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Magnetically supported tori in active galactic nuclei

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Abstract. We explore the concept that the gas tori inferred to exist around the central black holes in many active galactic nuclei (AGN) have significant magnetic pressure support. We derive the basic equations for magnetically supported tori with a general axisymmetric $\mathbf{B}$ field involving toroidal and poloidal components. We treat strong and weak field limits assuming the aspect ratio R/a larger than unity, where R is the major radius of the torus, and a is the minor radius. For strong fields, the major radius is influenced by the $\mathbf{B}$ field, whereas for weak fields only the minor radius is affected by the field. In the strong field limit, stability to kink perturbations is possible only for R/a approaching unity, just where our theory breaks down. The weak field tori tend to be kink stable for $R/a \lesssim 13$. This suggests that magnetically supported tori in nature need to be "fat" in order to be stable. This appears to be consistent with observations of tori in AGN. We note that some fat disks around young stars and around more evolved stars represent magnetically supported tori.

Key words: accretion, accretion disks – galaxies: active – magnetic fields – galaxies: nuclei

1. Introduction

Many Active Galactic Nuclei and quasars show evidence of molecular cloud tori in their central regions. In Seyfert galaxies of the type 1, a Broad Line Region (BLR) is observed, while in the Seyfert galaxies of the type 2, it is not directly observed but becomes visible in the polarized light (Antonucci 1993). It is widely thought that the reason for the absence of the BLR in type 2 Seyfert galaxies is the torus, located around the central black hole with the typical size of the order of $0.1 - 1$ pc (Krolik & Begelman 1988). Earlier, it had been shown with X-ray spectral data, that X-ray absorption due to a torus appears to be quite common in Seyfert galaxies (Lawrence & Elvis 1982; Mushotzky 1982). There is now a first direct image of an obscuring torus (Gallimore, Baum, & O'Dea 1997). However, the nature of this torus is not known.

In low power radio galaxies, the central source is thought to be enshrouded by a dusty torus, which may cover a large fraction of 4π, as seen from the central source. The role of this dust torus has been the subject of a number of investigations (Falcke & Biermann 1995; Falcke, Malkan, & Biermann 1995; Falcke, Gopal Krishna, & Biermann 1995; Falcke & Biermann, 1996), and has been discussed at some length in Falcke, Gopal Krishna, & Biermann 1995), where many earlier references are given [see also Barthel (1989a, 1989b)].

From early on, there were attempts to unify the various classes of active galactic nuclei (Scheuer & Readhead 1979). The most successful have been models which unify the radio-loud AGN, where the two main parameters are the aspect angle and the total power of the source (Orr & Browne 1982). Attempts to also include the radio-weak AGNs in such a model have become more complicated (Strittmatter et al. 1980; Falcke & Biermann 1995; Falcke, Sherwood, & Patnaik 1996; Falcke, Patnaik, & Sherwood 1996b). The aspect angle is the angle between the line of sight to the observer and the symmetry axis of the AGN, presumably the axis of the powerful jet which is thought to be perpendicular to the inner part of the accretion disk around a black hole. The total power of the source is the radiative luminosity, integrated over all frequencies (sometimes dominated by GeV γ-rays), plus the kinetic and Poynting energy fluxes of the jets.

It was recognized early on that the aspect angle plays a crucial role in the observed properties of AGNs, on one hand with respect to relativistic boosting possible for small angles between the jet and the line of sight (Scheuer & Readhead 1979; Orr & Browne 1982; Antonucci & Miller 1985; Wills & Browne 1986), and on the other hand with respect to absorption and scattering by surrounding material. The scattering material is usually in the form of a torus (Mushotzky 1982; Lawrence & Elvis 1982; Antonucci & Miller 1985).

Since the far infrared emission of quasars was recognized as arising from molecular material (Chini, Kreysa, & Biermann 1989; Chini et al. 1989), this emission of AGNs has become a focus of attempts to model the structure and emission regions of AGNs (Sanders et al. 1989; Pier & Krolik 1992b; Niemeyer & Biermann 1993). It now appears possible to begin mapping at least some part of the molecular cloud distribution with H_2O-maser lines such as in the galaxies NGC 4945, NGC 4258, and

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NGC 1068 (Barvainis 1995; Miyoshi et al. 1995; Greenhill, Moran, & Herrnstein 1996; Herrnstein, Greenhill, & Moran 1996; Greenhill et al. 1997).

