Cepheid metallicity and Hubble constant

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Abstract. This study combines data from a set of seven Cepheid-calibrated galaxies in Virgo, Leo I and Fornax and four supernovæ with known metallicities, including metallicity corrections to the Cepheid distances, in order to determine a consistent mean value for the Hubble constant. We derive Virgo and Coma distances over different paths, starting from the metallicity corrected Cepheid distances of Virgo and Leo I galaxies, and we relate them to the Virgo and the Coma recession velocities, paying special attention to thereby introduced correlations. We also determine the velocity of the Fornax cluster and relate this velocity to its distance. In addition we use the standard constant maximum brightness SN Ia method, as well as results from multicolor light curve shape (MLCS) analysis of SNe Ia, to derive the Hubble constant. Using the requirement of statistical consistency, our data set determines the value of the metallicity coefficient (the change in magnitude per factor of ten in metal abundance) to be $0.31^{+0.15}_{-0.14}$ in good agreement with other known determinations. We also use consistency as a criterion to select between mutually exclusive data sets, such as the two known, but conflicting, Virgo recession velocities (in favor of the lower value), as well as between two different analyses of (partly) the same supernovæ (in favor of MLCS analysis). When combining data we pay attention to the statistically correct handling of common errors. Our result is a Hubble constant with the value $H_0 = 68 \pm 5$ km s^{-1} Mpc^{-1}.

Key words: distance scale – stars: variables: cepheids – methods: statistical – galaxies: distances and redshifts – galaxies: abundances

1. The metallicity dependence

The value of the Hubble constant H_0 has been notoriously controversial, not only when determined by different methods, but even values resulting from the application of a single method to different objects are often in glaring conflict. The statistical error is of course large for small data samples, but in addition most determinations depend on secondary parameters which are ill known or guessed and which lack reliable error estimates, systematic errors for short.

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In this paper we address ourselves to the problem of determining H_0 using data on galaxies with Cepheid-calibrated distances. Then two important sources of systematic errors can be discerned: the effects of the metallicity of the Cepheid on the period-luminosity (PL) relation, and the determination of the distance to the Large Magellanic Cloud (LMC). Our main interest in this paper is to examine the metallicity effect, and we keep the LMC distance modulus at its classical value 18.5 ± 0.1 mag (Madore & Freedman, 1991). In Sect. 7 we discuss recent developments of determining LMC distance, and check how these will affect our results.

The Cepheid PL relation has been calibrated in the LMC by Madore & Freedman (1991). For quite some time it has been speculated that the Cepheid brightness and therefore the zero point of the PL relation is affected by the metallicity of the calibrating Cepheids (for a good review see Sasselov et al., 1997). Sometimes observations have refuted and sometimes confirmed the existence of this effect. The idea is that metal richer Cepheids would be brighter and therefore lie further away than what the standard PL analysis implies. However, the brighter Cepheids are also thought to be intrinsically redder, which will complicate the analysis, and make the change of distance to depend on wavelength, or in practise, the bandpasses used in the photometric analysis. Throughout this paper we talk about the metallicity effect to Cepheid distance modulus obtained using V and I bands, because that is the choice of Hubbe Space Telescope teams.

Although the shift in distance depends in theory (Stothers, 1988) on the abundance of heavy elements, the only actually observable "metallicity" is usually the oxygen abundance of surrounding HII regions. Therefore in this paper we actually address ourselves to the systematic shift due to the [O/H] dependence of the distance moduli. Assuming that the ratio of oxygen abundance of HII regions to the corresponding value of LMC, equals the corresponding ratio of heavy element abundances, we interpret the [O/H] - effect as a metallicity effect.

For the metallicity correction of the distance modulus of a Cepheid we use a form

$$\delta\mu = \gamma \left([O/H] - [O/H]_{LMC}\right). \tag{1}$$

$\delta\mu$ is the correction needed to the distance modulus obtained by standard VI photometry ($\mu_{true} = \mu_{standard\,VI} + \delta\mu$). [O/H]

denotes the logarithm of the abundance of oxygen (by number) relative to hydrogen, in Solar units. The logarithmic dependence in the above formula follows Sasselov et al., (1997). The constant γ gives the correction needed for the distance modulus in magnitude per factor of ten in metal abundance (mag/dex).

The early results (Freedman & Madore, 1990) indicated a small effect, which was later contested by Gould (1994) who found a much larger γ. At that time the color variation was not accounted for. More recently, three new determinations have become available: Beaulieu et al. (1997) and Sasselov et al. (1997) in the EROS project analyzed very large samples of Cepheids in the LMC and the SMC and published a γ value for the Z difference. We use $\gamma = 0.48^{+0.1}_{-0.2}$ mag/dex (Sasselov, priv.comm.) which corresponds to the [O/H] difference between LMC and SMC. In statistical agreement with this Kennicutt et al. (1998) have reported $\gamma = 0.24 \pm 0.16$ mag/dex from a study targeting Cepheids in two fields in M101. The third result, $\gamma = 0.4 \pm 0.2$ mag/dex, is due to Kochanek (1997) who used multicolor photometry to determine 694 Cepheid distances in 17 galaxies, including the effects of temperature, extinction and metallicity. His simultaneous fitting approach to actual photometry data is perhaps the currently best way to address the question of metallicity effects on the cepheid PL relation, but it is rather complicated (a multi-parameter fitting procedure in a Bayesian approach). The approach adopted by us is different, we do not consider the systematic errors of the actual photometry, but instead we put emphasis on the error correlations of photometry results (magnitudes) and on error correlations arising from the metallicity correction in the averaging procedure.

