Astron. Astrophys. 339, 15–18 (1998)

ASTRONOMY AND ASTROPHYSICS

A test of some luminosity functions of quasars

Yi-Ping Qin[1], **G.Z. Xie**[1], **and Xue-Tang Zheng**[2]

[1] Yunnan Observatory, The Chinese Academy of Sciences, Kunming, Yunnan 650011, P.R. China
[2] Department of Physics, Nanjing University of Science and Technology, Nanjing, Jiangsu 210014, P.R. China

Received 11 August 1997 / Accepted 4 June 1998

Abstract. The n/n_{max} test from our previous work is applied to some luminosity functions of quasars. The CFHT and AAT samples are employed. For a subsample of AAT, all the luminosity functions except one pass the n/n_{max} test, while for two subsamples of CFHT, none of them passes the test. It shows that, if the luminosity functions adopted are correct in the concerned redshift and magnitude ranges, the CFHT sample must be incomplete while the AAT is complete to $z = 2.2$. From the study we find that the n/n_{max} test is useful for testing the correctness of luminosity functions, and hence useful for testing the correctness of theoretical models of quasars, as long as complete samples are available. It is also useful for testing the completeness of samples if the true luminosity function has been known.

Key words: methods: statistical – galaxies: luminosity function, mass function – galaxies: quasars: general – galaxies: statistics – cosmology: observation)

1. Introduction

In the study of the luminosity function of quasars, the statistical method is a fundamental tool. Among the many methods applied so far, the V/V_{max} test has played a very active role. The test was first introduced by Schmidt (1968) as a measure of the uniformity of the space distribution of radio sources. A revised V/V_{max} test, the V'/V'_{max} test, which takes into account the density function of quasars, could be used to test the fit of density evolution models (Schmidt 1968). A shortcoming of the V'/V'_{max} test (and so the V/V_{max} test) is that it assumes objects of all luminosities increase in density at the same rate as one another, and therefore its application is limited to pure density evolution models (Hartwick & Schade 1990). For this reason, our previous work generalized the V/V_{max} test so that it could be applicable to any kind of luminosity function models (Qin & Xie 1997; Paper I). In our generalization, the statistic V/V_{max} was replaced by n/n_{max}, which takes into account the luminosity function of quasars. When the adopted luminosity function of quasars is correct, the values of n/n_{max} in a complete sample are expected to be uniformly distributed in the interval [0, 1]. In this paper, we apply the n/n_{max} test to some luminosity functions of quasars.

Send offprint requests to: Yi-Ping Qin

In Sect. 2, we present the samples and luminosity functions adopted. In Sect. 3, the n/n_{max} test is carried out. A summary and the discussion of the results are given in Sect. 4.

2. Samples and luminosity functions

Among the many samples, the CFHT sample (Crampton et al. 1989) and the AAT sample (Boyle et al. 1988; Boyle et al. 1990) are both relatively big-sized and claimed complete. The CFHT sample is the result of a slitless spectra survey. It contains 270 quasars in two large fields and four small ones, and has been believed to be complete to $m < 20.5$ (Crampton et al. 1989; Hartwick & Schade 1990). Following Hartwick & Schade (1990), we adopt a conservative magnitude limit ($m \leq 20.0$) for the CFHT survey and get a sample of 194 quasars (called the CFHT1 sample). For the sake of comparison (see the following), we further construct a subsample of the CFHT1 by adopting a redshift limit ($z \leq 2.2$), and have a sample with 171 quasars (called the CFHT2 sample). Different from the CFHT sample, the AAT sample comes from a UVX survey. It contains 420 quasars and the objects detected could be as faint as $B \leq 21mag$. The UVX technique is well known to be free of substantial bias or incompleteness in identifying QSOs with redshifts less than 2.2 (Véron 1983; Boyle et al. 1990). Therefore, we construct a subsample from the AAT sample by limiting redshifts to 2.2 ($z \leq 2.2$) and, for the sake of comparison, magnitudes to 20.0 ($B \leq 20.0$). This sample contains 211 quasars (called the AAT2 sample). In converting b magnitudes in the data into B magnitudes, we adopt $B = b - 0.14mag$ (Boyle et al. 1990).