As a result of the mentioned studies, there is now a fairly established model which suggests (Antonucci 1993; Urry & Padovani 1995) that the BL Lacs are the low power end of radio galaxies in a distribution of intrinsic luminosities viewed at various aspect angles. Due to relativistic boosting, the blazars belong to the most luminous class of sources observed. A key aspect of the model is the inferred property that the molecular cloud tori cover a larger fraction of 4π with decreasing luminosity of the source (Falcke, Gopal Krishna, & Biermann 1995).

Thick disks or tori may also be present in other, very different systems. Menshikov & Henning (1997) discuss a model where the dusty disk around a young star shows extreme flaring (their Fig. 16). Also, Osterbart et al. (1997) discuss the "Red Rectangle," for which a similar model may be implied.

A central problem of the gas tori in AGN is to understand how the torus can subtend a large solid angle as seen from the central source. Several proposals have been made to account for this inferred property. Sanders et al. (1989) suggested that warping could produce such an effect from an otherwise thin molecular cloud disk. Indeed, some warping is observed in the water-maser line emission in for example NGC 4258 (Miyoshi et al. 1995; Herrnstein, Greenhill, & Moran 1996). NGC 4258 is particularly interesting in that the warp of the accretion disk observed in the water maser line is slightly warped with respect to a plane, but at nearly 90 degrees to the outer galaxy; at the same time, in the water maser line the disk thickness is unresolved. Another proposal to explain the thickness of disks so as to model a torus is to invoke radiation pressure support from the central source (Pier & Krolik 1992a).

A complete model of gas tori in AGNs should explain where the torus is in radius, why it is "thick," and why the solid angle of the torus as seen from the central object increases with decreasing power of the central source.

Here, we explore a model in which the torus is magnetically supported by the combination of a toroidal and weaker poloidal magnetic field. The overall effect of the magnetic field will be to push matter away from the center with the result that the equilibrium will have rotational velocities smaller than the Keplerian. Both, toroidal and poloidal magnetic field are observed in different parts of galaxies. Toroidal magnetic field is the dominant field observed in our Galaxy and in many spiral galaxies (see review by Kronberg 1994). On the other hand, a region of strong poloidal field is observed in the central part of our Galaxy (Yusef-Zadeh & Morris 1987a; Yusef-Zadeh & Morris 1987b; Yusef-Zadeh et al. 1996; Morris 1996; Morris & Serabyn 1996). The topology of these apparently poloidal magnetic fields is likely to be in bundles as discussed by Mezger, Duschl, & Zylka (1996). The strength of the field is of order mG, and thus a factor of more than 100 higher than in the local Galactic disk (Beck et al. 1996).

In Sect. 2 we develop a simple model for magnetically supported tori. In Sect. 3 we discuss the possible non-axisymmetric kink instability of magnetically supported tori. In Sect. 4 we summarize conclusions of this work.

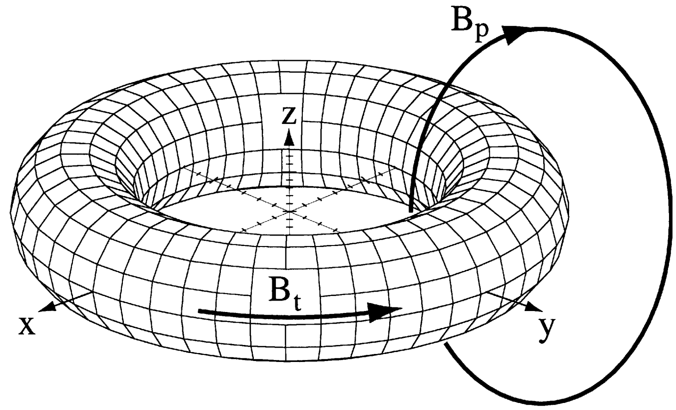

Fig. 1. Drawing of a magnetically supported torus of partially ionized gas and dust in an Active Galactic Nucleus. A massive black hole is located at the origin. The poloidal (B_p) and toroidal (B_t) field components are shown. For this drawing the ratio of the major to minor radii of the torus is $R/a = 3.33$. Outside of the torus there is assumed to be a lower density ionized medium.