Beaulieu et al. (1997) and Sasselov et al. (1997) applied their metallicity correction to reevaluate the Hubble constant for Cepheids in HST-observed galaxies with reported metallicities, and demonstrated that some of the previous disagreement between the H_0 values was indeed removed.

In the present paper we investigate whether the inclusion of the metallicity correction is enough to produce a statistically consistent H_0 average for a larger sample of galaxies, or whether inconsistencies remain, indicating the presence of yet further systematic errors. Our sample consists of 7 galaxies with known distances, velocities and metallicities, and 4 supernovæ with known V and B peak absolute magnitudes and metallicities.

2. The data sets

The galaxies we have selected for this study fall into five sets due to differences in the method of analysis. All have Cepheid-calibrated distances to which we apply metallicity corrections, and their metallicities are all known.

Most of the metallicity values are from Kochanek (1997) taking the positions of the Cepheids and the metallicity gradient across the galaxies into account. We have estimated the metallicities for NGC 4571 and NGC 1365 from Skillman et al. (1996) and Zaritsky & al. (1994), respectively. To all the metallicity values we assign an error of 0.15 dex, following Kennicutt et al. (1998). This value also encompasses all the observational [O/H] errors of these galaxies (Zaritsky et al., 1994).

The five data sets (with our code names) are briefly summarized below; more detail can be found in Table 1 and in Sect. 4.

Virgo (V) Four galaxies in the Virgo cluster, NGC 4321 (M100), NGC 4496A, NGC 4536, and NGC 4571, for which we take the distance moduli and errors from Freedman (1996) (however, M100 from Ferrarese et al. 1996).

Leo (L) Two galaxies in the Leo I group, NGC 3368 (M96) and NGC 3351. We take the distance moduli from Tanvir et al. (1995) and Graham et al. (1997), respectively.

Fornax (F) One galaxy in the Fornax cluster, NGC 1365, for which the distance modulus (adding the 3 % uncertainty of the distance of NGC 1365 from the Fornax center) is given by Freedman (1996).

Sandage (S) Four galaxies hosting recent Ia type supernovæ (ignoring historical ones which may be dubious), NGC 4496A (SN 1960F), NGC 4536 (SN 1981B), NGC 4639 (SN 1990N), and NGC 5253 (SN 1972E). The V and B peak absolute magnitudes are from Sandage et al. (1996).

Riess (R) Three of the supernovæ listed above (all except SN 1960F) have been analysed by Riess et al. (1996) by an empirical method that uses multicolor light-curve shapes.

All the distance moduli depend on the LMC modulus which has commonly been taken to be 18.50 ± 0.10 mag. We keep this value, but we discuss the recent developments of measuring the LMC modulus, and its effect on our results in Sect. 7.

In Table 1 we give the distance moduli and metallicities of the galaxies used as well as the V and B peak absolute magnitudes of the supernovæ. The error due to the LMC modulus error has not been included in the values tabulated.

For most of the time we work with the logarithm of the Hubble constant rather than with H_0 itself. For the galaxies in the sets V, L, F (each galaxy subscripted g) $\log H_0$ has the form

$$\log H_{0,g} = \log(zc) - 0.2(\mu_g + \delta\mu_g) + 5 , \qquad (2)$$

where zc is the nonrelativistic recession velocity, μ_g is the distance modulus, and $\delta\mu_g$ is the metallicity correction to the distance modulus from Eq. (1). In this formula we have suppressed all the numerous error terms mentioned before.

The treatment of the supernovæ in set S is different. We have

$$\log H_{0,g} = 0.2[M_V(max)_g - \delta\mu_g] + \alpha_V , \qquad (3)$$

where $M_V(max) = m - \mu$ is the V peak absolute magnitude, and analogously for the B data. The constant α takes the values $\alpha_V = 5.658 \pm 0.011$ and $\alpha_B = 5.637 \pm 0.011$ (Tammann & Sandage, 1995). The two errors just happen to be equal, but are uncorrelated.

The treatment of the supernovæ in set R is the following. Riess et al. (1996) determine accurate relative distances to 20 SNe Ia using their MLCS method, tying their distances to the absolute Cepheid distance scale via SNe 1972E, 1981B and 1990N, leading to their result H = 64 $\pm$6 km/s/Mpc. We take into account the metallicity values of these three SNe , correcting the Riess et al. average distance to them, and thus correcting also their value of the Hubble constant starting from their value of $\log H_0(\gamma = 0)$, thus

$$\log H_0 = \log H_0(\gamma = 0) - 0.2\langle\delta\mu\rangle . \qquad (4)$$

The supernova data in the sets S and R are to some extent overlapping, thus the derived H_0 values are correlated and should not be averaged. The methods are, however very different, and so are the results. Using our consistency requirement we want to test whether either set can be combined with sets V, L, and F.

3. The strategies

With the given data sets there are several questions that we can answer. For each question a different strategy has to be followed; in particular, the treatment of common errors is different.

