

# Photometry and asteroseismology of $\delta$ Scuti stars in Praesepe

T. Arentoft<sup>1,2</sup>, H. Kjeldsen<sup>1,3</sup>, J. Nuspl<sup>4</sup>, T.R. Bedding<sup>5</sup>, A. Frontó<sup>4</sup>, M. Viskum<sup>1</sup>, S. Frandsen<sup>1</sup>, and J.A. Belmonte<sup>6</sup>

<sup>1</sup> Institute of Physics and Astronomy, Aarhus University, Bygn. 520, DK-8000 Aarhus C, Denmark  
(toar@obs.aau.dk; hans@obs.aau.dk; mv@obs.aau.dk; srf@obs.aau.dk)

<sup>2</sup> University of Brussels (VUB), Pleinlaan 2, B-1050 Brussels, Belgium (tarentof@vub.ac.be)

<sup>3</sup> Theoretical Astrophysics Center, Danmarks Grundforskningsfond, Aarhus University, DK-8000 Aarhus C, Denmark

<sup>4</sup> Konkoly Observatory, H-1525 Budapest, Hungary (nuspl@ogyalla.konkoly.hu; fronto@ogyalla.konkoly.hu)

<sup>5</sup> School of Physics, University of Sydney 2006, Australia (bedding@physics.usyd.edu.au)

<sup>6</sup> Instituto de Astrofísica de Canarias, E-38200 La Laguna, Tenerife, Spain (jba@iac.es)

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**Abstract.** We discuss observations and seismic analyses for three multi-periodic  $\delta$  Scuti stars in the Praesepe cluster: BU, BN and BV Cnc. For two stars (BU and BN Cnc) we re-analyse published photoelectric photometry and show that using statistical weights results in a dramatic reduction in the noise level. We detect seven frequencies in BU Cnc and eight in BN Cnc, with three of the latter being previously unknown. We also present new time-series CCD observations of BN and BV Cnc that successfully demonstrate high-precision differential photometry on strongly defocussed images. The observations show that BV Cnc is a multi-periodic variable, with at least four oscillation frequencies. Finally, we compare the observed oscillation frequencies in the three stars with models and derive luminosities, effective temperatures and masses.

**Key words:** stars: variables:  $\delta$  Scuti – stars: oscillations – open clusters and associations: individual: Praesepe – methods: data analysis

## 1. Introduction

Multi-periodic  $\delta$  Scuti variables represent an important opportunity to apply asteroseismology to “ordinary” main sequence F stars. Unfortunately, this goal has been hampered by our inability to identify which oscillation modes are being observed. For reasons which are not understood, only a seemingly random subset of possible modes are excited in any given star. In addition, many  $\delta$  Scuti stars rotate rapidly, which further complicates the interpretation of their frequency spectra.

One approach is to concentrate on stars in which many frequencies are detected. Attempts to match observations to models have recently been made for XX Pyx by Pamyatnykh et al. (1998) and for FG Vir by Viskum et al. (1998). In the latter case, mode identification was achieved by combining simultaneous photometric and spectroscopic observations.

A complementary approach is to study  $\delta$  Scuti stars in open clusters. This allows several variables to be observed simultaneously, especially if a CCD detector is used. A second advantage is that modelling can be carried out for all cluster members at once, aided by the assumption of a single set of values for distance, age and metallicity.

In this paper we discuss observations and models for three  $\delta$  Scuti stars in Praesepe. The Praesepe cluster (M44, NGC 2632) contains 18 known  $\delta$  Scuti stars (Rodríguez et al. 1994) in different evolutionary stages, offering a good opportunity for testing theories of stellar structure and evolution. Our hope is that identifying modes in a few stars will make mode identification easier for the remainder.

## 2. The target stars

Four stars are discussed in this paper:

BU Cnc (= HD 73576 = KW 207) is a well-studied  $\delta$  Scuti star which has multiple frequencies. We present a re-analysis of published photoelectric photometry in Sect. 4.

BN Cnc (= HD 73763 = KW 323) is also a well-studied multi-periodic pulsator. We present a re-analysis of published photoelectric photometry in Sect. 4 and new observations using CCD photometry in Sect. 5.

BV Cnc (= HD 73746 = KW 318) has been reported by Jakisch (1972) to have an oscillation period of 5 hr. We are not aware of any other time-series measurements. In Sect. 5 we present new CCD photometry which shows this star to be a multi-periodic pulsator.

HD 73712 (= KW 284) is a more evolved low-frequency  $\delta$  Scuti variable which is discussed briefly in Sect. 4.1

Finally, in Sect. 6 we compare the observed frequencies with theoretical models.

Table 1 lists the properties of these stars. All photometric and rotational velocity values are from García et al. (1995), except the value of  $v \sin i$  for HD 73712, which is taken from the SIMBAD database. We estimated  $T_{\text{eff}}$  from the photometry and took bolometric corrections from Schmidt-Kahler (1982) on

**Table 1.** Properties of the stars discussed in this paper.

Star	$V$	$B - V$	$b - y$	$\beta$	$v \sin i$ (km/s)	$T_{\text{eff}}$ (K)	$BC$	$M_{\text{bol}}$	$L$ ( $L_{\odot}$ )
BU Cnc	7.68	0.19	0.10	2.812	200	7800	-0.13	$1.31 \pm 0.12$	$21.5 \pm 2.4$
BN Cnc	7.80	0.22	0.13	2.796	130	7650	-0.12	$1.44 \pm 0.12$	$19.1 \pm 2.1$
BV Cnc	8.66	0.29	0.18	2.748	110	7250	-0.09	$2.33 \pm 0.12$	$8.4 \pm 0.9$
HD 73712	6.78	0.27	0.16	2.756	40	7300	-0.10	$0.44 \pm 0.12$	$38.9 \pm 4.3^*$

\*luminosity of the brighter component, adopting a magnitude difference of  $1.6 \pm 0.1$  (see Sect. 2.1).

a system in which the Sun has  $M_{\text{bol}} = 4.64$  and  $BC = -0.15$ . Note that we have not included any correction for the rotation (the radiation observed from a rapidly rotating star will depend on the inclination of the axis to the line of sight). The absolute bolometric magnitudes and luminosities are based on a distance modulus for Praesepe of  $6.24 \pm 0.12$ , as measured by Mermilliod et al. (1997) using Hipparcos data. Following these authors, we also adopt zero reddening as measured by Crawford & Barnes (1969).