2. Model

We investigate the influence of an ordered magnetic field on the equilibrium configurations of ionized gas around a gravitating central black hole in the nucleus of a galaxy. Even a small degree of ionization will be sufficient to couple the ionized and neutral particles and effectively give conditions of perfect conductivity, $\mathbf{E}+\mathbf{v}\times\mathbf{B}/c = 0$, where $\mathbf{v}$ is the bulk velocity of the matter. For simplicity, we consider an axisymmetric, circular cross-section torus of mass M_t, minor radius a, and major radius R from the central gravitational object of mass M_{bh}. The geometry is shown in Fig. 1. Outside the torus we assume that there is a lower density ionized medium. (In Sect. 3 we however consider non-axisymmetric kink perturbations of the torus.) We suppose that the magnetic field is general with both toroidal and poloidal magnetic field components,

$$\mathbf{B} = B_\phi \hat{\boldsymbol{\phi}} \ + \mathbf{B}_p \ , \tag{1}$$

where the subscript p denotes the poloidal (r, z) component in cylindrical (r, ϕ, z) coordinates.

2.1. Equilibrium of Torus

We consider that the forces acting on the torus are: (1) the gravitational force due to the central object (that is, we neglect the self-gravity of the torus); (2) the magnetic force; (3) the centrifugal force due to rotation of the torus; and (4) the kinetic pressure force. We do not include the radiation pressure force.

In order to obtain tractable formulae for the different forces, we assume that the torus aspect ratio R/a is larger than unity, say, $(R/a)^2 \gg 1$. The observed tori have $R/a \sim 2-3$, but our expressions should give a reasonable first approximation.

We first calculate the gravitational energy of the torus in the field of the central black hole. As mentioned we neglect the self-gravity of the torus. Further, the major radius of the torus is assumed sufficiently small that the stellar gravitational potential ($\sim 2\pi G\rho_*(0)R^2/3$) is negligible compared with GM_{bh}/R. Thus,

$$\mathcal{U}_{grav} \;=\; -GM_{bh}\int d^3r\;\frac{\rho(\mathbf{r})}{|\mathbf{r}|} \approx\; -\frac{GM_{bh}M_t}{R}+\frac{GM_{bh}M_t\;a^2}{16R^3}\;, \tag{2}$$

where we have assumed that the density is uniformly distributed within the torus. It is evident that the gravitational field acts to reduce the major radius,

$$F_R^{grav} = -\frac{\partial \mathcal{U}_{grav}}{\partial R} \approx -\frac{GM_{bh}M_t}{R^2}\;, \tag{3}$$

where we have neglected the term of order $(a/R)^2$. The radial force acting on a segment δs of the circumference of the torus, which has mass $\delta M_t = [M_t/(2\pi R)]\delta s$, is

$$\frac{\delta F_R^{grav}}{\delta s} = -\frac{GM_{bh}M_t}{2\pi R^3}\;. \tag{4}$$

It is also evident that the gravitational (tidal) field acts to reduce the minor radius of the torus,

$$F_a^{grav} = -\frac{\partial \mathcal{U}_{grav}}{\partial a} = -\frac{GM_{bh}M_t\;a}{8R^3}\;. \tag{5}$$

The force F_a acts normal to the surface of the torus.

We now consider the magnetic forces acting on the torus. We assume that the current-density,

$$\mathbf{J} = J_\phi\hat{\boldsymbol{\phi}}\;+\mathbf{J}_p\;, \tag{6}$$

is non-zero only within the torus. The total toroidal current is

$$I_t = \int_{(R-r)^2+z^2\le a^2} dr dz\; J_\phi(r,z)\;. \tag{7}$$

The total poloidal current is

$$I_p = -2\pi\int_R^{R+a} rdr\; J_{pz}(r,z=0)\;, \tag{8}$$

where the minus sign gives $B_\phi > 0$ for $I_p > 0$.

The ‘poloidal’ magnetic energy due to I_t can be expressed as

$$\mathcal{U}_{Bp} = \int d^3r\;\frac{\mathbf{B}_p^2}{8\pi} = \frac{1}{2c^2}L_pI_t^2\;, \tag{9}$$

where

$$L_p \approx 4\pi R\left[\ln\left(\frac{8R}{a}\right)-\frac{7}{4}\right] \tag{10}$$

is the poloidal self-inductance of a current loop with $R/a \gg 1$. See for example Landau and Lifshitz (1960, p. 139). The volume integral in Eq. (9) extends over all space.

