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Magnification cross-sections of gravitational lensing by galaxies in general FLRW cosmologies

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Abstract. For a wide variety of cosmological models characterized by the cosmic mass density Ω_M and the normalized cosmological constant Ω_Λ, we derive an analytic expression for the estimate of magnification cross-sections by an ensemble of isothermal spheres as models of galactic mass distributions. This provides a simple approach to demonstrate how the lensing probability by galaxies depends on the cosmological parameters. An immediate consequence is that, while a non-zero cosmological constant indeed leads to a significant increase of the lensing probability as it has been shown in the literature, only a small fraction of sky to $z \sim 3$ can be moderately ($\mu \sim 1.3$) lensed by galaxies even in a Λ-dominated flat universe. Therefore, whether or not there is a nonzero cosmological constant, it is unlikely that the overall quasar counts have been seriously contaminated by the presence of galactic lenses.

Key words: gravitational lensing – galaxies: general

1. Introduction

It is well known that the gravitational lensing by the foreground objects (e.g., galaxies) can alter the apparent brightness of background objects (e.g., quasars), which may contaminate our observations. Three decades ago, Barnothy and Barnothy (1968) proposed that all the quasars were nothing but the gravitationally magnified images of Seyfert galactic nuclei. Press and Gunn (1973) showed that the probability of occurrence of gravitational lensing in an $\Omega = 1$ universe is nearly unity. For many years there had been a lack of both convincing observational and theoretical supports for these speculations. However, numerous and unprecedented deep galaxy surveys have recently revealed a considerably large population of faint galaxies (Metcalfe et al. 1996; references therein). This motivates one to readdress the question if the observations of background objects are seriously affected by the gravitational lensing effect of foreground galaxies? For this purpose, Zhu & Wu (1997) have calculated the lensing cross-sections of background quasars by the foreground galaxies, and concluded that, despite the fact that there is a considerably high surface number density of faint galaxies the total lensing cross-sections by galaxies towards a distant source are still rather small, when only a special cosmological model of $\Omega_M = 1$ is considered.

Nevertheless, the optical depth (probability) of gravitational lensing depends sensitively on the cosmological models. It is worthy of examining whether the above claim is valid under general cosmological models. In this paper, we extend our previous work to a variety of cosmological models, which are characterized by the mass density parameter Ω_M and the normalized cosmological constant Ω_Λ (cf., Carroll, Press & Turner 1992). This is well motivated because cosmological models with nonzero cosmological constant have become quite popular recently. Many years ago, Gott, Park & Lee (1989) have given the general expressions for the optical depth and mean image separation in general Friedman-Lemaître-Robertson-Walker (FLRW) cosmological models. Yet, these expressions are complicated and thereby hard to use in practice. One of the purposes of this paper is thus to simplify the formula. Furthermore, we would like to investigate how the cosmological parameters affect the estimate of the lensing cross-section.

2. Distance and volume measures in general FLRW cosmologies

We assume a homogeneous and isotropic universe described by a Robertson-Walker metric (Weinberg 1972):

$$ds^2 = -c^2dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

or in the form

$$ds^2 = -c^2dt^2 + R^2(t)\left[d\chi^2 + f(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \quad (2)$$

where

$$r = f(\chi) = \mathrm{sinn}(\chi) \equiv \begin{cases} \sin\chi & k = +1\,, \\ \chi & k = 0\,, \\ \sinh\chi & k = -1\,. \end{cases} \quad (3)$$

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$$\chi = \frac{c}{H_0 R(t_0)} \int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda}}$$
$$= \begin{cases} \frac{c}{H_0 R(t_0)} \int_0^z \frac{dz}{\sqrt{\Omega_M (1+z)^3 - \Omega_M + 1}}, & \Omega_M + \Omega_\Lambda = 1 \ (k=0)\,, \\ |\Omega_M + \Omega_\Lambda - 1|^{1/2} \int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda}}, & \Omega_M + \Omega_\Lambda \neq 1 \ (k=\pm 1)\,. \end{cases} \quad (6)$$

