

A direct method to determine the geometry of the hollow cones in pulsars

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Abstract. The small dispersion in the observed relation between period and intrinsic opening angles of the conal emission regions in pulsars can be used to study their geometry. Applying this direct method shows the corresponding hollow cones to be consistent with being circular; an upper limit of approximately 10% is derived for the deviation from a circular geometry.

Previous investigations have relied on assumptions regarding the statistical properties of the pulsar sample used in the analysis. The disparate conclusions reached in the past concerning the geometry of pulsar beams is traced to the sensitivity of such indirect methods to the validity of these assumptions. It is further argued that there is no observational evidence for an evolution with pulsar age of the angle between the cone axis and rotation axis.

Key words: pulsars: general

1. Introduction

An understanding of the geometry of the pulsar emission beam would provide clues to the generation and dynamical evolution of the emitting plasma. Several attempts have been made to deduce the salient features of the beam from the observed properties of pulsar emission profiles. However, very different answers have been arrived at; for example, Narayan & Vivekanand (1983a) concluded that the pulsar beam is elongated in the latitudinal direction, Lyne & Manchester (1988) found no evidence for a non-circular beam, while Biggs (1990) argued for a beam which is compressed in the latitudinal direction. Due to the inability to extract sufficient information from the observations, the geometry could not be determined directly but relied on assumptions regarding the properties of the pulsar sample; for example, all pulsars were assumed to be basically the same. The geometry was then inferred from a statistical analysis of the distribution of various observational parameters. In order to derive reliable results from such a statistical approach, the sample must be well defined and its internal properties known; for example, the analyses of Lyne & Manchester (1988) and Biggs (1990) used the same pulsar sample and their different conclusions regarding the geometry of the pulsar beam stem mainly

from different assumptions about the importance of selection effects.

Rankin (1983) has shown that all pulsars are not intrinsically equal and, thus, should not be lumped together in a statistical analysis; for example, the relative strengths of the three constituents of the pulsar profile, the narrow, centre filled core component and the two conal pairs, can vary dramatically between different pulsars. The main parameter which determines their prominence in the pulsar profile is the period; on average, the core component is most prominent in short period pulsars, while the relative strengths of the conal pairs increase with period with the outer pair dominating for long period pulsars. Furthermore, Rankin (1990, 1993a) found that the intrinsic angular width of the core component as well as the opening angles of the hollow cones, which give rise to the observed conal pairs, depend, with remarkably small scatter, only on period.

The objective of this paper is to point out that Rankin's analysis of the properties of the core component provides the additional information needed for a direct study of the geometry of the hollow cones. Hence, for pulsars with core as well as conal emission, the geometry of the latter can be revealed without assumptions regarding the statistical properties of the sample. Some of the problems inherent in a statistical approach to the hollow cone geometry are briefly discussed in Sect. 2.1; in particular, it is emphasized that observational biasing is likely to play a role even when selection effects due to survey sensitivity limits are well understood. It is shown in Sect. 2.2 that the small dispersion of opening angles deduced from the conal pairs sets very stringent limits to deviation from circularity of the hollow cones. In Sect. 3 the implications of the results are discussed and a comparison is made between previous investigations and the present one. A few concluding remarks are given in Sect. 4.

2. The geometry of hollow cones

The standard assumption made regarding the magnetic field in the emission region is that it is dipolar and, thus, has a projection perpendicular to the line of sight, which is purely radial with respect to the magnetic axis. If the polarization of the radiation is either parallel or perpendicular to the projected magnetic field, the observed position angle (Ψ) is given by

sample. To illustrate the type of observational biasing which can influence the conclusions of a statistical approach to the cone geometry, consider a sample of pulsars assumed to have a uniform distribution of β values for $|\beta| \leq \Delta\rho_o$ (see Fig. 2; cf. Narayan & Vivekanand 1983a; Biggs 1990). The cone ellipticity is deduced from a comparison between the observed distribution of β_n and the expected probability distribution $P(\beta_n) \propto d\beta/d\beta_n$. After normalization and neglect of spherical effects (i.e., $\sin\rho \approx \rho$)

$$P(\beta_n) = \frac{1}{R(1 + \beta_n^2(R^{-2} - 1))^{3/2}}, \quad (6)$$

where R is the axial ratio of the ellipse; hence, $R > 1$ and $R < 1$ corresponds, respectively, to elongation of the beam in the latitudinal and longitudinal direction.