The ‘toroidal’ magnetic energy due to I_p can be expressed as

$$\mathcal{U}_{Bt} = \int d^3r\;\frac{B_\phi^2}{8\pi} = \frac{1}{2c^2}L_tI_p^2\;. \tag{11}$$

We assume that the toroidal magnetic field is trapped in the torus and negligibly small outside the torus. Consequently, $B_\phi \approx 2I_p/(cR)$, and $\mathcal{U}_{Bt} \approx 2\pi^2a^2RB_\phi^2\;/(8\pi) = L_tI_p^2/(2c^2)$, so that

$$L_t \approx \frac{2\pi a^2}{R} \tag{12}$$

is the toroidal self-inductance of the torus.

It is useful to introduce the poloidal and toroidal fluxes

$$\Phi_p = \frac{1}{c}L_pI_t\;, \quad \text{and} \quad \Phi_t = \frac{1}{c}L_tI_p\;. \tag{13}$$

The mean poloidal and toroidal fields can be defined such that

$$\Phi_p = \pi R^2\overline{B}_p\;, \quad \text{and} \quad \Phi_t = \pi a^2\overline{B}_t\;. \tag{14}$$

Thus

$$\overline{B}_p = \frac{4\left(\ln\frac{8R}{a}-\frac{7}{4}\right)I_t}{cR}\;, \tag{15}$$

$$\overline{B}_t = \frac{2I_p}{cR}\;. \tag{16}$$

For comparison with Eq. (15), note that the poloidal magnetic field at the center of the torus is of magnitude similar to $\overline{B}_p$,

$$B_z(\mathbf{0}) = \frac{2\pi I_t}{cR} = \frac{I_{Amp}}{5R_{cm}}\;(\text{Gauss})\;. \tag{17}$$

On the other hand, the magnitude of the poloidal field on the torus surface,

$$B_{ps} \approx \frac{2I_t}{ca}\;, \tag{18}$$

is larger than $B_z(0)$ by a factor $R/(\pi a)$.

We can write the total magnetic energy as

$$\mathcal{U}_B = \frac{L_p}{2c^2}I_t^2+\frac{L_t}{2c^2}I_p^2 = \frac{\Phi_p^2}{2L_p}+\frac{\Phi_t^2}{2L_t}\;. \tag{19}$$

The different magnetic forces on the torus can be expressed as

$$F_R^{mag} = -\frac{\partial\mathcal{U}_B}{\partial R}\bigg|_\Phi\;, \quad \text{and} \quad F_a^{mag} = -\frac{\partial\mathcal{U}_B}{\partial a}\bigg|_\Phi\;, \tag{20}$$

where the Φ subscript indicates derivatives with Φ_p and Φ_t kept constant. This corresponds to the assumed high conductivity of the torus plasma. We find

$$F_R^{mag} = \frac{\left(\ln\frac{8R}{a}-\frac{3}{4}\right)\Phi_p^2}{8\pi R^2\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2}-\frac{\Phi_t^2}{4\pi a^2}\;. \tag{21}$$

The poloidal flux acts to expand the major radius whereas the toroidal flux has the opposite influence owing to the tension in the toroidal field lines. Also,

$$F_a^{mag} = -\frac{\Phi_p^2}{8\pi Ra\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2}+\frac{R\Phi_t^2}{2\pi a^3}\;. \tag{22}$$

Thus the poloidal field acts to compress the minor radius while the toroidal field has the opposite effect.

The energy associated with the rotation of the torus is

$$\mathcal{U}_{rot} = \frac{\mathcal{L}_t^2}{2M_tR^2} , \tag{23}$$

where $\mathcal{L}_t$ is the total angular momentum of the torus matter which is a constant in that the plasma behaves as an ideal fluid (at least for short time scales).

The angular momentum of the electromagnetic field is negligible if $v_A \ll c$, where $v_A = |\mathbf{B}|/\sqrt{4\pi\rho}$ is the Alfvén speed and c is the speed of light. The corresponding force is

$$F_R^{rot} = -\frac{\partial \mathcal{U}_{rot}}{\partial r}\bigg|_{\mathcal{L}_t} = \frac{\mathcal{L}_t^2}{M_tR^3} , \tag{24}$$

and $F_a^{rot} = 0$. In the absence of magnetic and pressure forces the major radius of the torus would be

$$R_o \equiv \frac{\mathcal{L}_t^2}{GM_{bh}M_t^2} . \tag{25}$$

If v_c denotes the azimuthal velocity of the torus matter, then $\mathcal{L}_t = M_tv_cR$, and

$$\frac{R_o}{R} = \left(\frac{v_c}{v_K}\right)^2 , \tag{26}$$

where $v_K \equiv (GM_{bh}/R)^{\frac{1}{2}}$ is the Keplerian velocity at radial distance R. Thus for a sub-Keplerian torus ($v_c < v_K$), R_o is less than the actual major radius R of the torus.