The first question is obviously what value of the Hubble constant each data set determines separately, let us call this Strategy I. If these values should be final, they must contain all known errors, including the common ones such as the LMC distance error. On the other hand, they can then not be combined into a final average, because that requires that common errors be included only after averaging. Thus final averaging requires a different strategy, let us call that Strategy IV.

The purpose of Strategy II is to distinguish between conflicting data sets. As already mentioned, the correlation between the data sets S and R does not permit both of them to be included in a final average, thus a choice must be made. Another choice which has to be made is between the two conflicting values of the Virgo velocity, in respect to the Local Group, 1179 ± 17 km/s (Sandage & Tammann 1990) and 1404 ± 80 km/s (Huchra 1988) which cannot be combined. One of them (at least) must be wrong. The small error of the low Virgo velocity is probably significantly underestimated (Huchra 1997), but for the time being, due to the lack of a better estimate, and since the distace error dominates over the velocity error for Virgo, we keep the published value.

Before defining Strategy III, let's make a brief digression. The coefficient γ in the metallicity correction term $\delta\mu$ is a quantity common to all data, thus causing correlations between different galaxies. To handle this, we treat γ (in Strategies I and II) as a variable rather than as a known input number with errors. If we let γ take values in the range 0.0 – 1.0 [mag/dex], we cover most of the published values. Thus the Hubble constant is a function $H_0(\gamma)$. This permits us to define a Strategy III where we use the likelihood function formed by all individual galaxies to "determine" our own best value of γ.

Clearly, the questions answered by Strategies II and III influence the way the final average is formed in strategy IV. There we combine our γ value with the independently determined value of Sasselov et al. (1997), and Kennicutt et al. (1998), and determine the best value of the Hubble constant from the position of the maximum of the likelihood function, or actually the minimum of the chi-square sum $d\chi^2(\gamma)/d\gamma = 0$. Finally we add all common errors neglected up to this point.

4. Calculations

4.1. Leo-Virgo and Fornax galaxies

The Hubble constant can be determined from the sets V and L by two routes: using the Virgo recession velocity or the Coma recession velocity. In addition, the distance to the Virgo center can be determined directly from Virgo galaxy distances, or from Leo galaxy distances using published values for the Leo-Virgo distance. The distance to Coma can be measured from the above determined Virgo distance, using published values for the Virgo-Coma distance, or from the Leo galaxy distances using published values for Leo-Coma distance.

Making use of all combinations requires a calculational scheme with quite complicated correlations which we must take into account exactly. Errors in common with the distance of the Virgo center are the uncertainty of the galaxy distances from the Virgo center, and a similar common error exists for the distance of the Leo center. All individual determinations of the Virgo distance are affected by the uncertainty due to the recession velocity error (as in Freedman et al. 1994).

All individual Cepheid distances are further affected by the common LMC distance error and the errors related to the metallicity correction term. As already said, to all the observed metallicity values we assign, in lack of better information, one and the same error of 0.15 dex, but that does not make it a common error. The error in γ is indeed common, but we do not introduce it until in Strategy IV.

The set F has its own errors which are not correlated to the previous ones, except for the LMC distance error and the γ error. Here the same comment applies as above. Note in particular that this set is independent of the conflicting Virgo velocities.

In order to describe the averaging procedures compactly, we introduce the notation $\langle A; B \rangle$ for the weighted mean of two quantities A and B with errors $\sigma(A)$ and $\sigma(B)$, variances $V(A) = \sigma(A)^2$ and $V(B) = \sigma(B)^2$, covariance $\mathrm{cov}(A, B)$, and correlation coefficient $\mathrm{corr}(A, B) = \mathrm{cov}(A, B)/\sigma(A)\sigma(B)$. For the weighted mean of a set of quantities A_i we write simply $\langle A \rangle$. Let us proceed in numbered steps.

1) Compute the mean metallicity-corrected distance moduli of the Virgo galaxies, set V

$$\langle \mu_V \rangle = \langle (\mu_g + \delta\mu_g)_V \rangle \tag{5}$$

and the mean metallicity-corrected distance moduli of the Leo galaxies, set L

$$\langle \mu_L \rangle = \langle (\mu_g + \delta\mu_g)_L \rangle . \tag{6}$$

The weights are the individual uncorrelated variances $V(\mu_g)$ of the distance moduli, adding the combined effect of [O/H] - $[O/H]_{LMC}$ errors and the γ errors in quadrature. In Table 1 we only include the $V(\mu_g)$ errors $\sigma(\mu_g)$.