There are two types of intrinsic biasing which can invalidate the assumption of a uniform distribution of the unobserved parameter β . First, in pulsars where the flux is dominated by the conal pair(s) a uniform distribution of β values requires the observed flux to be independent of β_n . However, the two components in a conal pair can show rather large variations in their relative flux indicating an uneven intensity distribution along the rim of the cone; for the most extreme cases it has been suggested that the cone is only partial (Lyne & Manchester 1988). Furthermore, a conal pair with equal flux in its two components does not preclude an intrinsic variation of the cone intensity in the latitudinal direction. Second, even for a uniform intensity distribution along the rim of the cone, biasing of the β values can occur for pulsars whose flux is dominated by core radiation. For some of them the flux in the conal pair(s) is smaller than the detection limit of a particular survey; hence, these pulsars are detected only because of their large flux in the core component. The inclusion of such pulsars in a sample depends on the intensity distribution of the core emission in the latitudinal direction, which is not well known. In analogy with its longitudinal distribution, it is usually assumed that the core emission decreases monotonically with increasing angular distance from the magnetic axis; if true, this would lead to detection of pulsars preferentially with small values of β , which, from Eq. (6), results in an artificially decrease of the R -value.

2.2. A direct method

Previous attempts to determine the beam geometry have been affected by unknown selection effects and/or biasing due to the lack of a third independent observable. This situation has changed with Rankin's (1990) finding that the observed width of the core component (W_{obs}) is determined only by α and the pulsar period. Hence, W_{obs} provides a third independent observable and for those pulsars which have a core component as well as conal pair(s), the geometry of the hollow cones can be investigated without the need for any assumptions about the statistical properties of the pulsar sample.

The method employed by Lyne & Manchester (1988) to derive separate values for α and β relies on the assumption of circular beams. Likewise, the use of W_{obs} as a third independent observable entails assumptions in addition to the validity of the

RC-model. However, in contrast to Lyne & Manchester (1988), the recognition of distinct beam components (core, inner and outer cone) makes it possible to judge the consistency of these assumptions; i.e., whether values obtained for one component give a consistent description of the others. Therefore, before using the values of W_{obs} to study the geometry of the hollow cones, such a consistency check is appropriate.

Rankin identified six pulsars which have both a measurable core component width and are believed to be orthogonal rotators (e.g., they all have interpulses). The relation between the observed width and period found for these pulsars provided an excellent lower bound to the distribution of the whole core component sample. Furthermore, the small scatter around this relation implies that the intrinsic width of the core component (W_c) depends only weakly, if at all, on β . As mentioned by Rankin (1990), a bivariate Gaussian form of the core emission is consistent with the observations. Assuming W_c to be independent of α as well as β allowed Rankin (1990) to calculate α from $\sin\alpha = W_c/W_{\text{obs}}$. Note that the assumption of no β dependence in W_c does not imply a circular core beam (i.e., isophotes), since W_c is a measure of the isophotal width in the longitudinal direction only. This is in contrast to the lack of a β dependence in $\Delta\rho$, which does imply a circular hollow cone, since $\Delta\rho$ measures (cf. Fig. 2) the width between the magnetic axis and the rim of the cone (i.e., it depends on the longitudinal as well as the latitudinal extension of the cone).

In pulsars with both core and conal pair(s), the width $\Delta\rho$ of the hollow cone(s) can be *calculated* from Eqs. (2) & (4) with the use of α , derived from the core component, and β from the value of $d\Psi/d\phi_m$. Rankin (1993a) found that both of the conal pairs follow a width–period relation ($\Delta\rho_{\text{obs}} \propto P^{-1/2}$), similar to the core component, with remarkably small scatter. Although it is hard to estimate the accuracy of the derived $\Delta\rho$ values (Rankin, private communication), the scatter is consistent with measurement errors. It should be noted from Eq. (4) that the scatter in $\Delta\rho$ provides an upper limit to the errors made in the determination of α ; hence, the small scatter in $\Delta\rho$ strengthens the validity of the method used to obtain the α values.