The thermal energy of the torus gas is

$$\mathcal{U}_{th} = \frac{1}{\gamma-1}Nk_BT , \tag{27}$$

where γ is the usual specific heat ratio and k_B is the Boltzmann constant. For the ideal plasma assumed, the forces are

$$F_R^{th} = -\frac{\partial \mathcal{U}_{th}}{\partial R}\bigg|_S , \quad \text{and} \quad F_a^{th} = -\frac{\partial \mathcal{U}_{th}}{\partial a}\bigg|_S , \tag{28}$$

where the S subscript indicates that the derivatives are taken with the entropy kept constant. For constant S, note that $\mathcal{U}_{th} \propto (2\pi^2a^2R)^{-(\gamma-1)}$. Thus we find

$$F_R^{th} = \frac{\gamma-1}{R}\mathcal{U}_{th} , \quad \text{and} \quad F_a^{th} = \frac{2(\gamma-1)}{a}\mathcal{U}_{th} , \tag{29}$$

so that the thermal forces tend to expand both the major and minor radii. These thermal forces are small compared with the gravitational, centrifugal, and possibly magnetic forces for observed tori in Active Galactic Nuclei. This corresponds to the sound speed being much smaller than both the rotational velocity of the matter v_c and the Alfvén velocity v_A. For this reason we do not consider the thermal forces further.

2.2. *Magnetically supported Tori*

We first consider the limit where the torus is supported against gravity entirely by the magnetic pressure. Thus $\mathcal{U}_{tot} = \mathcal{U}_{grav} + \mathcal{U}_B$, and the conditions for equilibrium are

$$\begin{aligned} 0 &= -\frac{\partial \mathcal{U}_{tot}}{\partial R} \\ &= -\frac{GM_{bh}M_t}{R^2} + \frac{\left(\ln\frac{8R}{a}-\frac{3}{4}\right)\Phi_p^2}{8\pi R^2\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2} - \frac{\Phi_t^2}{4\pi a^2} , \end{aligned} \tag{30}$$

and

$$\begin{aligned} 0 &= -\frac{\partial \mathcal{U}_{tot}}{\partial a} \\ &= -\frac{GM_{bh}M_t\, a}{8R^3} - \frac{\Phi_p^2}{8\pi Ra\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2} + \frac{R\Phi_t^2}{2\pi a^3} . \end{aligned} \tag{31}$$

The tidal gravitational term $GM_{bh}M_ta/(8R^3)$ in Eq. (31) is of order $(a/R)^2 \ll 1$ times the remaining terms and can be neglected. Combining (30) and (31) gives

$$GM_{bh}M_t = \frac{\left(\ln\frac{8R}{a}-\frac{5}{4}\right)\Phi_p^2}{8\pi\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2} , \tag{32}$$

and

$$\frac{R}{a} = \frac{|\Phi_p/\Phi_t|}{2\left(\ln\frac{8R}{a}-\frac{7}{4}\right)} . \tag{33}$$

Notice that R is *not* determined by Eqs. (19) and that a fixed value of $\Phi_p^2/(GM_{bh}M_t)$ is required for equilibrium. The indeterminancy of R is removed by inclusion of the centrifugal forces as discussed below.

In view of Eq. (18), Eq. (31) gives

$$|\overline{B}_t| \approx |B_{ps}| . \tag{34}$$

This relation is pertinent to the stability of the torus to non-axisymmetric perturbations which we discuss in Sect. 3.

We can rewrite Eq. (30) in terms of the magnetic field $B_z(0)$ at the center of the torus which will support a torus with parameters M_t, a, and R against collapse or expansion,

$$\begin{aligned} B_z(0) &= \left[\frac{2\pi GM_{bh}M_t}{R^4\left(\ln\frac{8R}{a}-\frac{5}{4}\right)}\right]^{\frac{1}{2}} , \\ &\approx 0.091G\left(\frac{M_{bh}}{10^7M_\odot}\right)^{\frac{1}{2}}\left(\frac{M_t}{10^5M_\odot}\right)^{\frac{1}{2}}\left(\frac{\text{pc}}{R}\right)^2 , \end{aligned} \tag{35}$$

where we have assumed $R/a = 4$ in the last expression. For these reference values, the poloidal flux is $\Phi_p \approx 3\times 10^{36}$ G cm^2, the total magnetic energy is $\mathcal{U}_B \approx 0.87\times 10^{53}$ erg, and $\Phi_t/\Phi_p \approx 0.073$.

