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Determination of temperature and chemical composition profiles in the solar interior from seismic models

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Abstract. The primary inversion of the solar oscillation frequencies coupled with the equations of thermal equilibrium and input physics enable us to infer the temperature and hydrogen abundance profiles inside the Sun. The inferred hydrogen abundance profile is smoother than that in a solar model with conventional treatment of diffusion, in the region just beneath the solar convection zone ($r \gtrsim 0.68R_\odot$). Such a mixing process could account for the observed low lithium abundance in the solar envelope. It is also possible to constrain the nuclear reaction rates using the inferred temperature and hydrogen abundance profiles. The helioseismically estimated cross-section for pp nuclear reaction turns out to be $(4.15 \pm 0.25) \times 10^{-25}$ MeV barns, where the error estimates include those from opacities arising from up to 50% uncertainty in heavy element abundance Z.

Key words: Sun: abundances – Sun: interior – Sun: oscillations

1. Introduction

The precisely measured frequencies of solar oscillations provide us with a powerful tool to probe the solar interior with sufficient accuracy. These frequencies are primarily determined by the dynamical quantities like sound speed, density or the adiabatic index of the solar material and a primary inversion of the observed frequencies yields the sound speed and density profiles inside the Sun (Gough 1985; Gough & Kosovichev 1990; Gough & Thompson 1991; Dziembowski et al. 1994; Antia & Basu 1994a; Basu et al. 1996; Gough et al. 1996). On the other hand, in order to infer the temperature and chemical composition profiles additional assumptions regarding the input physics such as opacities, equation of state and nuclear energy generation rates are required (Gough & Kosovichev 1988; Shibahashi 1993; Antia & Chitre 1995; Shibahashi & Takata 1996; Kosovichev 1996). Although the primary inversions can yield the sound speed to an accuracy of 0.1%, the opacities and nuclear reaction rates are hardly known to comparable accuracy and consequently, more systematic errors are introduced in these secondary inversions for temperature and chemical composition.

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There are a number of approaches adopted for secondary inversions. Gough & Kosovichev (1988) and Kosovichev (1996) have employed the equations of thermal equilibrium to express the changes in primary variables (ρ, Γ_1) in terms of those in secondary variables (Y, Z) and obtained equations connecting the frequency differences to variations in abundance profiles. It should be noted that modifications in Z profile mainly affect the opacities in the solar interior, while the equation of state and nuclear energy generation rates are affected to a much lesser extent. Such a procedure is essentially equivalent to finding the Y profile along with the necessary opacity modifications. It is not clear if the X and Z profiles can indeed be determined independently, as this may result in non-unique inferred profiles. Shibahashi and Takata (1996, hereinafter ST96) adopt the standard opacities and nuclear reaction rates to obtain the temperature and hydrogen abundance profiles with the use of only the inverted sound speed profile.

Antia and Chitre (1995) used a similar approach to estimate the central temperature of the Sun and the corresponding neutrino fluxes. They adopted the inverted sound speed and density profiles to obtain the temperature (T) and helium abundance (Y) profiles in the solar core. The main thrust of this study was to estimate the central temperature of the Sun and the neutrino fluxes. On the other hand, Roxburgh (1996) and Antia & Chitre (1997) studied various possible abundance profiles without any bounds on the opacity variations to study the implication of helioseismic constraints on the solar neutrino problem.