### 2.1. Binarity of the targets stars

Mason et al. (1993) have conducted a speckle survey of stars in Praesepe to check for binarity. For BN, BU and BV Cnc they found no evidence for companions. Their observations were sensitive to separations in the range 0.035–1.0 arcsec and magnitude differences less than about 3.0.

The star HD 73712, on the other hand, is known to be a close binary from both lunar occultation and speckle observations. The only reported measurement of the magnitude difference is about 1.6, derived from observations of a lunar occultation by Peterson et al. (1989). By examining published speckle and occultation observations spanning the years 1982–95 (Hartkopf et al. 1997), we have found that the binary system has a roughly circular 28-yr orbit which is seen almost exactly edge-on. The apparent separation was close to zero in 1991 and will be so again in about 2005. However, the angular diameter of the primary is expected to be about 0.002 arcsec, so the chances of eclipse are very small.

## 3. Method of time series analysis

Searching for periodic signals in a time series usually involves Fourier analysis. In practice, the data quality often varies considerably during an observing run due to changes in observing conditions (airmass, extinction, cloud cover, seeing, scintillation, etc.) and changes or drifts in the telescope and instrument (position of star on CCD, gain of detector, etc.). In addition, one may wish to combine observations from more than one site.

The conventional Fourier transform is equivalent to a least-squares fit of sine and cosine functions, with equal weight given to all data points. The final noise level in the power spectrum will therefore be dominated by the noisiest parts of the time series. We overcome this by assigning a statistical weight to

each data point according to its quality and then calculating the power spectrum as a *weighted* least-squares fit.

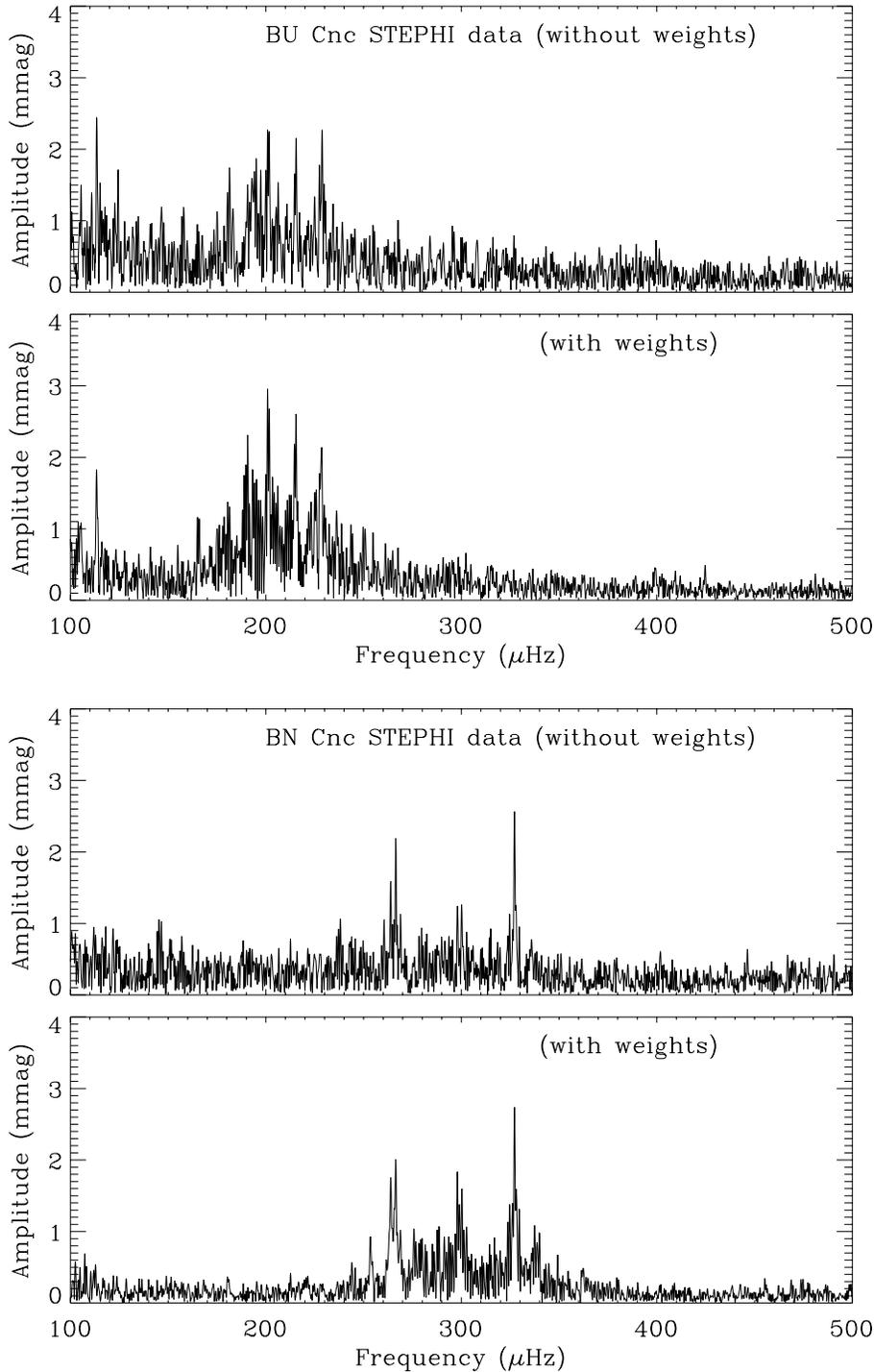
The first step is to remove bad data points (equivalent to assigning them zero weight) by performing a  $4\sigma$  clipping, where  $\sigma$  is the local rms scatter in the time series. We then calculate the weight of each data point as the inverse of the local variance, which we measure in a copy of the time series that has been high-pass filtered. In this way, we are basing the calculation of weights on a part of the power spectrum where no signal is expected and also where we are more sensitive to fast variations in the quality of the time series. It is important to ask whether the noise at these high frequencies has the same statistical properties as the noise at lower frequencies, where the signal is located. This can be tested by calculating the ratio between the noise at low and high frequencies for different segments of the time series. If this ratio is constant, we can conclude that the high-frequency weights can be used directly. In practice, we find that this ratio is constant within a night but can vary from one night to the next. We therefore adjust the weights on a night-by-night basis by a simple multiplicative factor so that they reflect the noise at lower frequencies (where they are used) rather than at the high frequencies (where they are calculated).