The equilibrium given by Eqs. (30) and (31) is stable to axisymmetric perturbations. This is shown by the fact that

$$\begin{aligned} \frac{\partial^2\mathcal{U}_{tot}}{\partial R^2} &= \frac{\left(\frac{3}{2}\ln\frac{8R}{a}-\frac{9}{8}\right)\Phi_p^2}{8\pi R^3\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^3} > 0 , \\ \frac{\partial^2\mathcal{U}_{tot}}{\partial a^2} &= \frac{\left(2\ln\frac{8R}{a}-\frac{3}{2}\right)\Phi_p^2}{8\pi Ra^2\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^3} > 0 , \end{aligned} \tag{36}$$

where the equilibrium Eqs. (30) and (31) have been used.

2.3. Magneto-centrifugally supported Tori

We now consider the more general case with both magnetic and centrifugal forces so that $\mathcal{U}_{tot} = \mathcal{U}_{grav} + \mathcal{U}_B + \mathcal{U}_{rot}$. Force balance gives, in place of Eqs. (30) and (31),

$$0 = -\frac{GM_{bh}M_t}{R^2}\left(1-\frac{v_c^2}{v_K^2}\right) + \frac{\left(\ln\frac{8R}{a}-\frac{3}{4}\right)\Phi_p^2}{8\pi R^2\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2} - \frac{\Phi_t^2}{4\pi a^2}\ , \qquad (37)$$

$$0 = -\frac{GM_{bh}M_t\ a}{8R^3} - \frac{\Phi_p^2}{8\pi Ra\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2} + \frac{R\Phi_t^2}{2\pi a^3}\ . \qquad (38)$$

Combining these two equations gives

$$1-\left(\frac{v_c}{v_K}\right)^2 + \left(\frac{a}{4R}\right)^2 = \frac{k\Phi_p^2}{GM_{bh}M_t}\ , \qquad (39)$$

where

$$k \equiv \frac{\ln(\frac{8R}{a})-\frac{5}{4}}{[\ln(\frac{8R}{a})-\frac{7}{4}]^2},$$

with $k \approx 0.753$ for $R/a = 4$. We have

$$|\overline{B}_t| = |B_{ps}|\left[1+\frac{\pi\left(\ln\frac{8R}{a}-\frac{7}{4}\right)^2 GM_{bh}M_t}{\Phi_p^2}\left(\frac{a}{R}\right)^2\right]^{\frac{1}{2}}, \qquad (40)$$

in place of Eq. (34).

There are two interesting limits of Eq. (39): The strong field limit where $1-(v_c/v_K)^2 \gg (a/4R)^2$ so that

$$1-\left(\frac{v_c}{v_K}\right)^2 \approx \frac{k\Phi_p^2}{GM_{bh}M_t}\ . \qquad (41)$$

In this limit, the torus is at least noticably sub-Keplerian due to the outward magnetic force, and the tidal gravitional force in the minor direction is negligible in (38). In this limit Eq. (40) gives $|\overline{B}_t| \approx |B_{ps}|$.

In the opposite, weak field limit where $1-(v_c/v_K)^2 \ll (a/4R)^2$, we have

$$\left(\frac{a}{4R}\right)^2 \approx \frac{k\Phi_p^2}{GM_{bh}M_t}\ . \qquad (42)$$

In this limit, the torus rotation is close to Keplerian, and the magnetic field does not appreciably influence the major radius. However, the magnetic pressure still provides the essential 'support' for the minor radius. In this limit Eq. (40) gives $|\overline{B}_t| \approx 10.6|B_{ps}|$ for $R/a = 4$.