2) Let us denote the Coma velocity $v_C = (7200 \pm 100)$ km/s (Freedman et al., 1994, and references therein) and the Virgo velocity v_V. In combining the Virgo and Coma center distances with the Coma and Virgo velocities, we use both conflicting v_V

Table 1. Input data

Galaxy (supernova)	Group	$\langle\mu\rangle$ [mag]	$[O/H]-$ $[O/H]_{LMC}$	$M_B(max)$ [mag]	$M_V(max)$ [mag]
NGC 4321	Virgo	31.04 ± 0.17	0.84		
NGC 4496A (SN 1960F)	Virgo	31.13 ± 0.10	0.00	-19.52 ± 0.14	-19.61 ± 0.20
NGC 4571	Virgo	30.87 ± 0.15	0.70		
NGC 4536 (SN 1981B)	Virgo	31.10 ± 0.13	0.00	-19.29 ± 0.13	-19.32 ± 0.12
NGC 4639 (SN 1990N)	Virgo		0.10	-19.30 ± 0.23	-19.39 ± 0.23
NGC 3368	Leo I	30.32 ± 0.16	0.69		
NGC 3351	Leo I	30.01 ± 0.19	0.94		
NGC 1365	Fornax	31.32 ± 0.20	0.10		
NGC 5253 (SN 1972E)	Centaurus		-0.25	-19.55 ± 0.23	-19.50 ± 0.21

values, thus we will have two sets of results: high v_V results (denoted by HI) and low v_V results (denoted by LO). Below we also need the three distance moduli, $\mu_{LV} = 0.99 \pm 0.15$ for the Leo–Virgo distance, $\mu_{LC} = 4.90 \pm 0.34$ for the Leo–Coma distance, and $\mu_{VC} = 3.75 \pm 0.1$ for the Virgo–Coma distance (Tanvir et al. 1995 and references therein). Since these three values have been determined independently, we are not surprised that $\mu_{LC} \neq \mu_{LV} + \mu_{VC}$, but we note that both sides of the equation are consistent within the errors. Then the logarithm of the combined H_0 measurements for the sets V and L can be written

$$\log H_{0,VL} = \langle\log H_{0,V}; \log H_{0,C}\rangle, \tag{7}$$

where

$$\log H_{0,V} = \log v_V - 0.2\langle\langle\mu_V\rangle; \langle\mu_L\rangle + \mu_{LV}\rangle + 5, \tag{8}$$

and

$$\log H_{0,C} = \log v_C - 0.2\langle\langle\langle\mu_V\rangle; \langle\mu_L\rangle + \mu_{LV}\rangle + \mu_{VC};$$
$$\langle\mu_L\rangle + \mu_{LC}\rangle + 5. \tag{9}$$

Here we do not spell out the various error terms: the LMC magnitude error ±0.10, the Virgo back-to-front position error of ±0.35 mag (Freedman et al. 1994), the distance uncertainty ±0.11 mag due to the Virgo recession velocity error, the uncertainty of the Leo galaxy distances from the center of Leo I group, ±0.04 mag (Tanvir et al. 1995), the observational uncertainty of the O-abundance of a HII region (O/H error), and the uncertainty involved in identifying the heavy element abundance of a Cepheid sitting in a HII cloud with the O-abundance of the surrounding HII cloud (HII-to-ceph error). At this stage we omit the last error, we check its effect in the Discussion.

3) The correlation coefficients needed are found by numerical integration of the expressions above. The correlation coefficient corr($\langle\langle\langle\mu_V\rangle; \langle\mu_L\rangle + \mu_{LV}\rangle + \mu_{VC}, \langle\mu_L\rangle + \mu_{LC}\rangle$) is of the order of 4–9% for the metallicity range $\gamma = 0.0$–1.0. The correlation coefficient corr($\langle\log H_{0,V}, \log H_{0,C}\rangle$) is of the order of 44–57% if the LMC error is neglected, and about 66% with the LMC error included.

The combined Virgo-Leo results containing the LMC error 0.10 mag (Strategy I) are tabulated in Table 2. Since we do not present results separately for sets V and L, we here introduce a different grouping: the results $H_{0,HI}$ on the first line are obtained by combining the Coma results with the results using high Virgo velocity; the results $H_{0,LO}$ on the second line by combining the Coma results with the results using low Virgo velocity.

4) The Fornax galaxy NGC 1365 forming the set F is by itself uncorrelated to the previous sets. Thus the Hubble constant can be worked out directly. The Fornax galaxy velocity data (kindly provided by J.Huchra), consist of 119 galaxies with velocities smaller than 3000 km/s. The mean velocity and its error from the velocity dispersion are 1459 ± 35 km/s (consistent with Madore et al. 1998). From this error we remove the "fictitious" effect of observational errors of galaxy velocities (Danese et al. 1980), yielding a true velocity error of 25 km/s. Correcting for the Virgo-centric flow with -40 ± 20 km/s, and converting the velocity to the Local Group by -147 ± 10 km/s, gives us the final expansion velocity of Fornax as 1272 ± 34 km/s. The results for the γ-dependent Hubble constant $H_{0,F}$ including the LMC error are tabulated on the third line of Table 2.

4.2. Supernovæ

5) The supernova set S is averaged as follows

$$\log H_{0,S} = \langle0.2\langle[M_V(max) - \delta\mu]\rangle + \alpha_V; \tag{10}$$
$$0.2\langle[M_B(max) - \delta\mu]\rangle + \alpha_B\rangle. \tag{11}$$

The results for the γ-dependent Hubble constant $H_{0,S}$ including the LMC error are tabulated on the fourth line of Table 2.

6) For the supernova set R we simply take the published H_0 value. Its error contains the effect of an LMC distance modulus error 0.15 mag, which we decrease to 0.10 mag to be consistent with the previous part of our analysis, and further we add the metallicity correction $\langle\delta\mu\rangle$. The γ-dependent averages

$$\log H_{0,R} = \log H_0 - 0.2\langle\delta\mu\rangle \tag{12}$$

are tabulated on the fifth line of Table 2.