Once the weights have been assigned, the power spectrum is calculated by performing a weighted least-squares fit of sine and cosine functions. The method is described in detail by Kjeldsen (1992) and Frandsen et al. (1995). In this paper, we display each result as an amplitude spectrum, which is the square root of the power spectrum.

## 4. STEPPI observations of BN and BU Cnc

The stars BN and BU Cnc were observed using the STEPPI network (STellar PHotometry International Project) by Belmonte et al. (1994; hereafter STEPPI94). The observations were made over three weeks in 1992 using photoelectric photometry from three sites. The resulting amplitude spectra had a noise level of about 0.75 mmag at 200–250  $\mu\text{Hz}$  and about 0.60 mmag at 250–350  $\mu\text{Hz}$ . Note that we give semi-amplitudes throughout this paper, whereas amplitudes in STEPPI94 are peak-to-peak values, which we have halved.

From their observations, Belmonte et al. (1994) identified six frequencies in BU Cnc and five in BN Cnc. We have used the techniques described in Sect. 3 to re-analyse the STEPPI data. The results are shown in Figs. 1 and 2, and the improvement gained by using statistical weights is evident. These fig-



**Fig. 1.** Amplitude spectra of observations of BU Cnc, calculated from the STEPHI data. Bad data points have been removed and decorrelation was performed. In addition, statistical weights were applied in calculating the lower panel, but not for the upper panel.

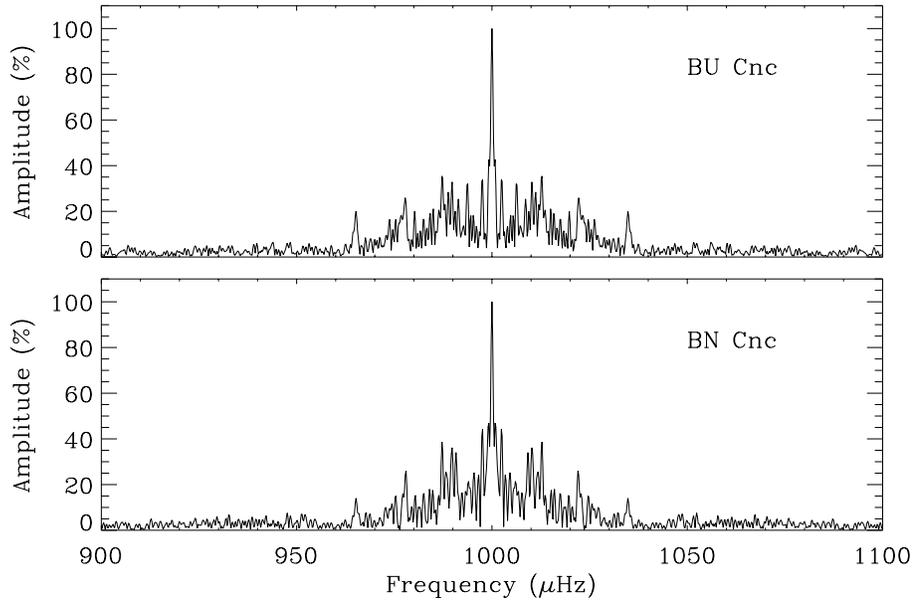
**Fig. 2.** Same as Fig. 1, but for the star BN Cnc.

ures should be compared with Fig. 2 of STEPHI94, where the amplitude scale in that paper should be divided by two.

The noise level in the BU Cnc amplitude spectrum (lower panel of Fig. 1) is 0.25–0.30 mmag at 200–250  $\mu$ Hz. This represents an improvement in signal-to-noise by more than a factor of two over the results in STEPHI94. For BN Cnc, the improvement is more than a factor of four.

The spectral window functions for the two stars are shown in Fig. 3. Compared with the original time series (Fig. 2c of

STEPHI94), there is more power in the sidelobes. This is to be expected because some of the data points have been given low weight or even discarded entirely. The question, of course, is whether the disadvantage of having a degraded window function is outweighed by the dramatic reduction in noise. For these data we believe this to be the case since the highest sidelobes in the spectral window after data weighting are still less than 45% in amplitude of the main peak (less than 20% in power).



**Fig. 3.** Spectral window for the STEPHI observations of BU Cnc and BN Cnc, based on our re-analysis (including statistical weights).