The main difference between our approach and that of ST96, is that we adopt both sound speed and density profiles obtained from primary inversion. By making use of only the inverted sound speed profile, ST96 determine the density profile along with those of temperature and hydrogen abundance by resorting to the equations of hydrostatic and thermal equilibrium. Thus the inferred density profile in their approach should depend on the opacities and nuclear reaction rates (Takata & Shibahashi 1997; Shibahashi et al. 1997). Similarly, Tripathy & Christensen-Dalsgaard (1996) have tried to estimate the opacity corrections by using only the inverted sound speed profile. In our approach we do not solve the equations of hydrostatic equilibrium directly, since all the relevant information about the mechanical equilibrium is already contained in the inverted sound speed and density profiles. It may be argued that in the radiative interior the density and sound speed profiles are not independent as they are related through the equation of hydrostatic equilibrium. The basis of this belief may be attributed to the assumption of adiabatic index Γ_1 remaining almost constant in this region. However, it turns out that in the radiative interior Γ_1 varies by about 0.004 in models constructed using OPAL or MHD equation of state. This variation is much more than the estimated errors in sound speed inversions and there is therefore no reason to believe that the density and sound speed are not independent properties. Even a small variation in Γ_1 can result in a large variation in the density. In fact, by varying the X profile and the nuclear reaction rates it is demonstrably possible to construct solar models which agree reasonably well with a given sound speed profile, but which have significantly different density profiles.

In the present study we extend the earlier work of Antia and Chitre (1995) to determine the temperature and hydrogen abundance profiles throughout the radiative interior of the Sun and investigate possible uncertainties that might exist in the basic nuclear energy generation rates. Recently, there has been a claim that the pp nuclear reaction rate should be revised upwards by a factor of 2.9 (Ivanov et al. 1997) and it would therefore be interesting to test this suggestion helioseismically (Degl'Innocenti et al. 1997). Further, in the earlier study we had restricted the composition profiles to smooth functions represented by a low degree polynomial, which constrained the class of admissible solutions. This restriction has been relaxed in the present study by directly solving the equations of thermal equilibrium without invoking any possible opacity modification or any representation for composition profile.

A possible drawback of this approach is that the sound speed and density profiles are constrained to match the inverted profiles exactly. It is therefore not possible to ensure that the resultant computed luminosity matches the observed value, unless the nuclear reaction rates are modified slightly. In order to overcome this limitation we have also attempted a direct approach where the composition profile taken from the standard calculation of static solar model is adjusted to match the inverted sound speed and density profiles. This complements the earlier approach, since in this process although the luminosity constraint is satisfied, the resulting sound speed and density profiles may not exactly match the inverted seismic profiles. Thus, essentially, the luminosity constraint is satisfied at the expense of the variance in sound speed and density profiles in the solar core.

The rest of the paper is organized as follows: the technique employed to obtain the temperature and chemical composition profiles is described in Sect. 2, and the results are set out in Sect. 3. Our attempts to constrain the cross-section for the pp reaction using the inferred profiles are outlined in Sect. 4, while Sect. 5 describes the direct approach. Finally, Sect. 6 summarizes the conclusions from our study.

2. The technique

The sound speed and density profiles inside the Sun are inferred from the observed frequencies using a Regularized Least Squares technique (Antia 1996). The primary inversions based on the equations of hydrostatic equilibrium along with the adiabatic oscillation equations, however, give only the mechanical variables like pressure, density and sound speed. This provides us with the ratio T/μ, where μ is the mean molecular weight. In order to determine separately T and μ, it becomes necessary to use the equations of thermal equilibrium, i.e.,

$$L_r = -\frac{64\pi r^2\sigma T^3}{3\kappa\rho}\frac{dT}{dr}, \qquad (1)$$

$$\frac{dL_r}{dr} = 4\pi r^2\rho\epsilon, \qquad (2)$$

where L_r is the total energy generated within a sphere of radius r, σ is the Stefan-Boltzmann constant, κ is the Rosseland mean opacity, ρ is the density and ϵ is the nuclear energy generation rate per unit mass. In addition, the equation of state needs to be adopted to relate the sound speed to chemical composition and temperature:

$$c = c(T, \rho, X, Z). \qquad (3)$$

Eq. (1) is applicable when there is no convective transport of energy. This is generally true in the region below the outer convection zone, and we have verified that all the models considered in this work are stable against convection. We again emphasize that the equations governing the hydrostatic equilibrium are not used while determining the thermal structure, since these equations are already employed while obtaining the inverted sound speed and density profiles from the seismic data. The mechanical equilibrium conditions are thus already implicit in the primary inversion profiles.