The main point of the present paper is that a non-circular hollow cone (i.e., a β dependence of $\Delta\rho$) contributes to the scatter of $\Delta\rho$ values around the deduced $\Delta\rho_{\text{obs}}$ -versus- P relation. However, the deviation of $\Delta\rho$ from $\Delta\rho_{\text{obs}}$, due to a non-circular hollow cone, is not random but depends on β and R . Hence, the value of R can be determined directly from observations. The expression for $\Delta\rho$ is given by (cf. Fig. 2)

$$\Delta\rho(\beta_n) = \frac{\Delta\rho_o}{R(1 + \beta_n^2(R^{-2} - 1))^{1/2}}, \quad (7)$$

where $\Delta\rho_o \propto \Delta\rho_{\text{obs}} \propto P^{-1/2}$. In principle, all the measured $\Delta\rho$ -values could be used together. However, in practice, the accuracy to which the width of a conal pair can be measured depends on its prominence relative to the other components; hence, the different classes of pulsars in Rankin's classification scheme are not equally good probes of the conal geometry. The most stringent constraints on the conal geometry are obtained from classes for which the measurement errors are expected

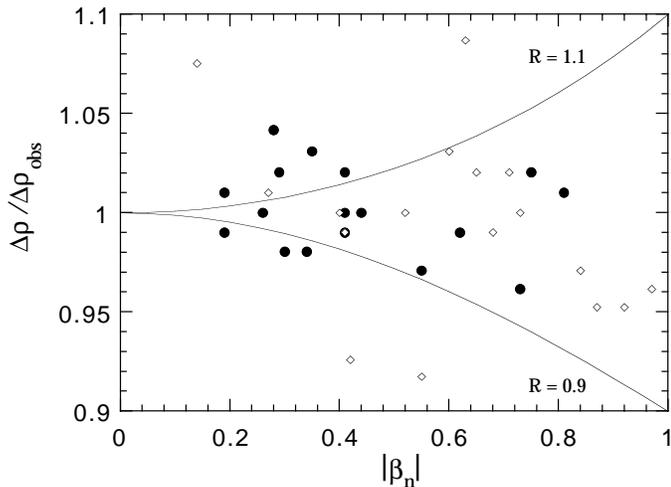


Fig. 3. The measured widths of the hollow cones ($\Delta\rho$) normalized to the relation ($\Delta\rho_{\text{obs}}$) found by Rankin versus the absolute value of the impact angle $|\beta_n|$ (see text). Dots correspond to **M**-pulsars and diamonds to **S_t**-pulsars. Data have been taken from Rankin (1993b). The curve $(1 + \beta_n^2 (R^{-2} - 1))^{-1/2}$ is drawn for $R = 0.9$ and 1.1 , respectively.

to be smallest while, at the same time, containing a sufficient number of pulsars for the result to be significant. The two most suitable classes of pulsars in this respect are the **M**-pulsars (outer cone) and **S_t**-pulsars (inner cone; note that roughly 2/3 of all **S_t**-pulsars have conal emission at frequencies above 1 GHz). In Fig. 3, $\Delta\rho$ is plotted as a function of $|\beta_n|$ for these two classes. Data are taken from Rankin (1993b) and contain all pulsars with measured β values. In order to guide the eye, the function $(1 + \beta_n^2 (R^{-2} - 1))^{-1/2}$ has been drawn in Fig. 3 for $R = 0.9$ and 1.1 . Least square fits of Eq. (7) to the data in Fig. 3 give $R^{-2} - 1 = 0.013 \pm 0.057$ ($R = 0.99^{+0.03}_{-0.02}$) and $R^{-2} - 1 = 0.103 \pm 0.091$ ($R = 0.95^{+0.04}_{-0.03}$), respectively, for **M**- and **S_t**-pulsars, where the errors correspond to 1σ .

The other pulsar classes give similar, although somewhat less restrictive, results. The pulsars used to estimate R have an average α of $\sim 40^\circ$. In the model considered by Biggs (1990) this gives a conal geometry corresponding to $R \approx 0.90$. As compared to the best measured pulsars (**M**-pulsars), such a value of R is excluded at the 4σ level.

3. Discussion

The relations found by Rankin between period and intrinsic component widths/opening angles offer a direct method to study the geometry of the hollow cones without the need to rely on assumptions about the statistical properties of the pulsar sample. Furthermore, her classification scheme opens the possibility for an internal consistency check on the assumptions, in addition to the validity of the RC-model, which are needed to derive the geometry. It is emphasized in Sect. 2.2 that Rankin's assumptions regarding the salient properties of the core component (at a given period, the intrinsic width is independent of both inclination angle (α) and impact angle (β)) are entirely compatible

with a geometric interpretation of the conal emission. The deduced (Sect. 2.2) circularity of the hollow cones accounts for the similar values of α and β derived by Lyne & Manchester (1988) and Rankin using different methods; Lyne & Manchester assumed a circular beam in order to derive α and β , while the values derived by Rankin's independent method imply circular cones.