The relation of the measurables to the unmeasurables is (Lightman et al. 1975; Carroll, Press & Turner 1992)

$$(1+z) = R(t_0)/R(t),\ D^A = R(t)r;\ D^M = R(t_0)r,$$
$$D^L = R^2(t_0)r/R(t). \quad (4)$$

where t_0 is the present time of the universe, and D^A, D^M and D^L are the angular diameter distance, the proper motion distance and the luminosity distance respectively. Distances used in lensing theory are the angular diameter distances (Schneider, Ehlers & Falco 1992). From Eqs. 3 and 4, one can derive an important relation

$$\frac{D^A{}_{ds}}{D^A{}_s} = \frac{f(\chi_s - \chi_d)}{f(\chi_s)}\,, \quad (5)$$

where D^A_{ds} and D^A_s are the angular distances from the lens to the source and from the observer to the source, respectively.

Using the Einstein field equation, it can be shown that the relation of the comoving distance χ to the redshift for the general FLRW cosmologies is (see Eq. 6 on top of this page)

For our end, the comoving volume dV is more convenient than the traditional physical volume. Within the shell $d\chi$ at χ, dV reads (Gott, Park & Lee 1989)

$$dV = 4\pi R^3(t_0) f^2(\chi) d\chi =$$
$$\left(\frac{c}{H_0}\right)^3 4\pi \left(\frac{c}{H_0 R(t_0)}\right)^{-3} f^2(\chi) d\chi\,. \quad (7)$$

3. Magnification cross-sections of gravitational lensing

First of all, we consider the lensing cross-section (Turner, Ostriker & Gott 1984) due to a specific galaxy. Following Turner et al. (1984), we model the mass density profile of the total galaxy matter as the singular isothermal sphere (SIS), whose magnification for a point source is given by (Schneider, Ehlers & Falco 1992; Wu 1996)

$$\mu = \frac{\theta}{\theta - \theta_E}, \quad \text{for } \theta > \theta_E \equiv 4\pi \left(\frac{\sigma}{c}\right)^2 \frac{D^A_{ds}}{D^A_s}\,, \quad (8)$$

where θ is the observed angular position of the source (image position), θ_E is the angular radius of Einstein ring and σ is the velocity dispersion of the lensing galaxy. Note that we only include the contribution of the primary image because here we will not deal with the statistics of multiple images. The dimensionless magnification cross-section for a point source located at z_s produced by a single SIS galaxy at z_d is

$$\hat{\sigma}(>\mu) = \frac{1}{(\mu-1)^2}\pi\theta_E^2 =$$
$$\frac{1}{(\mu-1)^2} 16\pi^3 \left(\frac{\sigma}{c}\right)^4 \left[\frac{f(\chi_s - \chi_d)}{f(\chi_s)}\right]^2\,, \quad (9)$$

where the relation of Eq. 5 has been employed.

Now, let's consider the contributions of an ensemble of galaxies having different luminosities and redshifts. The present-day galaxy luminosity function can be described by the Schechter function (Peebles 1993)

$$\phi_i(L)dL = \phi_i^*(L/L_i^*)^{-\alpha} \exp(-L/L_i^*) d(L/L_i^*), \quad (10)$$

where i indicates the morphological type of galaxies: i=(E, S0, S). The above expression can be converted into the velocity dispersion distribution through the empirical formula between the luminosity and the central dispersion of local galaxies $L/L_i^* = (\sigma/\sigma_i^*)^{g_i}$. We keep the same parameters ($\phi_i^*, L_i^*, \alpha; \sigma_i^*, g_i$) as those adopted by Kochanek (1996) based on the surveys (Loveday et al. 1992, Marzke et al. 1994), which yield $\sigma_i^* = (220, 220, 144)$ km/s and $g_i = (4, 4, 2.6)$ for $i = (E, S0, S)$ galaxies, and the morphological composition $\{\gamma_i\} = (44, 56)$ for $(E+S0, S)$. For the spatial distribution of galaxies, we use a general FLRW cosmological model parametrized by Ω_M and Ω_Λ, which has been outlined in Sect. 2. Finally, the total dimensionless magnification cross-section by galaxies at redshifts ranging from 0 to z_s for the distant sources like quasars at z_s is

$$\hat{\Sigma}(z_s, >\mu) = 4\pi \frac{1}{(\mu-1)^2} \left(\sum_{i=E,S0,S} F_i\right) T(z_s)\,, \quad (11)$$

