rm cz}$	ΔR	$\frac{\Delta u}{u}(0.2)$	$\frac{\Delta u}{u}(0.67)$	$\frac{\Delta u}{u}(0.72)$	χ^2
S120	1586	5.1E-03	3.1E-04	-2.4E-3	5.1E-3	1.3E-4	6.3
S140	1709	5.1E-03	-4.7E-06	-3.2E-3	4.2E-3	-1.0E-3	7.6
S160	1809	5.0E-03	-1.1E-04	-3.5E-3	3.9E-3	-1.3E-3	7.6
S194	1889	4.0E-03	-5.7E-04	-3.5E-3	3.9E-3	-1.3E-3	8.7
B150	2253	5.7E-03	4.4E-04	-0.1E-4	5.6E-3	6.3E-4	2.6

The S*** denote the SOHO/MDI sets truncated at $\ell = \ell_{\max}$ values given by the number; B150 is a combination of the BBSO (Libbrecht et al., 1990) and BISON (Elsworth et al., 1994) data sets.

(1998) from the same SOHO/MDI data and with models using OPAL opacities. The difference is within our estimate of the uncertainty of the inversion. The uncertainty Basu gives is 0.001 reflects the spread of the results obtained with use of different data. Here again we agree. There, however, also remarkable differences.

Firstly, our value for B150 set (see comment in Table 2) is by 1.7E-3 higher than that for S194 set , whereas, the difference Basu finds is -1.2E-3. This supports the point made in the previous section that the difference obtained for different sets reflects, at least partially, inadequacies of the inversion method. Secondly, we find $Y_\odot = 0.242$ using a model calculated with the MHD EOS which is by 0.006 less than for models calculated with the OPAL EOS. The corresponding difference according to Basu is -0.004. We do not know why we have very good agreement only if we use OPAL and MDI data. Is this only an accident or rather evidence for a superiority of the SOHO/MDI and the OPAL EOS. This has to be clarified. There is certainly a need for further work to improve the methodology of inversion, since in our view, it is the most important source of the uncertainty in helioseismic determination of helium abundance. Perhaps, it would be useful to reconsider the alternative approach to $\Delta Y_{\rm cz}$ determination which relies on the frequency dependence of the near surface contribution to $\Delta\nu$ (Christensen-Dalsgaard and Pérez Hernández, 1991; Vorontsov et al., 1991).

There is evidence that solar models calculated with uninhibited gravitational settling have a lower photospheric helium abundance than the Sun. The difference is most likely in the range 0.004-0.006. To explain Li depletion, Richard et al. (1996) include rotation-induced mixing below the convective zone. They found that the effect leads to the $Y_\odot$ reduction by about 0.007 and improves also the agreement in the sound-speed in the outer layers of the radiative interior.

The cause of the mixing is a rotation-induced turbulence, which is not a well understood phenomenon. Thus, constraints from helioseismology are very much needed. With current accuracy the value $Y_\odot$ is at the verge of yielding such a constraint. We believe that there is a chance to improve it.

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