In the weak field limit, Eq. (42) can be used to obtain

$$v_{Ap} = \frac{v_K}{\sqrt{32\pi k}}\left(\frac{a}{R}\right)^2\ , \qquad (43)$$

where $v_{Ap} \equiv |\overline{B}_p|/\sqrt{4\pi\rho}$ is the poloidal Alfvén velocity and we have used $M_t = 2\pi^2 a^2 R\rho$. This equation shows the dependence of the thickness a on the magnetic field. Even for $R/a \sim 1$ the poloidal Alfvén velocity is small compared with the Keplerian velocity v_K. Even the Alfvén velocity based on the toroidal magnetic field, v_{At}, is smaller than v_K.

3. Non-axisymmetric stability of Tori

A magnetic field line near the torus surface may wrap around the small direction of the surface more than once as the azimuth ϕ increase from 0 to 2π. In fact, the total 'wrapping angle' is

$$\iota \approx 2\pi\left|\frac{B_{ps}}{\overline{B}_t}\right|\frac{R}{a}\ , \qquad (44)$$

which is termed the rotational transform in treatments of tokamak plasmas. Thus, magnetically supported tori in the strong field limit violate the Kruskal-Shafranov *linear* stability condition, $\iota < 2\pi$, for non-axisymmetric kink perturbations (see for example Bateman 1980). This stability condition, derived for tokamak plasmas, does not include the influence of significant plasma rotation. However, the same condition applies including the plasma rotation if $\rho v_c^2 < \mathbf{B}^2/(4\pi)$ (Lovelace 1976). The linear kink perturbations are characterized by a toroidal mode number $m = 1, 2, ..$ with the radial shift of the major axis of the torus $\delta R = \delta R_o\cos(m\phi)$, say, and the axial shift of this axis $\delta z = R_o\sin(m\phi)$.

For the strong field tori, $\iota \approx R/a$. Only in the limit $R/a \sim 1$ do the tori approach the condition for stability, and in this limit our theory breaks down. On the other hand, the weak field tori are linearly stable to kink perturbations for $R/a \lesssim 13.1$. The threshold value of $\Delta \equiv 1-(v_c/v_K)^2$ for Kruskal-Shafranov kink stability is $\Delta \approx 0.0771$ for $R/a = 3$ and $\Delta \approx 0.0251$ for $R/a = 4$ as follows from Eqs. (25) and (27). (These equations give a threshold $\Delta > 0$ only for $R/a \leq 13.1$ which includes the aspect ratios of interest here.) For Δ larger than these values there is instability.

The *nonlinear* behavior of the kink perturbation of a AGN torus is unknown. The geometrical nature of a finite amplitude kink perturbation of a magnetically supported torus is shown in Fig. 2. The $m = 1$ kink corresponds to both a tilt and a radial shift of the torus. For the probable conditions of significant, supersonic, but sub-Keplerian rotation, the kink perturbation would be important for the outward transport of angular momentum and the consequent enhanced accretion of the torus matter. Trailing spiral, slow-magnetohydrodynamic shocks can be expected to form and to have the effect of deflecting azimuthally moving matter into inward radial motion.

Other non-axisymmetric instabilities may be important on smaller length-scales. A plasma β dependent ballooning instability related to that of tokamak plasmas (Bateman 1980) may occur in that for the galactic tori $\rho\mathbf{v}^2/2$ acts as an effective pressure (Lovelace 1976). It is possible that the magnetic field of the torus actually consists of a set of many separated flux tubes. In this case, the bouyancy instablity of the flux tubes may lead to their escape from the torus (Parker 1979).

3.1. Magnetic field decay and regeneration

The torus matter will of course *not* be perfectly conducting as assumed in the above treatment. However, the important question is the decay time for the ordered field. The field decay can result from ambipolar diffusion and/or turbulent diffusion.

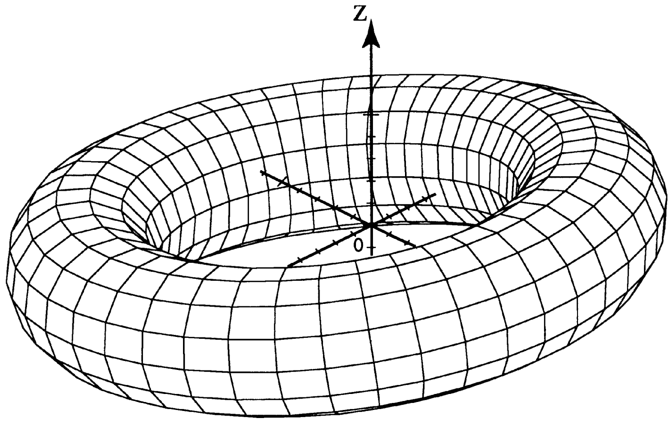

Fig. 2. Drawing of finite amplitude kink perturbation of a magnetically supported torus with aspect ratio $R/a = 3.33$ for toroidal mode number $m = 1$. The amplitude of the major radius shifts $\delta R = 0.2R$. Note that the perturbation involves both a radial shift and a tilt of the torus.