**Table 2.** Oscillations in BU Cnc, based on re-analysis of the STEPHI data.

Frequency ( $\mu\text{Hz}$ )	Amplitude (mmag)	S/N	Identification
113.37	1.8	4.1	
193.25	1.4	5.2	$n = 3, \ell = 0$
195.19	1.9	6.7	
200.88	2.9	10.5	$(n = 5, \ell = 0)^*$
215.44	1.8	6.6	
228.53	2.1	7.8	$(n = 6, \ell = 0)^*$
229.85	1.4	5.2	$n = 4, \ell = 0$

\*from Pérez Hernández et al. (1995)

From the re-analysis of the BU Cnc data, we identify six frequencies with S/N greater than 5, all of which agree with those reported by STEPHI94 (and subsequently adopted by Pérez Hernández et al. 1995 for comparison with model calculations). The frequencies are given in Table 2 (the mode identifications are discussed in Sect. 6). A seventh peak at 113.37  $\mu\text{Hz}$ , already present in the STEPHI94 analysis, is detected at  $S/N = 4.1$  (see also Fig. 4). Throughout this paper, we do not attempt to apply tests of statistical significance because the noise is not white.

Five of the frequencies in the STEPHI data agree with those found by Breger et al. (1993), with an rms difference of 0.08  $\mu\text{Hz}$ . This gives us a measure of the accuracy with which the frequencies have been determined. However, Breger et al. (1993) also found a peak at 276.7  $\mu\text{Hz}$  which is absent in the STEPHI data, even after our re-analysis. The strongest peak within 1  $\mu\text{Hz}$  of this frequency has an amplitude of 0.45 mmag, only half of that found by Breger et al. (1993). A possible peak for BU Cnc at 401.4  $\mu\text{Hz}$  suggested by STEPHI94 is not present after our re-analysis.

For BN Cnc we identify eight frequencies, five of which agree with those given by STEPHI94 and Pérez Hernández et al. (1995). The three new frequencies (at 279.87, 307.01 and 323.56  $\mu\text{Hz}$ ) have smaller amplitudes and their detection is a result of the lower noise level. The frequencies and amplitudes are given in Table 4 (the mode identifications are discussed in Sect. 6).

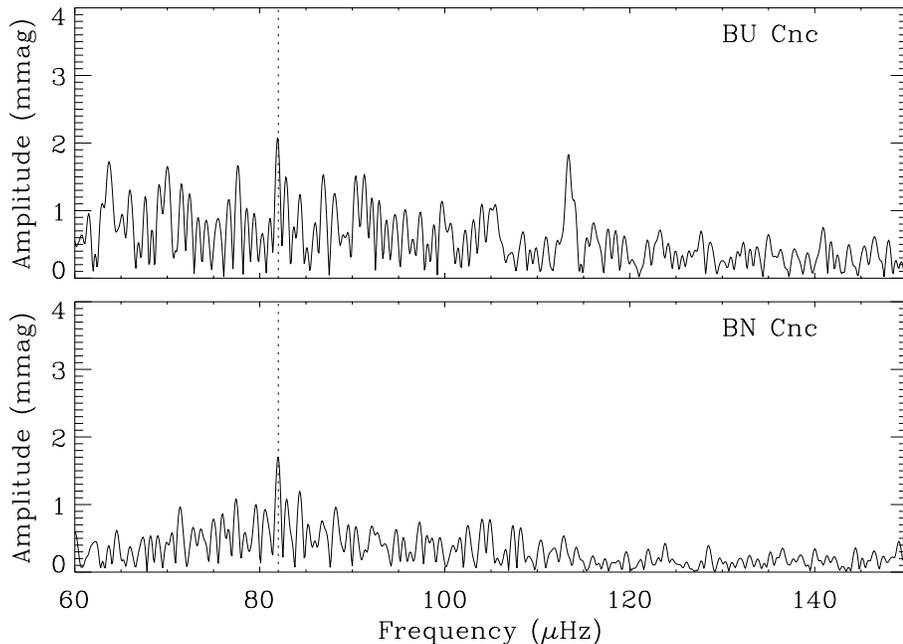
#### 4.1. Oscillations in HD 73712

The comparison star used for the STEPHI observations was HD 73712, which is also a member of the Praesepe cluster. At the time, the STEPHI team were unaware that this star had been reported by Rolland et al. (1991) to be a  $\delta$  Scuti variable, with an oscillation frequency of 79  $\mu\text{Hz}$  and an amplitude of 2 mmag (4 mmag peak-to-peak).

After becoming aware of the earlier results, Michel et al. (1995) re-examined their STEPHI observations by calculating amplitude spectra at low frequencies. They identified a peak at 69.5  $\mu\text{Hz}$  which was common to the spectra of both BN and BU Cnc, having amplitudes of 0.9 and 1.5 mmag, respectively. Michel et al. attributed this common peak to variability in the comparison star HD 73712. However, this conclusion must be treated with caution for two reasons. Firstly, 69.5  $\mu\text{Hz}$  is almost equal to six times 1/day (11.57  $\mu\text{Hz}$ ). Secondly, as pointed out by Michel et al., the noise level in the STEPHI data at low frequencies was rather high.

Our re-analysis of the STEPHI data has reduced the noise significantly. Fig. 4 shows the amplitude spectra of BU and BN Cnc in the frequency range 60–150  $\mu\text{Hz}$ . In both spectra, the strongest peak occurs at 82.0  $\mu\text{Hz}$  and we identify this as an oscillation frequency of HD 73712. The peak at 69.5  $\mu\text{Hz}$  has disappeared, implying that it was due to noise.

Can we reconcile our detection at 82.0  $\mu\text{Hz}$  in the STEPHI data (taken 1992 Jan) with the value of 79  $\mu\text{Hz}$  reported by Rol-



**Fig. 4.** Comparison of the amplitude spectra of BU Cnc and BN Cnc at low frequencies, based on our re-analysis of the STEPPI data (including statistical weights). In both spectra, the highest peak is at  $82.0 \mu\text{Hz}$  (dotted lines).

land et al. (1991) from observations in 1991 Feb? The Rolland et al. result was a clear detection with good signal-to-noise, even in the raw light curves, but had aliasing problems because it was based on two 5-hour observing runs centred 8 nights apart. The STEPPI data, on the other hand, have an excellent window function. We have attempted to reconcile the two frequencies, taking into account the window function of the Rolland et al. data, but are unable to do so.