This concludes the calculations in Strategy I, which are summarized as values of $H_0(\gamma)$ including the LMC error in Table 2.

Table 2. $H_0(\gamma)$ for all data sets (H in units of km s^{-1} Mpc^{-1}, γ in units of mag/dex)

γ	0.0	0.2	0.4	0.6	0.8	1.0
$H_{0,HI}$	80.5 ± 5.0	77.9 ± 4.9	75.2 ± 4.8	72.2 ± 4.7	69.1 ± 4.6	65.9 ± 4.6
$H_{0,LO}$	73.8 ± 4.1	71.5 ± 4.0	69.1 ± 3.9	66.5 ± 3.9	63.6 ± 3.9	60.6 ± 3.8
$H_{0,F}$	69.2 ± 7.4	68.6 ± 7.4	68.0 ± 7.5	67.3 ± 7.7	66.7 ± 8.0	66.1 ± 8.4
$H_{0,S}$	58.3 ± 3.3	58.4 ± 3.3	58.4 ± 3.4	58.5 ± 3.4	58.6 ± 3.5	58.7 ± 3.7
$H_{0,R}$	64.0 ± 4.6	64.8 ± 4.7	65.5 ± 4.8	66.1 ± 5.1	66.6 ± 5.3	67.0 ± 5.6

By inspection of the results for the different sets one easily convinces oneself that there is a range of overlap near $\gamma = 0.4 - 0.8$ for the sets LO, F and R. Does that mean that the two other results, $H_{0,HI}$ and $H_{0,S}$, can be coldly ignored as being in statistical disagreement and probably affected by systematic errors? To answer this question we turn to Strategy II in the next section.

5. Testing

How important are the disagreements in Table 2? Let us first turn to the supernova set S and test it against the Virgo-Leo sets HI and LO. In this test we do not use the Fornax results, because F is just a single galaxy with a fairly large H_0 error, and we further neglect the set R because it anyway does not come into question to combine it with S. It is important here to remove the LMC error from the Table 2 results, because it is common to all sets and completely correlated.

The first result is that $H_{0,HI}$ disagrees with $H_{0,S}$ by 5.4σ at $\gamma = 0.0$, diminishing to 3.4σ at $\gamma = 0.6$. Note that no published estimate of γ exceeds 0.6 within 1σ errors (except Gould 1994, but in that work the color variation was not accounted for and therefore his γ is not comparable to ours), so it is difficult to defend solutions requiring larger γ–values. Thus we conclude that it is not statistically defendable to combine $H_{0,HI}$ with $H_{0,S}$.

The second result is that also $H_{0,LO}$ disagrees with $H_{0,S}$, by 4.9σ at $\gamma = 0.0$, diminishing to 2.4σ at $\gamma = 0.6$.

Since ultimately we have to choose between combining either S or the other supernova set, R, with one of the Virgo-Leo sets, let us see how much better the $H_{0,R}$ fares. The $H_{0,HI}$ value disagrees with $H_{0,R}$ by 3.5σ at $\gamma = 0.0$, but at $\gamma = 0.6$ the disagreement is only 1.3σ. The $H_{0,LO}$ value disagrees with $H_{0,R}$ by 2.4σ at $\gamma = 0.0$, but from about $\gamma = 0.35$ up the disagreement is no longer significant. We conclude from this that the set R is strongly favored over S on the basis of our consistency requirement. Part of the reason for this is that the errors of set S are very small.

Another test is possible in Strategy III: first we compute for each data set the mean values of H_0, as a function of γ. While averaging H_0-values at a given γ-value, we use only uncorrelated errors as weight, (See Fig. 2 for the LO + R + F set). We then construct a χ^2 sum as a function of γ, comparing the above mean $H_0(\gamma)$-values with the individual $H_0(\gamma)$-values from different groups: each galaxy in the sets V, L, F is represented by

one term, the average S by one term, and the average R by one term.

This complicated process of constructing a χ^2 sum from correlated terms makes it rather problematic to relate it to probability and to interpret the sum as a goodness-of-fit test. However, different data combinations can be compared and relative figures of merit can be established.

The resulting $\chi^2(\gamma)$ functions are shown in Fig. 1 for the four data set combinations $HI+R+F$, $LO+R+F$, $HI+S+F$, $LO+S+F$. The common LMC error is again set to zero. One notes that the combination $LO + R + F$ attains the lowest minimum value 5.9. The minima of all the four combinations of data are

$$
\begin{aligned}
HI + R + F : & \quad 10.8 \,@\, \gamma = 0.53 \,, \\
LO + R + F : & \quad 5.9 \,@\, \gamma = 0.31 \,, \\
HI + S + F : & \quad \sim 20.7 \,@\, \gamma \geq 1.0 \,, \\
LO + S + F : & \quad 16.0 \,@\, \gamma = 0.69 \,.
\end{aligned}
$$

If we interpret the differences as standard variances, every combination is at least 2.2σ worse than $LO + R + F$. In particular, every combination including the set S is at least 3.2σ worse. From the curves in Fig. 1 one can deduce confidence intervals for γ (cf. Sect. 6).