The ambipolar diffusion is thought to be important for dense molecular clouds (away from the galactic center), and it can be described in terms of a diffusion coefficient $\mathcal{D} = v_A^2 t_{ni}$, where $v_A = |\mathbf{B}|/\sqrt{4\pi\rho_n}$ is the Alfvén velocity and $t_{ni} = 1/(\gamma\rho_i)$ is the mean collision time of a neutral molecule with an ion, where ρ_n is the neutral mass density, $\rho_i = C\sqrt{\rho_n}$ ion mass density, and C and γ are constants (Shu, Adams, & Lizano 1987). The ambipolar decay time for the field is $t_{ap} \sim R^2/\mathcal{D}$. This time is an order of magnitude larger than the dynamical time $t_{dyn} \sim R/v_K$ for typical molecular clouds. The tori in galactic nuclei exist in a intense enviorment where X-ray heating is important so that the matter is thought to be in a two phase structure, a high temperature atomic phase and a lower temperature molecular phase (Neufeld, Maloney, & Conger 1994). For these conditions $t_{ap} \gg t_{dyn}$. The shortest lifetime for the $\mathbf{B}$ field is that expected for turbulent motion of the torus gas where the magnetic diffusivity can be written as $\eta_T \sim \alpha_t c_s a$ with α_t a dimensionless constant $\mathcal{O}(0.01 - 0.1)$ (Shakura 1973; Shakura & Sunyaev 1973). The field lifetime in this case $t_T \sim R^2/\eta_T$ although longer than the dynamical time could be relatively short.

If the field decay time is short, then it is necessary in the present picture to assume efficient regeneration of the field. The problem of regeneration of $\mathbf{B}$ in tori is analogous to that for the distributed magnetic field of galaxies where the basic picture involves an $\alpha - \Omega$ dynamo (Parker 1979). The Ω effect transforms poloidal field into a toroidal field due to differential rotation of the matter ($d\Omega/dr$) and the α effect due to small scale cyclonic motion makes poloidal field from toroidal field. However, the mechanism of the α effect remains problematic (Parker 1992; Vainshtein, Parker, & Rosner 1993).

4. Discussion

This work has developed a model for magnetically supported gas tori surrounding central black holes in AGNs. In view of this model we return to discuss the three critical questions posed in Sect. 1.

1.) What determines the major radius R of the torus? The magnetic field may be enhanced by dynamo action in the region where the rotation curve changes from Keplerian at small r to approximately rigid rotation at larger r. In this region the gravitational field of the stellar distribution merges into the field of the central black hole (Duschl 1988). This is typically $\sim 0.1 - 1$ parsec from the central black hole (Faber et al. 1997).

2.) Why is the solid angle of the torus large as seen from the central source? This may be due to the fact that a "thin" torus with large R/a is unstable to kink formation. Kink instability of a "thin" torus and the associated nonlinear evolution can give rise to a "thick" torus (Kadomtsev 1963). Further, turbulence triggered by instability in a torus can act to amplify the magnetic field which in turn thickens the torus until stability is attained.

3) Why is there an inverse relation between the aspect angle and the power of the central source? This could happen in two ways, by the major radius R changing or by the minor radius a changing. In the first case R would be an inverse function of the central power with a approximately constant. Alternatively, the magnetic field, which supports the torus, may be inversely correlated with the central power.

The thickness of the torus to obscuration depends on the state of ionization and temperature of the gas. At higher central powers, a larger opening angle (about the symmetry axis) may be ionized and "burned" away, thus explaining the observed relation between opening angle and central power. However, the details are outside the scope of this paper. Another possibility is that as a result of accretion of matter with non aligned angular momentum, the axis of the jet precesses and "cleans out" a conical region of the torus around the average axis of symmetry.

This paper has shown that a thick torus may result from magnetic field pressure. The most likely conditions are in the weak field limit where the field provides essential support for the minor radius a but is unimportant for the major radius R. The magnetic kink instability may be important for large values of R/a.

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