Since we have only three equations, namely, Eqs. (1–3) to determine the variables T, L_r and the chemical abundances, it becomes possible to determine only one parameter specifying the composition, e.g., the mean molecular weight. Clearly, the solution cannot be unique and therefore, in this work we assume the heavy element abundance, Z to be prescribed and attempt to determine the profile of hydrogen abundance (X) as also the temperature. It should be stressed that Z mainly affects the opacity in the solar interior, since the bulk of the energy generation takes place through the pp chain. One reason to keep the Z profile fixed is that the value of Z/X in the convection zone is known (Grevesse & Noels 1993) and the change in the interior due to diffusion is not expected to be very large, being of the order of 10% (Proffitt 1994) or even less depending on treatment of diffusion (Richard et al. 1996). Apart from diffusion there could be other uncertainties arising from possible inhomogeneities in the initial composition of the Sun (Levy & Ruzmaikina 1994). Recently, using the results from solar neutrino experiments Fukugita & Hata (1998) have shown that the heavy element abundance in the solar interior is within the range 0.4–1.4 times that at the surface. In this work we therefore, adopt an uncertainty of 50% in the heavy element abundance in solar interior. This is comparable to combined uncertainties in Z and opacities assumed by Bahcall & Pinsonneault (1992) to calculate the uncertainties in neutrino fluxes.

It may be noted that instead of ρ, c we could have used some other suitable pair of independent profiles obtained from primary inversions. For example, we could have used ρ, P, where P is the pressure. We have tried the calculations using P instead of c, to find that the results are not significantly affected, thus confirming that these constraints arising from primary inversions are equivalent. This follows from the fact that $P = c^2\rho/\Gamma_1$ and the profile of Γ_1 inferred from helioseismic inversions (Elliott 1996; Antia 1996; Basu & Christensen-Dalsgaard 1997) agrees with that calculated from OPAL equation of state (Rogers et al. 1996) to within 2σ of estimated errors in inversion. Thus, by matching c we should be automatically able to match P to desired accuracy. There may be small differences which are probably due to errors in inversion and uncertainties in the equation of state of solar material. It is thus not possible to use more than two constraints from primary inversions to determine any extra variables, like Z. It should be noted that a stellar model may be uniquely specified if the mass and composition profiles are known. It is not clear, however, if this is applicable to the models of present Sun, as these models have to satisfy additional constraints imposed by the observed luminosity and radius. In the usual prescription for calculating the static present day solar model the X profile is scaled to satisfy these constraints. Thus it is not obvious that an acceptable solar model can be constructed with a specified X and Z profile along with fixed input physics, as the resulting model may not have correct mass, radius or luminosity.

It may be noted that Eqs. (1, 2) used to calculate the hydrogen abundance and temperature profiles are the same set of equations that are used in constructing a static solar model. This calculation thus does not involve solution of ill-posed integral equations, as is the case with inversion problems. As a result, no smoothing or regularization is required and one does not expect significant error magnification in this process. Of course, the computation of sound speed and density profiles from solar oscillation frequencies involves solution of ill-conditioned integral equations. But that is by now a well studied problem and the inversion techniques have been tested extensively by comparison with other workers and through a Hare and Hound exercise (Antia et al. 1997).

We use the nuclear reaction rates from Bahcall & Pinsonneault (1995, hereinafter BP95) to calculate the nuclear energy generation rate, ϵ using a subroutine kindly provided by Bahcall. However, the estimated value of the cross-section for the pp reaction which has a dominant influence on nuclear energy generation rate in the Sun, has been found to be lower by about 4.5% in the recent calculations (BP95) as compared to earlier value (Bahcall 1989), with a quoted 1σ uncertainty of 1.1% (BP95). Most of the calculations have been performed using the older reaction rate for pp reaction, as that is found to give the computed luminosity closer to the observed value, $L_\odot$. We adopt the recent OPAL opacities (Iglesias and Rogers 1996) and OPAL equation of state (Rogers et al. 1996).