Of the many models of luminosity functions, the pure luminosity evolution model for which the V/V_{max} test has been applied by Mathez (1976), is the most successful one and has been frequently used (Mathez 1978; Marshall et al. 1984; Boyle et al. 1988, 1990; Pei 1995; Van Waerbeke et al. 1996). In the following, we employ only the luminosity functions belonging to this kind of models to study.

The luminosity functions are adopted from Pei (1995) and Boyle et al. (1990). Pei studied only the double-power-law and

Table 1. The adopted luminosity functions

Name (1)	Model (2)	Evolution (3)	(h, q_0, α) (4)	References (5)
D1	Double-power-law	$\frac{exp[-(z-z_*)^2/2\sigma_*^2]}{(1+z)^{1+\alpha}}$	$(0.5, 0.5, -0.5)$	1
D2	Double-power-law	$\frac{exp[-(z-z_*)^2/2\sigma_*^2]}{(1+z)^{1+\alpha}}$	$(0.5, 0.1, -1.0)$	1
E1	Exponential $L^{1/4}$ law	$\frac{exp[-(z-z_*)^2/2\sigma_*^2]}{(1+z)^{1+\alpha}}$	$(0.5, 0.5, -0.5)$	1
E2	Exponential $L^{1/4}$ law	$\frac{exp[-(z-z_*)^2/2\sigma_*^2]}{(1+z)^{1+\alpha}}$	$(0.5, 0.1, -1.0)$	1
B	Double-power-law	$(1+z)^k$	$(0.5, 0.5, -0.5)$	2
D	Double-power-law	$e^{k\tau}$	$(0.5, 0.5, -0.5)$	2
J	Double-power-law	$(1+z)^k$	$(0.5, 0.5, -0.5)$	2
K	Double-power-law	$e^{k\tau}$	$(0.5, 0.5, -0.5)$	2

References to Table 1: 1. Pei 1995; 2. Boyle et al. 1990.

the exponential $L^{1/4}$ law models. He combined the Hartwick & Schade (1990) sample (HS sample) and the Warren et al. (1994) sample (WHO sample) to get a big sample containing more than 1200 quasars and covering a large range of redshifts $0 \leq z \leq 4.5$, and adopted a simple Gaussian form of the luminosity evolution for the study. The parameters of the models were determined from fits to the data for two sets of constants (h, q_0, α) =$(0.5, 0.5, -0.5)$ and (h, q_0, α) =$(0.5, 0.1, -1.0)$, where h is a relative value of the Hubble constant H_0 ($h \equiv H_0/100kms^{-1}Mpc^{-1}$), q_0 is the deceleration parameter, and α is the spectral index ($S_\nu \propto \nu^\alpha$). We denote the two specific luminosity functions of the double-power-law model in the above two sets of constants as D1 and D2, respectively, and the two of the exponential $L^{1/4}$ law model as E1 and E2, respectively. There are 14 specific luminosity functions presented by Boyle et al. (1990). For simplicity, we adopt only the B, D, J and K luminosity functions there and denote them with the same letters B, D, J and K, respectively. These luminosity functions all belong to the double-power-law model. The form of the luminosity evolution adopted in cases B and J is $(1+z)^k$, while that in cases D and K is $e^{k\tau}$. The parameters of B and D were determined from a combined sample, and those of J and K were determined only from the AAT sample. They were all computed in the set of constants (h, q_0, α) =$(0.5, 0.5, -0.5)$. The above luminosity functions are listed in Table 1. The columns are: (1) luminosity function name; (2) luminosity function model; (3) luminosity evolution form; (4) constant set (h, q_0, α); (5) references.

3. The n/n_{max} test

The value of n/n_{max} for each object in each sample is computed with each luminosity function and the corresponding constant set (h, q_0, α). For sample CFHT1, z_{max} is determined by the following equation

$$M_B + 5logA(z_{max}) - 2.5(1+\alpha)log(1+z_{max}) + 23.89 = 0. \quad (1)$$

Equation (1) is adopted from Schmidt & Green (1983) with $B_{lim} = 20.0$, where $h = 0.5$ and $A(z)$ is given by

$$A(z) = z[1 + \frac{z(1-q_0)}{1+q_0 z + \sqrt{1+2q_0 z}}]. \quad (2)$$

For samples CFHT2 and AAT2, when the value of z_{max} computed from equation (1) is greater than 2.2, we take $z_{max} = 2.2$ instead. We notice that, in equation (3) of Paper I, z_{max} could be both luminosity-dependent and fixed. The former corresponds to a flux-limited sample while the latter to a redshift-limited sample. the above determination of z_{max} in the two samples is suitable for a sample limited by both flux and redshift.