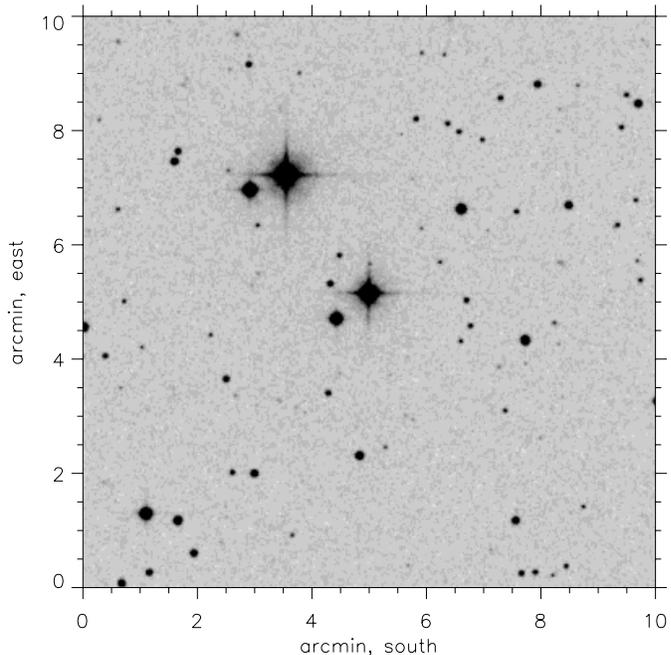
## 5. CCD photometry of BN and BV Cnc

### 5.1. Observations and reductions

We have used CCD photometry to observe a field in Praesepe containing the  $\delta$  Scuti stars BN and BV Cnc, which are separated by 3 arcmin (see Fig. 5). The first set of observations was carried out in 1996 February using the IAC 80-cm telescope at Teide Observatory, Tenerife, Spain. We used a  $1024 \times 1024$  pixel Thomson CCD and a standard Johnson  $B$  filter. The field of view was  $8' \times 8'$ .

A second set of observations was made a year later using the 1-m RCC telescope at Piszkestető, the mountain station of Konkoly Observatory, Hungary. We used a  $770 \times 1152$  pixel EEV CCD and standard Johnson  $V$  filter. The field of view was  $4' \times 6'$ . A log of both observing runs is shown in Table 3.

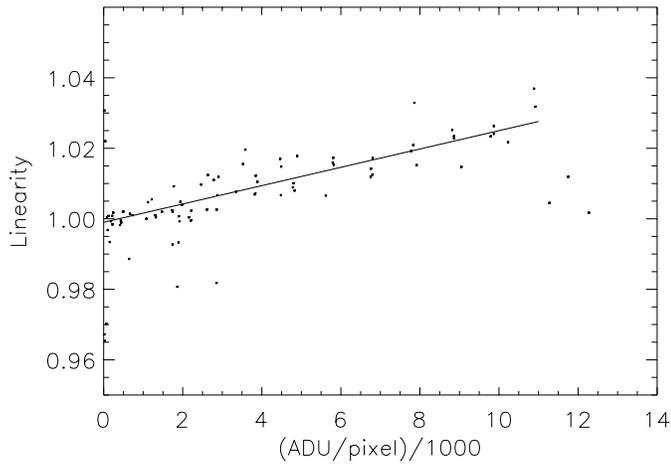
Several comparison stars were available in the field to allow differential photometry. These comparison stars are much fainter than the two targets — the brightest two were  $V = 10.6$  and  $V = 11.6$  — so the telescopes were strongly defocussed in order to prolong the integration times. The defocussed images typically had radii of 30–40 pixels (at a scale of  $0.34''$  per pixel and typical seeing of  $2\text{--}3''$ ), which was possible because the field was not crowded. Care was taken to keep the image drift at a very low level (a few pixels), in order to minimize flat-field errors.



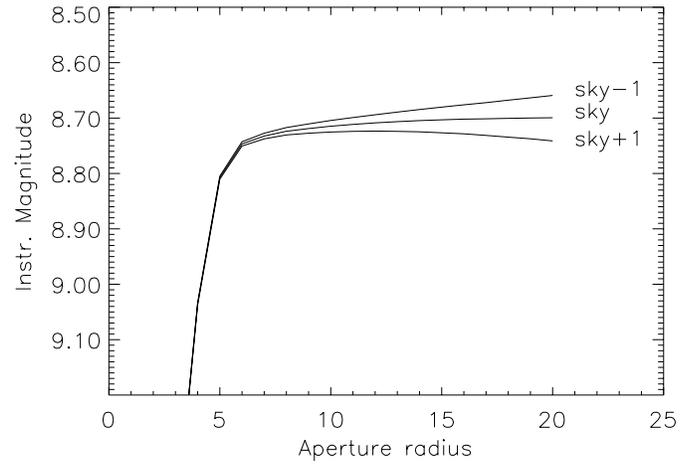
**Fig. 5.** Digital Sky Survey image of the field in Praesepe. The two brightest stars are BV Cnc (centre) and BN Cnc.

We performed linearity tests on the CCD at Konkoly Observatory using a series of flat-field exposures (see Gilliland et al. 1993, Sect. 4.2). Fig. 6 shows the results, which indicate that the system was non-linear at the level of a few percent. We fitted the non-linearity with a straight line and applied the correction to each pixel of every frame. The CCD at Teide Observatory was not checked for linearity.