An important conclusion is that the overall consistency is worse in the combinations including the set S than including the set R.

Note that this method also permits us to choose between the high and low Virgo velocities, although the significance of the test is only 2.2σ in favor of the low Virgo velocity.

6. Results

From the χ^2 sum constructed (Strategy III) in the previous section and plotted in Fig. 1. we can determine a value of γ. Keeping both the high and low Virgo velocities, we have two results. For the set $LO + R + F$ we find

$$\gamma = 0.31^{+0.15}_{-0.14} \text{ mag/dex} \,, \tag{13}$$

and for $HI + R + F$

$$\gamma = 0.53^{+0.17}_{-0.15} \text{ mag/dex} \,. \tag{14}$$

The low Virgo velocity result is in good agreement with previous results quoted in Sect. 1, whereas the high Virgo velocity result
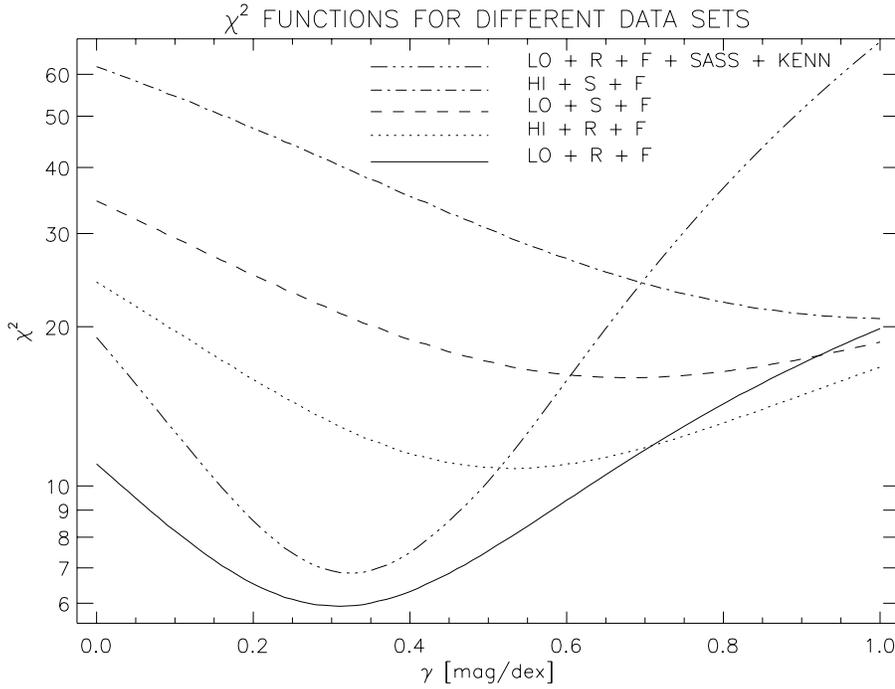
Fig. 1. The χ^2 sum as a function of γ for the four different data sets, and for the LO + R + F set including Sasselov's and Kennicutt's γ as additional terms.
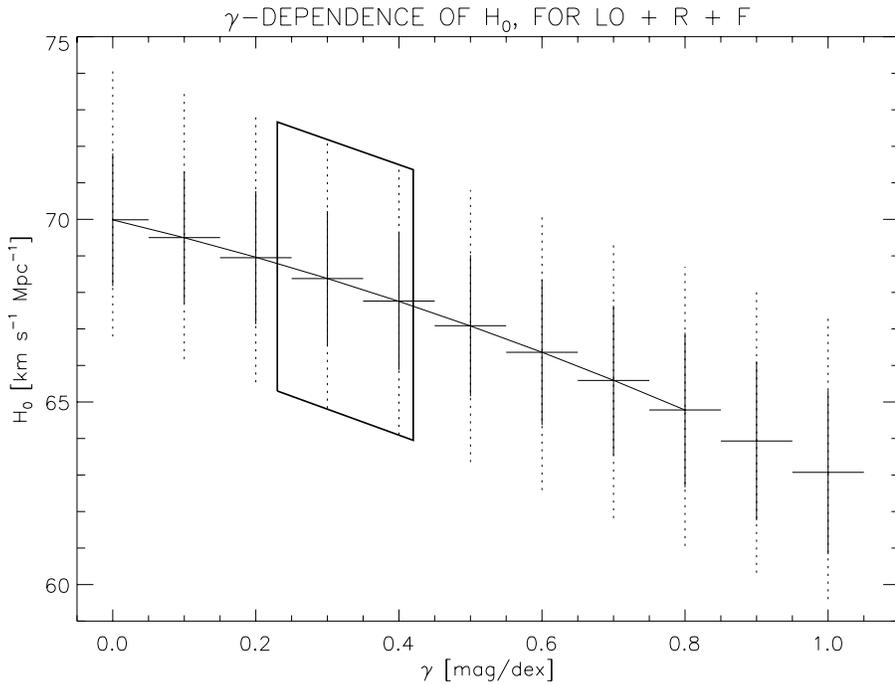

Fig. 2. The mean $H_0(\gamma)$-values for the data set including Leo-Virgo-Coma data using low Virgo velocity, Fornax data and SN Ia data from Riess. The errors of the mean (including only uncorrelated errors) are indicated with vertical solid lines. The total errors (including LMC error) are indicated by vertical dotted lines. The best 2nd order polynomial fit is indicated by a solid curve. The γ-value from our analysis, combined with Sasselov's and Kennicutt's value, and the corresponding allowed H_0 values are indicated as a thick quadrangle.

is only marginally so. At this point we have to choose between the two.