The mean value of n/n_{max}, $< n/n_{max} >$, in a complete sample is expected to be 0.5 when the luminosity function adopted is correct, and the variance of $< n/n_{max} >$ for a given total number N of objects of a sample was adopted as $1/12N$ (Avni & Bahcall 1980; Qin & Xie 1997). It is noticed that, when the size of a sample is big, the value of $| < n/n_{max} > -0.5|$ is small for a correct luminosity function, while the variance of $< n/n_{max} >$, $1/12N$, is also small. It is clear that the value of $| < n/n_{max} > -0.5|$ alone cannot tell the fineness of a result. Instead, the ratio of $| < n/n_{max} > -0.5|$ to $1/\sqrt{12N}$ will be suitable for judging the fineness of a result. Therefore, we define the following relative error

$$\sigma_r \equiv \frac{| < n/n_{max} > -0.5|}{1/\sqrt{12N}}. \quad (3)$$

With the definition of σ_r, the basic n/n_{max} test becomes that, when $\sigma_r \geq 1$ for a complete sample, the luminosity function adopted is not acceptable.

For each sample and each luminosity function, the values of $< n/n_{max} >$ and σ_r are calculated. The results are given in Table 2.

4. Discussion and conclusions

Of the three adopted samples, the CFHT1 is a pure flux-limited sample, while the rest are both redshift-limited and flux-limited samples. The n/n_{max} test is applicable to these two kinds

Table 2. The values of $< n/n_{max} >$ and σ_r

Sample (1)	Luminosity Function (2)	$< n/n_{max} >$ (3)	σ_r (4)
CFHT1	D1	0.400	4.84
	D2	0.401	4.76
	E1	0.400	4.84
	E2	0.397	4.97
	B	0.373	6.13
	D	0.396	5.03
	J	0.364	6.55
	K	0.378	5.90
CFHT2	D1	0.439	2.77
	D2	0.431	3.13
	E1	0.438	2.79
	E2	0.426	3.36
	B	0.439	2.76
	D	0.462	1.73
	J	0.424	3.46
	K	0.438	2.79
AAT2	D1	0.509	0.46
	D2	0.502	0.11
	E1	0.509	0.44
	E2	0.497	0.14
	B	0.510	0.51
	D	0.534	1.69
	J	0.496	0.23
	K	0.511	0.57

of samples (see section 3) as well as to pure redshift-limited samples. For pure redshift-limited samples, the computation of n/n_{max} is easier, where, the value of z_{max} in equation (3) of Paper I is fixed. One can verify that, when redefining the statistics n and n_{max} (see Paper I) in the following way

$$n(M,z) \equiv \int_{z_{\min}}^{z} \Phi(M,z)dV(z) \tag{4}$$

and

$$n_{max}(M) \equiv \int_{z_{\min}}^{z_{max}(M)} \Phi(M,z)dV(z), \tag{5}$$

the properties of the statistic n/n_{max} are maintained. For a redshift-limited, $z_{\max} \geq z \geq z_{\min}$, sample, the values of n/n_{max} are easily computed by applying equations (4) and (5) with the values of z_{min} and z_{max} being fixed.

In most cases, the samples supplied are always flux-limited. Constructing a pure redshift-limited sample requires some techniques, e.g., some basic assumptions. For a specific discussion on this issue one may refer to Van Waerbeke et al. (1996).

We notice that, the V/V_{max} analysis is completely independent of the luminosity function. The completeness of a sample can be tested by its V/V_{max} distribution when and only when the true distribution of V/V_{max} is known. When a given couple (cosmology, evolution) is chosen, the luminosity function will be known to an arbitrary constant, e.g., Φ_*. Thus, the distribution of V/V_{max} will also be known (by computation) to an arbitrary constant and can be used to test the sample completeness to a constant.