The parameter F_i represents the effectiveness of the i-th morphological type of galaxies in producing double images (Turner et al. 1984), which reads

$$F_i \equiv 16\pi^3 \left(\frac{c}{H_0}\right)^3 \langle n_{0i} \left(\frac{\sigma}{c}\right)^4 \rangle =$$
$$16\pi^3 \left(\frac{c}{H_0}\right)^3 \phi^* \gamma_i \left(\frac{b_i \sigma_i^*}{c}\right)^4$$
$$\cdot \int (L/L_*)^{\alpha + 4/g_i} \exp(L/L_*) dL/L_*\,, \quad (12)$$

where b_i is the velocity bias between the velocity dispersion of stars and of dark matter particles. The above equation can be further written as

$$F_i = 16\pi^3 \left(\frac{c}{H_0}\right)^3 \phi^* \gamma_i \left(\frac{b_i \sigma_i^*}{c}\right)^4 \Gamma(-\alpha + 4/g_i + 1),$$
$$\text{if} \quad L \in (0, \infty)\,, \quad (13)$$

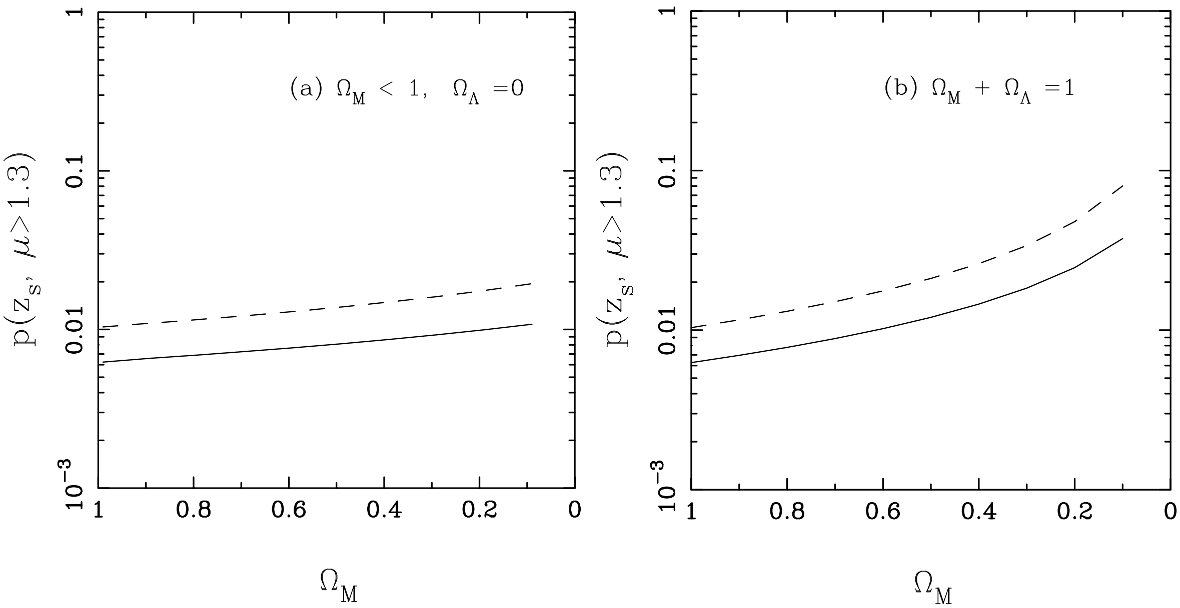

Fig. 1a and b. The probability, $p(z_s, \mu > 1.3)$, that the sources located at $z_s = 2$ (solid-lines) and $z_s = 3$ (dashed-lines) are magnified by the factor greater than $\mu = 1.3$ as a function of Ω_M for **a** an open universe with $\Omega_M < 1$, $\Omega_\Lambda = 0$, and **b** a flat universe with $\Omega_M + \Omega_\Lambda = 1$.