Gil and Han (1996) made a statistical study of pulsar properties by assuming all pulsars to be intrinsically equal. Combinations of different trial distributions for various parameters were used together with Monte Carlo simulations to calculate the distributions of observable quantities. The intrinsic limitations of such an indirect method were made less severe by a large set of data and the simultaneous prediction of several observables. Although the precision with which the different parameters could be determined was somewhat restricted, it is interesting to note that the highest probability of occurrence was given to combinations in which the hollow cones were assumed not to deviate too much from circular.

Previous studies regarding the geometry of pulsar beams have mostly been of a statistical nature. One exception is the direct method suggested by Manchester & Lyne (1977); for pulsars in which the interpulse arises from the same magnetic pole as the main pulse and the line of sight passes through the same beam component twice (i.e., $R > 1$ is a prerequisite), fitting Eq. (1) to the observed variation of the polarization angle provides enough information to study the beam geometry directly for individual pulsars. Narayan & Vivekanand (1983b) and Phillips (1990) deduced from such an analysis large R -values for PSR B0950+08 and B1929+10, respectively. However, the basic assumption of these conclusions has been questioned; for example, Gil (1983, 1985) argued for a model in which the main- and interpulse of PSR 0950+08 come from different beam components and Rankin (1990) has interpreted the main- and interpulse in PSR 1929+10 as coming from different poles.

The distinction between the core component and conal pairs and the realization that they have very different intrinsic properties can also be used to discuss a few related issues regarding the pulsar beam geometry. The observed variation of Ψ in the core component is generally much smaller than expected from the RC-model (e.g. Rankin 1990). Together with the fact that the width of the core component is considerably smaller than either of the conal pairs', this mocks a situation where pulsars with only core emission (**S_t**) represent lines of sight which just touch the outer part of a hollow cone. The unawareness of this special property of the core emission leads to a misclassification of **S_t**-pulsars which, in turn, causes an overabundance of small values of $\Delta\Psi$ or equivalently, from Eq. (3), to an overestimation of the value of β ; the resulting distribution of either $\Delta\Psi$ or β values then implies an artificial elongation of the beam in the latitudinal direction (cf. Eq. 6).

Although the large values of R deduced by Narayan & Vivekanand (1983a) are due partly to an underestimation of the observed values of $\Delta\Psi$ (Lyne & Manchester 1988), an important contribution comes from the above effect; in particular,

this is true for the period dependence of R found by Narayan & Vivekanand (1983a). Their sample of pulsars was divided into three groups according to the period. In the group with the shortest periods, 6 out of 10 pulsars are classified as S_t by Rankin (1993a), in the middle group 3 out of 10, while there is no S_t -pulsar in the long period group. Hence, the increase of R with decreasing period found by Narayan & Vivekanand (1983a) is likely due to the increasing fraction of S_t -pulsars. It is worth noticing (cf. Sect. 2.2) that Narayan & Vivekanand (1983a) obtained $R \sim 1$, albeit with large uncertainties, for the long period group, which is uncontaminated by S_t -pulsars.