From our calculations of the mean $H_0(\gamma)$ for LO + R + F and HI + R + F we find that the difference $\gamma = 0.53 - 0.31$ corresponds to a shift in H_0 of only about 1.4 km s^{-1} Mpc^{-1} which is negligible in comparison with the H_0 error flags. Thus the disagreement between the two Virgo velocities, which really is of the order of 2.8σ in velocity space, has now shrunk to much less than one standard deviation in H_0 space. We conclude that

we can safely trust the γ value from the low Virgo velocity χ^2 fit which was favored on the basis of its minimum χ^2 value.

We can now improve the precision of our γ value by combining it with other independent determinations. The value of Beaulieu et al. (1997) and Sasselov et al. (1997) is clearly independent of our result, because it is derived from Cepheids in the LMC and SMC only. Similarly, the Kennicutt et al. (1998) value is derived from observations of M101 only, and therefore independent of our result. The Kochanek (1997) value is derived

using some of the galaxies we also use, thus we cannot consider that completely independent of our analysis.

Combining our value $\gamma = 0.31^{+0.15}_{-0.13}$ mag/dex with the Sasselov et al. (1997) and Sasselov (priv. comm.) result $\gamma = 0.48^{+0.1}_{-0.2}$ mag/dex from $[O/H]_{SMC} / [O/H]_{LMC}$, and with the Kennicutt et al, (1998) result $\gamma = 0.24 \pm 0.16$ mag/dex, we obtain the fifth curve in Fig. 1 which has its minimum, $\chi^2 = 6.8$ at

$$\gamma = 0.33^{+0.09}_{-0.10} \text{ mag/dex} . \tag{15}$$

Note that this implies that the value $\gamma = 0$ is rejected by 3σ.

We finally arrive at Strategy IV. We transform the above best γ value via the fitted function $H_0(\gamma)$ to a H_0 value, as in Fig. 2. At this step we have to add the two correlated errors which are common to all our data sets: the error of 0.10 mag in the LMC distance modulus, and the above error in γ. The effect of the LMC error we find by redoing all calculations at the fixed LMC distance modulus values 18.40 and 18.60. Finally we treat the Virgo velocity error conflict as a systematic error of size 1.4 km s^{-1} Mpc^{-1} which we combine in quadrature with the other error terms. Our result is then

$$H_0 = 68 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1} . \tag{16}$$

We note that the dominant component in the error is due to the LMC distance modulus as long as the HII-to-ceph error is absent (see however Sect. 7 for a discussion of this point).

7. Discussion

To get an idea of the effect of the HII-to-ceph error (explained in Sect. 4), we rerun our computations, using an abundance error arbitrarily increased to 0.30 dex for each galaxy, instead of 0.15 dex used above. The corresponding χ^2-minima become so close to each other that we cannot reject any of the data combinations, and therefore we cannot get the best values of γ or H_0. However, including Sasselov's γ value as one term in the χ^2 sum, we find that the parameters are in the ranges $\gamma = 0.25$ to 0.63 mag/dex and $H_0 = 58$ to 74 km s^{-1} Mpc^{-1}. We note that the error ranges are naturally larger than in our results in the previous section, but that our value of H_0 in the previous section is totally within the new range here.

There exists quite a range of measured values of the distance modulus of LMC. The Barnes-Evans infrared surface brightness technique to calibrate period-radius and period-luminosity relations of Galactic Cepheids lead to a value $\mu_{LMC} = 18.46 \pm 0.02$ (Gieren et al., 1998), quite close to the classical 18.5 ± 0.1.

The recent HIPPARCOS measurements of the Galactic Cepheids lead to a value $\mu_{LMC} = 18.7 \pm 0.1$ (Feast & Catchpole, 1997). The reanalysis of the same data gives lower values $\mu_{LMC} = 18.57 \pm 0.11$ (Madore & Freedman, 1997) and $\mu_{LMC} = 18.56 \pm 0.08$ (Oudmaijer et al., 1998). The most recent determination from the expanding ring around the SN 1987a is $\mu_{LMC} = 18.58 \pm 0.03$ (Panagia et al., 1997). HIPPARCOS parallax analyses of local subdwarfs lead to values $\mu_{LMC} = 18.61 \pm 0.07$ (Gratton et al, 1997), or $\mu_{LMC} = 18.65 \pm 0.10$ (Reid, 1997), who speculates that the RR Lyrae distance scale

is too small. These observations form a quite impressive body of evidence for the real LMC distance modulus to be slightly higher than the standard 18.5 mag that we have assumed in this work.

However, the situation is more complicated due to observations of low values of μ_{LMC}: calibrating Galactic RR Lyrae variables using HIPPARCOS data leads to $\mu_{LMC} = 18.37 \pm 0.23$ (Luri et al., 1998) and $\mu_{LMC} = 18.26 \pm 0.15$ (Fernley et al, 1998). The HIPPARCOS calibrated red clump star method (Paczyński & Stanek, 1998), leads to low values $\mu_{LMC} = 18.07 \pm 0.10$ (Stanek et al, 1998) and $\mu_{LMC} = 18.08 \pm 0.12$ (Udalski et al., 1998).