It should be pointed out that the distribution of the values of the statistic n/n_{max} of a sample, $\{n/n_{max}\}$, is far more efficient than its mean value $< n/n_{max} >$. To get a more efficient n/n_{max} test, the methods concerning $\{ n/n_{max}\}$ rather than $< n/n_{max} >$, such as the famous Kolmogorov-Smirnov method or the maximum likelihood method applied to the V/V_{max} statistic by Van Waerbeke et al. (1996), should be applied.

Even though the $< n/n_{max} >$ method is not so efficient, it does supply a rather strict test for luminosity functions. From Table 2 we find that: a) if samples CFHT1 and CFHT2 are complete, none of the above luminosity functions passes the test; b) if sample AAT2 is complete, all the luminosity functions except D pass the test; c) if samples CFHT1 and CFHT2 are complete, the adopted luminosity functions predict more objects at lower redshifts than at higher redshifts; d) if most of the luminosity functions are correct in the concerned redshift and magnitude ranges, sample AAT2 must be complete and the others must be incomplete; e) if most of the luminosity functions are correct in the concerned redshift and magnitude ranges, the completeness of sample CFHT must be better in lower redshift ranges than in higher redshift ranges.

The n/n_{max} test is designed to test the correctness of luminosity functions. In this paper, we apply it to test various luminosity functions published in the literature, which unfortunately are estimated from essentially the same data used here. Applied in this way, the test is like a requirement for internal consistency of empirically derived luminosity functions. Some theoretical models of quasars or AGNs are elaborated enough that they predict a theoretical luminosity function (see, e.g., Siemiginowska & Elvis 1997). Thus, to test such theoretical luminosity functions can be thought as allowing one to test the correctness of theoretical models of quasars, which appears more interesting.

We then come to the following conclusions. a) The n/n_{max} test is useful for testing the correctness of luminosity functions, and hence useful for testing the correctness of theoretical models of quasars, as long as complete samples are available. b) The n/n_{max} test could be used to check the completeness of a sample when the luminosity function adopted is correct in the concerned redshift and magnitude ranges. c) If the luminosity functions adopted are correct in the concerned redshift and magnitude ranges, our results provide an evidence for the incompleteness of the CFHT sample. d) With the same condition, i.e., the luminosity functions adopted are correct in the concerned redshift and magnitude ranges, the AAT sample must be complete to $z = 2.2$.

References

Avni, Y., & Bahcall, J. N. 1980, ApJ, 235, 694

Box, T. C., & Roeder, R. C. 1984, A&A, 134, 234

Boyle, B. J., Fong, R., Shanks, T., & Peterson, B. A. 1990, MNRAS, 243, 1

Boyle, B. J., Shanks, T., & Peterson, B. A. 1988, MNRAS, 235, 935
Crampton, D. Cowley, A. P., & Hartwick, F. D. A. 1989, ApJ, 345, 59
Hartwick, F. D. A., & Schade, D. 1990, ARA&A, 28, 437
Marshall, H. L., Avni, Y. Braccesi, A., Huchra, J. P., Tananbaum, H., Zamorani, G., & Zitelli, V. 1984, ApJ, 283, 50
Mathez, G. 1976, A&A, 53, 15
Mathez, G. 1978, A&A, 68, 17
Pei, C. P. 1995, ApJ, 438, 623
Qin, Y. P., & Xie, G. Z. 1997, ApJ, 486, 100 (Paper I)
Schmidt, M. 1968, ApJ, 151, 393
Schmidt, M., & Green, R. F. 1983, ApJ, 269,352
Scott, D. 1991, A&A, 242, 1
Siemiginowska, A., & Elvis, M. 1997, ApJ, 482, L9
Van Waerbeke, L., Mathez, G., Mellier, Y., Bonnet, H., & Lachièze-Rey, M. 1996, A&A, 316, 1
Véron, P. 1983, in: Quasars and Gravitational Lenses, Proc. 24th Liège International Astrophysical Colloquium, p.210
Warren, S. J., Hewett, P. C., & Osmer, P. S. 1994, ApJ, 421, 412