The photometric reductions were done using a program written in IDL. Because of the large amount of defocussing, we



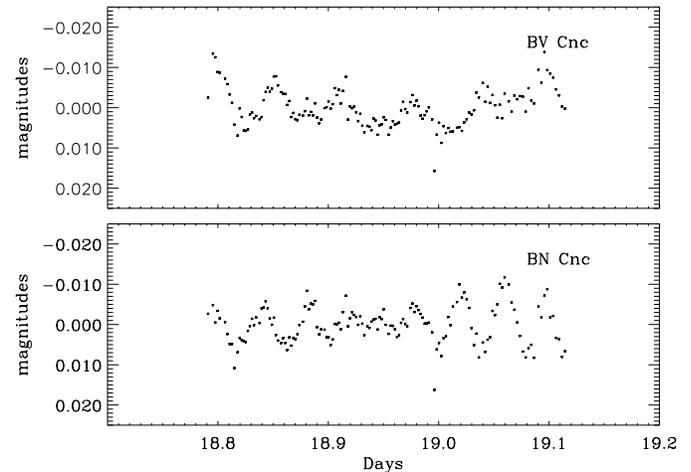
**Fig. 6.** Linearity curve for the CCD-detector at the 1.0-m RCC telescope at Konkoly Observatory. The vertical axis plots the ratio of observed to expected counts. The straight line is the fit used to correct for the non-linearities. The saturation-limit of the detector is approximately 11000 ADU, where the gain was 10.2 electrons per ADU.



**Fig. 7.** Magnitude measurements for one of the comparison stars as a function of aperture radius, using a CCD frame obtained at Teide Observatory. The middle curve uses an accurate value for the sky background. The lower and upper curves show the result when the sky background is deliberately over- and underestimated by 1 ADU (2 electrons).

**Table 3.** Log of CCD observations of BN and BV Cnc.

Observatory	Start date	Exp. times (s)	No. of frames	Length (hr)
Teide	1996 Feb 10	60-70	30	1
	1996 Feb 11	80-120	83	5
	1996 Feb 12	70-120	50	2.5
	1996 Feb 13	70-120	60	3
	1996 Feb 14	70-140	44	2.5
	1996 Feb 17	90-140	45	3
Konkoly	1997 Feb 14	160-200	97	5.5
	1997 Feb 19	150-200	121	8
	1997 Feb 28	160-240	97	6
	1997 Mar 1	160	132	7
	1997 Mar 2	130-160	150	7.5
	1997 Mar 4	160-270	95	7
	1997 Mar 5	140-210	151	8



**Fig. 8.** Light curves from the night 1997 March 5 for BN and BV Cnc. For each star, the mean magnitude has been subtracted from the measurements.

used aperture photometry. The sky values were determined locally from pixels around each star using a  $3\sigma$ -clipping procedure Kjeldsen & Frandsen (1992). Fig. 7 shows the measured magnitude for one of the comparison stars as a function of aperture radius. When the sky is estimated accurately, the measurement approaches a constant value (middle curve). However, this does not happen if the sky value is overestimated (lower curve) or underestimated (upper curve), and errors arise of several hundredths of a magnitude. This illustrates the importance of obtaining an accurate value for the sky background when performing photometry on highly defocussed images.

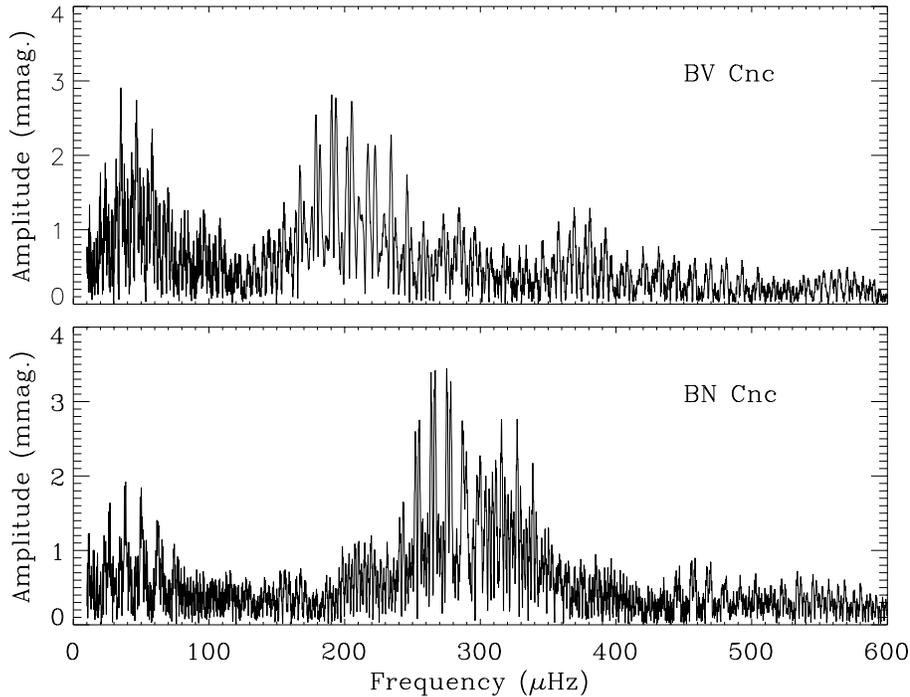
## 5.2. Analysis

The photometric series for the two target stars were analysed using the methods described in Sect. 3, including the use of statistical weights. The noise levels in the data from Teide Observatory

were about 8 times higher than expected from scintillation and photon noise alone. It is possible that detector non-linearities were the cause. The data from Konkoly Observatory had lower noise levels and for the rest of this paper we confine our discussion to those data.

Fig. 8 shows light curves for the two stars on one night. The complex patterns clearly indicate that several frequencies are present. Fig. 9 shows the amplitude spectra of the full data sets and we again see clearly that oscillations are present. The fact that the oscillations occur in different frequency bands for the two stars rules out the possibility that they are due to variability in one of the comparison stars.