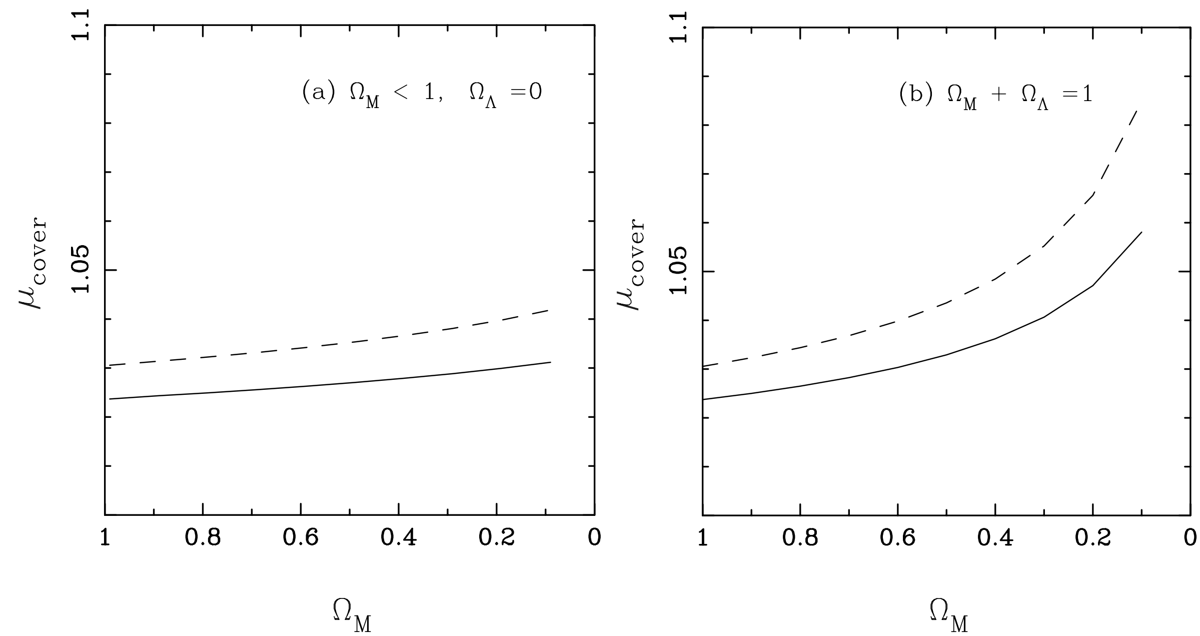

Fig. 2a and b. The magnification μ_{cover}, which makes the probability of the sources located at $z_s = 2$ (solid-lines) and $z_s = 3$ (dashed-lines) $p(z_s, > \mu) = 1$, as a function of Ω_M for **a** an open universe with $\Omega_M < 1$, $\Omega_\Lambda = 0$, and **b** a flat universe with $\Omega_M + \Omega_\Lambda = 1$.

if the integral is performed from 0 to ∞. In practice, the galaxy luminosities have the minimum and maximum limits, and, therefore, Eq. 13 is the maximum estimate of F_i. The z_s dependent factor $T(z_s)$ is

$$T(z_s) = \left(\frac{c}{H_0 R(t_0)}\right)^{-3} \int_0^{\chi_s} \left[\frac{f(\chi_s - \chi_d)}{f(\chi_s)}\right]^2 f^2(\chi_d) d\chi_d \,. \quad (14)$$

For general FLRW cosmologies, an analytic expression is found (see Eq. 15 on top of next page):

where χ_s can be calculated through Eq. 6. For a flat universe ($\Omega_M + \Omega_\Lambda = 1$), it reduces to (Turner 1990)

$$T(z_s) = \frac{1}{30}\left[\int_0^{z_s} \frac{dz}{\sqrt{\Omega_M(1+z)^3 - \Omega_M + 1}}\right]^3, \quad \Omega_M + \Omega_\Lambda = 1\,. \quad (16)$$