It is obvious that all these observations are not consistent with each other, so we may be obliged to make a selection. One possible way out would be to rely on the revised red clump values $\mu_{LMC} = 18.28 \pm 0.18$ (Girardi et al, 1998) who take metallicity effect into account, and $\mu_{LMC} = 18.36 \pm 0.17$ (Cole, 1998), who considers a luminosity dependence of the red clump stars on age and metallicity. Our motivation is that the red clump method is relatively new, and as such prone to more systematic errors than other methods. One point worth mentioning is that the weighted mean of the LMC modulus remains robustly 18.50 to within better than 0.02 regardless of whether we use the published variances as weights or an arbitrarily chosen weight, common to all data. Also, the mean is unsensitive to whether we include or exclude Feast & Catchpole (1997), which have already been revised by Stanek et al. (1998) and Udalski et al. (1998). Taking all the above into account (with only the revised data) we propose a very conservative estimate for the LMC distance modulus of $18.50^{+0.15}_{-0.25}$. This confidence interval is chosen to cover all the above quoted central values. We note that the data set used here is not totally consistent, and therefore this estimate for the error range is an overestimate, containing some underlying systematic errors that have not been estimated properly.

Another attempt to reduce some of this bias is by "weighing" the data sets with the merits of the methods used to obtain them. Considering the strong evidence for high distance an argument in disfavor of the red clump method, we are left with three independent methods (HIPPARCOS cepheids, local subdwarf parallaxes and SN 1987a) giving an LMC distance exceeding 18.50, one giving an intermediate value (IR surface brightness) and only one (RR Lyrae method) giving a low distance. This would point to a significant systematic error in RR Lyrae method, (see Reid's (1997) discussion on this). Excluding RR Lyrae results we end up with our "less conservative estimate", a slightly higher value of about $\mu_{LMC} = 18.55 \pm 0.1$.

We conclude that the issue of the LMC distance is not settled yet. Our two estimates above, a conservative one and a less conservative one are two different ways to look at the same problem. We note that both distance estimates are consistent with the classical value of Freedman & Madore. In the following we check how our best value would be affected by adopting either one of the above two "conservatisms".

Recalculating with the conservative estimate $\mu_{LMC} = 18.50^{+0.15}_{-0.25}$ we note that γ - determination and therefore the

best value of H_0 remains unchanged, since the central value of this estimate equals the classical one, and since LMC distance errors are not included in χ^2-calculations. However, the errors of the final value increase so that we obtain $H_0 = 68^{+8}_{-6}$ km s^{-1} Mpc^{-1}. Since this error range is conservative and an over-estimate, and since our best value is totally in this range, this value should only be taken as and indicator that our best value is valid.

Recalculating with the less conservative estimate 18.55 ± 0.1 we find out that all our conclusions remain unchanged, the values of γ remain virtually unchanged and the Hubble constant decreases marginally, by 2%, or 1.5 km s^{-1} Mpc^{-1}. This is our suggestion for the systematic error of H_0 due to the LMC distance uncertainty. However, adding this value in quadrature to the errors of our best estimate has no effect, therefore also this estimate supports our best value.

8. Conclusions

The existence of well measured distances to galaxies for which also the metallicities are known, permits one to attempt to form a consistent set of data from which a combined value of the Hubble constant can be obtained. We have used four Virgo galaxies, two Leo I galaxies, one galaxy in Fornax, and two partially over-lapping analyses of four supernovæ (all listed in Sect. 2. and in Table 1) to study the metallicity dependence of the Hubble constant derived from these data.

The analysis introduces several types of correlations, and some of the input errors are correlated as well. We take care of handling all such correlations statistically correctly. We conclude that we can select between the two supernova analyses: the MLCS set (Riess et al, 1996) is consistent with all our other data sets, whereas the standard constant maximum brightness analysis of type Ia SNe (Sandage et al, 1996) set is not. We can also select between the two conflicting Virgo velocity values in the literature (in favor of the low velocity).

Our main results are two (using $\mu_{LMC} = 18.5 \pm 0.1$): a new value for the Hubble constant, $H_0 = 68 \pm 5$ km s^{-1} Mpc^{-1}, and a new value for the metallicity coefficient (the correction for distance modulus in magnitude per factor of ten in metal abundance) of $0.31^{+0.15}_{-0.14}$ mag/dex, in excellent agreement with previous results. Thus the metallicity effect is clearly significant – combining our γ value with that of Sasselov et al. (1997) for [O/H], and Kennicutt et al. (1997), we obtain $\gamma = 0.33^{+0.09}_{-0.10}$ which is 3σ away from zero. Note the the analysis by Kochanek, yielding $\gamma = 0.4 \pm 0.2$, is in excellent agreement with this.

Taking into account the current knowledge of the LMC distance, we see that our best value is even more robust (see Sect. 7).

The general agreement of the final data set with our criterion of statistical consistency permits us to conclude that no further systematic errors are needed beyond those accounted for here.

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