The noise level in the amplitude spectrum of BN Cnc is about 0.31 mmag at 150–250  $\mu$ Hz and about 0.15 mmag at 700–900  $\mu$ Hz. In the following discussion, we shall refer to these frequency ranges as low and high, respectively. The contri-



**Fig. 9.** Amplitude spectra for BN and BV Cnc, based on the CCD photometry taken at Konkoly Observatory.

bution from scintillation noise is expected to be the same at both frequencies, with a value of about 0.04–0.08 mmag (e.g., Young 1967). Thus, at least at high frequencies, we are close to achieving the scintillation limit in these observations. This demonstrates the feasibility of obtaining high-precision differential photometry from strongly defocussed images, and hence that it is possible to use a CCD to observe targets that are much brighter than the available comparison stars. Correcting for scintillation gives a residual noise of 0.30 mmag at low frequencies and 0.14 mmag at high frequencies.

We can compare these noise levels for BN Cnc with those from the much longer set of STEPPI photoelectric data (after re-analysis). We must keep in mind that the scintillation noise for the STEPPI observations should only be about 0.01–0.02 mmag, since they were made with larger telescopes at higher altitudes. After correction for scintillation noise, the STEPPI noise levels in the low and high frequency bands are 0.17 mmag and 0.11 mmag, respectively.

The total amounts of useful observing time for STEPPI and Konkoly data were 228 hr and 47 hr, respectively. However, to compare the data we should calculate the effective observing times by taking the weights into account. We do this by integrating under the spectral windows, which gives 110 hr versus 25 hr. We would therefore expect the STEPPI data to have a noise level about 2.1 times lower than the Konkoly data. In fact, this ratio is 1.8 at low frequencies and 1.3 at high frequencies. In other words, the CCD observations have achieved a slightly better noise level per unit time than the photoelectric STEPPI data at low frequencies and a significantly better noise level at high frequencies. This confirms that differential CCD photometry is able to get closer to the scintillation-noise limit than is photoelectric photometry.

**Table 4.** Frequencies and amplitudes of oscillations in BN Cnc.

Frequency ( $\mu$ Hz)	STEPPI		Konkoly		Identification
	(mmag)	S/N	(mmag)	S/N	
263.69	1.2	7.8	2.5	8.2	$(n = 5, \ell = 0)^*$
266.42	2.1	13.8	2.3	7.6	
279.87	0.7	4.8	0.6	1.9	$n = 5, \ell = 0$
298.01	2.0	13.5	2.3	7.7	
300.24	0.9	6.5	1.9	6.4	$(n = 6, \ell = 0)^*$
307.01	0.5	3.2	0.6	2.1	
323.56	0.5	3.6	0.5	1.7	$n = 6, \ell = 0$
327.16	2.7	19.4	2.6	8.6	

\*from Pérez Hernández et al. (1995)

### 5.3. Results for BN Cnc

By prewhitening the amplitude spectrum of BN Cnc, using the procedure described by Frandsen et al. (1995), we found the same five frequencies as STEPPI94. Not surprisingly, the three weaker modes detected in our re-analysis of the STEPPI data (see Sect. 4) have low S/N in our CCD observations. The amplitudes for all eight modes are given in Table 4 (the mode identifications are discussed in Sect. 6). For most modes, there is good agreement between the two amplitude measurements. In two cases there is disagreement at a level more than five times the scatter in the amplitude differences, which may be due to changes in the star, but could also be caused by beating between unresolved modes.

**Table 5.** Oscillations in BV Cnc, based on observations at Konkoly Observatory.

Frequency ( $\mu\text{Hz}$ )	Amplitude		Identification
	(mmag)	S/N	
190.47	2.6	8.2	
204.83	1.5	5.0	
222.65	2.5	7.8	$n = 2, \ell = 0$
369.10	1.6	6.6	$n = 5, \ell = 0$

#### 5.4. Results for BV Cnc

Our observations provide the first detection of multi-periodic behaviour in BV Cnc. After prewhitening, we detected four oscillation frequencies, as listed in Table 5 (the mode identifications are discussed in Sect. 6). Note that, due to the single-site window function, the frequencies are susceptible to 1/day aliasing.

Finally, we note that Jakisch (1972) reported a 5-hr period ( $56 \mu\text{Hz}$ ) with a peak-to-peak amplitude in  $V$  of 17 mmag. We do not find evidence for this in our data.

## 6. Asteroseismology

Asteroseismology involves comparing observed oscillation frequencies with those calculated using theoretical models. However, this requires knowledge of which modes are being observed, which is notoriously problematic for  $\delta$  Scuti variables. In the present case, matters are complicated by the fact that these stars are rotating rapidly (see Table 1), which will affect the mode frequencies.

We therefore adopt the following approach. We assume that at least some of the observed modes are radial ( $\ell = 0$ ) and attempt to identify these. We only consider radial modes because those of higher degree have more complicated dependencies on stellar density and rotation (Christensen-Dalsgaard 1993; Frandsen et al. 1995). Even for radial modes, however, systematic errors will be introduced by ignoring the effect of rotation.

As our starting point we take a series of existing model calculations by Christensen-Dalsgaard (1993) for a  $\delta$  Scuti star having solar metallicity. In applying these models, we are adopting solar metallicity for Praesepe, consistent with recent results (e.g., Mermilliod et al. 1997). From the models we find following relation, which holds to better than 1% for luminosities in the range  $10\text{--}25 L_{\odot}$  and effective temperatures in the range  $7400\text{--}8000$  K:

$$\frac{L}{L_{\odot}} = 0.9543 \left( \frac{M}{M_{\odot}} \right)^{4.78} \left( \frac{T_{\text{eff}}}{5777 \text{ K}} \right)^{-0.709}. \quad (1)$$

This result will be used below.