If $\Omega = 1$ and $\Omega_\Lambda = 0$, it reads (Turner et al. 1984)

$$T(z_s) = \frac{4}{15}\frac{[(1+z_s)^{1/2} - 1]^3}{(1+z_s)^{3/2}}, \quad \Omega_M = 1, \, \Omega_\Lambda = 0. \quad (17)$$

$$T(z_s) = \begin{cases} \left(\frac{c}{H_0 R(t_0)}\right)^{-3} \frac{\chi_s^3}{30}, & \Omega_M + \Omega_\Lambda = 1\,, \\ |\Omega_M + \Omega_\Lambda - 1|^{-3/2} \left[\frac{1}{8}(1 + 3\cot^2\chi_s)\chi_s - \frac{3}{8}\cot\chi_s\right], & \Omega_M + \Omega_\Lambda > 1\,, \\ |\Omega_M + \Omega_\Lambda - 1|^{-3/2} \left[\frac{1}{8}(-1 + 3\coth^2\chi_s)\chi_s - \frac{3}{8}\coth\chi_s\right], & \Omega_M + \Omega_\Lambda < 1\,, \end{cases} \tag{15}$$

We should point out that the expression of Eq. 11 is very useful. Dividing the expression by 4π, one gets the fraction of the sky within redshift z_s which is magnified by the factor greater than μ:

$$p(z_s, > \mu) = \frac{1}{(\mu - 1)^2}\, F\; T(z_s)\,. \tag{18}$$

We employ $F = \sum_{i=E,S0,S} F_i$ denoting the total effective parameter of all galaxies in producing multiple images. Further omitting the μ-dependent term in Eq. 18, one obtains the conventional optical depth for multiple images.

In the above calculations, we have assumed that the comoving number density of galaxies is constant. However, this may not hold true for the realistic situation. The influence of galaxy evolution on the lensing cross-section should also be taken into account. Zhu and Wu (1997) have include this effect by using the galaxy merging model proposed by Broadhurst et al. (1992), since the scenario of galaxy merging can account for both the redshift distribution and the number counts of galaxies at optical and near-infrared wavelengths (Broadhurst et al. 1992). There are two effects arising from the galaxy merging: The first is that there are more galaxies and hence more lenses in the past. The second is that galaxies are typically less massive in the past and hence less efficient as lenses. As a result of two effects, the total magnification cross-section remains roughly unchanged (Zhu & Wu 1997).

4. Results and discussions

Knowing the analytic expressions for the lensing cross-sections in general FLRW cosmologies, we can explore in detail the influences of cosmological parameters by numerical computation. We compute the probability $p(z_s, > \mu)$ so as to investigate whether the background objects (like quasars) counts are significantly contaminated. Eq. 18 contains three factors, namely, the μ-dependent term, the galaxies term and the cosmological term, associated with $p(z_s, > \mu)$. Here we concentrate on the cosmological term by adopting the maximum value $F \sim 0.028$ (Kochanek 1996)and a moderate magnification of $\mu \sim 1.3$.

Fig. 1 shows how the probability $p(z_s, \mu > 1.3)$ depends on the normalized cosmological parameters Ω_M and Ω_Λ for an open ($\Omega_M < 1$, $\Omega_\Lambda = 0$) and a flat ($\Omega_M + \Omega_\Lambda = 1$) universe respectively. In our calculations, the source has been set at $z_s = 2$ or $z_s = 3$ respectively, and a moderate magnification of $\mu = 1.3$ has been used. Indeed, the probability $p(z_s, > \mu)$ depends sensitively on cosmological parameters Ω_M and Ω_Λ. However, even for a Λ-dominated ($\Omega_\Lambda = 0.9$) flat universe, only a small fraction ($< 10\%$) of the sky can be moderately ($\mu \sim 1.3$) lensed by galaxies.

Of course, by taking somewhat lower value for μ the probability of the magnification can siginificantly increase. In order to get a more robust conclusion, we now estimate what value of magnification would affect the current observations of quasar count, i.e., we calculate the value of μ_{cover} which makes $p(z_s, > \mu) = 1$. The resulting magnification, which depends on both the cosmological model and the source redshift, is shown in Fig. 2. Since the magnification is generally much lower than 1.1, our result reinforces the hypothesis that the quasar counts are not seriously contaminated by the galactic lenses.

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