Lyne & Manchester (1988) calculated $\Delta\rho_{90} \equiv \Delta\rho(\alpha = \pi/2)$ from Eqs. (2) & (4) and plotted it versus period (P) without distinguishing the different classes as defined by Rankin. A lower bound to this distribution was determined by an eyeball fit. Since the calculated value of $\Delta\rho$ increases with α (cf. Eq. 2), the lower bound corresponds to pulsars which actually have $\alpha = \pi/2$ and, consequently, it also measures the intrinsic width ($\Delta\rho$) of the beam. Assuming further that $\Delta\rho$ is independent of β (i.e., a circular beam) as well as α , the spread in $\Delta\rho_{90}$, at a given value of the period, is due to variation in α and the value of α can be obtained from Eqs. (2) & (4). As mentioned in Sect. 1, Rankin's pulsar classification is based on the relative prominence of the core component, inner and outer conal pair. Neglect of the fact that these constituents of the pulsar profile have different intrinsic widths causes two spurious effects to occur. First, in a $\Delta\rho_{90}$ -versus- P diagram the lower bound will be shallower than the lower bound appropriate for each class of pulsars separately. This effect contributes to the shallower $\Delta\rho_{\text{obs}} \propto P^{-1/3}$ relation obtained by Lyne & Manchester (1988) as compared to $\Delta\rho_{\text{obs}} \propto P^{-1/2}$ derived by Rankin for both the hollow cones and core component. Second, for a given period, pulsars dominated by core emission (S_t), which has the smallest intrinsic width, will, on average, lie closer to the lower boundary than do pulsars with prominent conal pairs, especially those with an outer conal pair. This results in an artificial correlation between the average value of α and pulsar class. In Fig. 4, pulsars classified by Rankin (1990, 1993a) have been used to plot fiducial age ($P/2\dot{P}$) versus period for three different classes of pulsars; in each of the classes the pulsar profile is dominated by one of its constituents. It is seen that, for a given period, the average fiducial age of pulsars tends to correlate with their classification; in particular, pulsars with an outer cone are on average considerably older than those without. Taken together with the above mentioned artificial correlation between the average value of α and pulsar class, this implies that treating all pulsars on an equal footing introduces an artificial, inverse correlation between α and pulsar fiducial age. The different conclusions regarding the evolution of α reached by Lyne & Manchester (1988) and Rankin (1990) can presumably be referred to this effect; hence, there is no evidence that the preferred value of α ($\sim 30^\circ - 40^\circ$) evolves with pulsar age; except that there is a secondary, narrow peak around $\alpha = \pi/2$ for young pulsars. Likewise, the finding of Biggs (1990) that the values of β decrease with decreasing α , need not indicate an increasing latitudinal compression of the beam for small val-

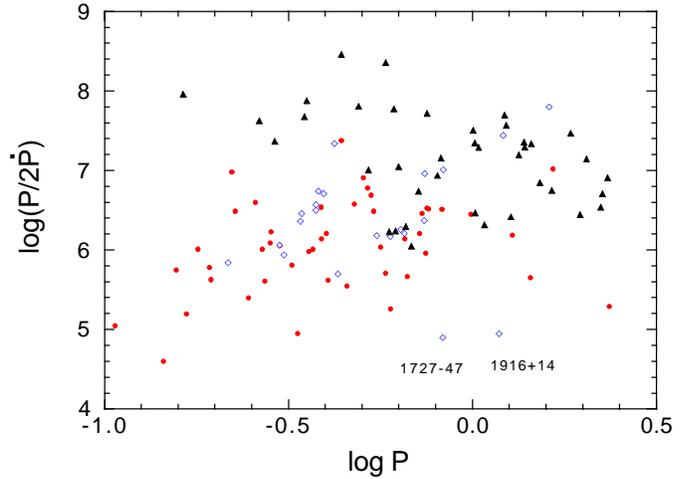


Fig. 4. Fiducial age ($P/2\dot{P}$) versus period (P) for pulsars classified by Rankin (1990, 1993a). Dots correspond to pulsars with only core emission at 1GHz, diamonds to pulsars with an inner but no outer cone, and filled triangles to pulsars with an outer cone. The data have been taken from Rankin (1993b) and include pulsars with $P > 0.1$ s. Note that PSR B1727-47 and PSR B1916+14 have uncertain classification

ues of α ; instead, the effect can be explained as a result of the artificially large α deduced for pulsars with intrinsically small widths.

4. Concluding remarks

The main result of the present paper is that there is little room for the geometry of the hollow cones to deviate from circular ($< 10\%$). This conclusion rests only upon the observed small dispersion of opening angles for the hollow cones at a given period and is not affected by possible incompleteness or biasing in the pulsar sample. The core emission is less straightforward to analyse, mainly because, in contrast to the cone geometry, the determination of the core geometry involves measuring and comparing intensities not angles; i.e., the geometry of a centre filled emission beam is deduced from the angular distribution of a set of isophotes. Hence, the accuracy by which the intrinsic latitudinal intensity distribution can be determined depends, for example, on the luminosity function of the core emission and its possible dependence on α . In order to discern the expected decrease of core intensity with β , a pulsar sample is needed which is large enough to overcome the intrinsic spread of core luminosities. A useful simplification in such an analysis is indicated by Rankin's result that the intrinsic longitudinal width of the core emission depends neither on α nor β . This restricts acceptable geometries and suggests a one parameter description of the intensity distribution to be appropriate also in the latitudinal direction.

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