We also find that the frequencies of radial modes scale with luminosity and effective temperature as follows:

$$f(n) = a_n \left( \frac{L}{L_{\odot}} \right)^{b_n} \left( \frac{T_{\text{eff}}}{5777 \text{ K}} \right)^{3.07}. \quad (2)$$

**Table 6.** The parameters  $a$  and  $b$  in Eq. 2.

$n$	$a_n$	$b_n$
	( $\mu\text{Hz}$ )	
1	344.9	-0.641
2	459.8	-0.645
3	529.5	-0.621
4	611.7	-0.611
5	710.0	-0.615
6	850.3	-0.626
7	989.2	-0.637

**Table 7.** Parameters derived from the asteroseismic analysis.

Star	$L$ ( $L_{\odot}$ )	$T_{\text{eff}}$ (K)	$M$ ( $M_{\odot}$ )	$R$ ( $R_{\odot}$ )	$\log g^*$
BU Cnc	22.1	7780	2.02	2.59	3.915
BN Cnc	19.6	7740	1.97	2.47	3.947
BV Cnc	8.6	7180	1.64	1.90	4.093
HD 73712	40.0	7240	2.10	4.03	3.550

\*  $g$  in units of  $\text{cm s}^{-2}$

The parameters  $a$  and  $b$  depend on the radial order  $n$  and are given in Table 6. This relation is similar to the normal PLC-relations such as those calculated by Stellingwerf (1979).

To apply these results to the three stars in Praesepe, we need preliminary estimates for their luminosities and effective temperatures, for which we use the values in Table 1. These values of  $L$  and  $T_{\text{eff}}$ , combined with Eq. 2, allow us to predict the frequencies of radial modes in the three stars. Comparison with the observations given in Tables 2, 4 and 5 makes it clear that if any of the observed modes are indeed radial, their  $n$  values must be as follows:  $n = 3$  or  $4$  for BU Cnc;  $n = 4, 5$  or  $6$  for BN Cnc; and  $n = 2$  or  $5$  for BV Cnc. This result is based on taking each observed frequency and using Eq. 2 plus  $L$  and  $T_{\text{eff}}$  (with uncertainties) to calculate allowed  $n$  values.

The next step is to calculate *ratios* between pairs of observed frequencies and compare with the models (Petersen 1978; Pérez Hernández et al. 1995). We do this because ratios are much less sensitive to changes in stellar parameters than are absolute frequencies. The identifications giving the best solutions are indicated in the last columns of Tables 2, 4 and 5. Indeed, they are the only solutions that give a good fit to these models.

Having made these identifications for each star, we can now adjust the luminosity in Eq. 2 (keeping  $T_{\text{eff}}$  constant) to achieve the best match between the observed and calculated frequencies of the radial modes. This gives the following estimates for the distance moduli of BU, BN and BV Cnc: 6.29, 6.21 and 6.32. The mean is  $6.27 \pm 0.03$ , which we can take as the distance modulus of the Praesepe cluster. This  $1\sigma$  uncertainty is based on the formal errors, but we stress again that we are neglecting rotation, which will certainly introduce systematic shifts to the frequencies.

With these luminosities, we now adjust the effective temperature in Eq. 2 for each star to achieve the best match between

the observed and calculated frequencies of the radial modes. The final results are shown in Table 7. We also show the masses estimated using Eq. 1 and the corresponding radii and surface gravities. The formal  $1\sigma$  uncertainties are about 4% in luminosity, 1% in effective temperature and 1% in mass. Again, note that uncertainties from neglecting the effects of rotation are not included.

HD 73712 is more luminous than the other three stars (Table 1) and is in the hydrogen shell burning phase. If we assume that the observed frequency of  $82.0 \mu\text{Hz}$  (Sect. 4.1) is a radial mode ( $\ell = 0$ ) then, based on an analysis similar to that described above, we identify this mode as  $n = 2$ . The resulting stellar parameters are given in Table 7.

Pérez Hernández et al. (1995) performed a seismic analysis of BU and BN Cnc by comparing the STEPPI frequencies with models constructed specifically for these stars. Their identifications of radial modes, shown in Tables 2 and 4, disagree with ours. The main reason is that Pérez Hernández et al. included a correction for rotation. This shows that rotation has an important effect on the interpretation of the data.

## 7. Conclusions

Our main conclusions are summarized below:

- By using statistical weights when calculating the amplitude (or power) spectrum of a time series, it is possible to reduce the noise level dramatically. This is demonstrated by our re-analysis of published STEPPI observations of BU Cnc, BN Cnc and HD 73712.
- Our CCD observations of a field in Praesepe demonstrate that, by obtaining defocussed images, it is possible to use a CCD to observe targets that are much brighter than the available comparison stars. We have also demonstrated the importance of measuring and correcting for any detector non-linearities.
- Results for the four stars are as follows:
  - BU Cnc: Our re-analysis of the STEPPI data improved the S/N by more than a factor of two. We recovered seven oscillation frequencies.
  - BN Cnc: Our re-analysis of the STEPPI data improved the S/N by more than a factor of four. We recovered five previously known oscillation frequencies and identified three new frequencies. We presented new CCD observations which confirmed the five strongest modes.
  - BV Cnc: We presented new CCD photometry which show this star to be a multi-periodic  $\delta$  Scuti star with four oscillation frequencies.
  - HD 73712: From our re-analysis of the STEPPI data, we detect a single peak at  $82.0 \mu\text{Hz}$ , confirming this star as a low-frequency  $\delta$  Scuti variable.
- Even in the absence of dedicated model calculations, it is possible to perform a simple seismic analysis by comparing observed frequencies with generic models. For this analysis we adopted solar metallicity for the Praesepe cluster and neglected the effects of rotation. For the three multi-periodic

variables, we derived luminosities with formal uncertainties of about 4% and effective temperatures and masses to about 1%.

It is important to note that our seismic analysis was based on assuming solar metallicity for Praesepe, and also on models which did not include rotation or overshoot. Future investigations should study the effects of these assumptions, and should also include all frequencies, not just those of the radial modes.

Further observational results for BN and BV Cnc will be produced by a large multi-site campaign in early 1998 with the STACC network (Small Telescope Array with CCD Cameras; Frandsen et al. 1997). This campaign will include spectroscopy to assist in mode identification (e.g., Viskum et al. 1998) and should provide valuable data for further theoretical modelling.

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