In this approach, it is not possible to ensure that the computed luminosity will necessarily match the observed solar luminosity $L_\odot$. Although the solar luminosity is known very accurately, the nuclear reaction rates and the inverted sound speed and density profiles are not known to sufficient accuracy. For example, uncertainties in the nuclear reaction rates can alter the computed luminosity by 2%, which is an order of magnitude larger than the errors in observed luminosity (Willson et al. 1984). Usually, while constructing standard solar models, the luminosity is adjusted by keeping the initial helium abundance as a free parameter. Since in our formulation we do not have any natural free parameter, it is not in general possible to reproduce the correct luminosity. This difficulty can be overcome if the nuclear energy generation rate itself is adjusted by multiplying it by a constant which is treated as a free parameter. With Z profiles normally used in standard solar models this factor comes out to be close to unity and hence the resulting nuclear energy generation is within the range normally used in solar model computations. Further, since the cross-section for the pp nuclear reaction has not been measured in laboratory, it is not unreasonable to keep it as a free parameter to be adjusted to obtain the correct luminosity. Moreover, it turns out that by adjusting other quantities, like the Z profile within reasonable limits it is not possible to adjust the luminosity significantly.

In principle, we can solve the Eqs. (1–3) with boundary conditions $L(0) = 0$ and $L(R_\odot) = L_\odot$, but it turns out that it is not possible to satisfy these boundary conditions with physically acceptable values of X (i.e., $0 < X < 1$). This is because, once the sound speed, density and Z profiles are known, it is possible to integrate these equations to calculate the T and L_r profiles, which will depend only on the central temperature T_c. Clearly, by adjusting T_c it is not possible to get both the correct luminosity and Y at the base of the convection zone. In fact, the solution is so sensitive to a choice of T_c that a change in T_c by mere 1000 K, results in the value of Y in the convection zone to increase from 0.25 to a value greater than 1! Thus, it is not possible to alter the luminosity even by $0.001L_\odot$ by adjusting T_c in the allowed range. The difference between the computed and the observed solar luminosity is thus essentially independent of the boundary conditions at the outer boundary. Consequently, the solution of Eqs. (1–3) will give an estimate of uncertainties in primary inversions or the input Z profile, or the basic microphysics, such as the equation of state, opacities and the nuclear reaction rates. It may be difficult to separate out the contributions from each of these sources. We will try to examine this question in some detail in Sect. 4.

For solving the Eqs. (1–3) we use the following boundary conditions:

$$L(0) = 0, \qquad L(r_b) = L_\odot, \qquad \left.\frac{dT}{dr}\right|_{r=r_b} = \left.\frac{dT}{dr}\right|_{ad}, \qquad (4)$$

where r_b is the radial position of the base of the convection zone, and the subscript ad denotes the adiabatic gradient. Of course, in order to satisfy these three conditions the nuclear energy generation rate is adjusted as explained earlier. It may be noted that the computed luminosity listed in various places in this paper is the value obtained without adjusting the nuclear reaction rates. The nuclear energy generation rate ϵ is adjusted by multiplying it with a constant, which is treated as a free parameter to be determined while solving the equations. With this prescription we need to solve two first order ordinary differential equations with one free parameter to satisfy the three boundary conditions and we therefore expect the solution to be unique. We use the fourth order Runge-Kutta method to integrate the equations numerically starting from the outer boundary. The initial values as well as the free parameter multiplying ϵ are determined iteratively to satisfy the required boundary conditions.

In principle, it may be possible to modify the Z profile to get the correct luminosity for any given nuclear reaction rates. However, in general very large modifications in Z are required to adjust the luminosity and in fact as we have shown in Sect. 4, for some nuclear reaction rates even after allowing for arbitrary modifications in opacity it is not possible to obtain the correct computed luminosity. In the absence of modification to the Z profile, it is possible to match all quantities except the observed solar luminosity. Thus, if the Z profile is to be modified to adjust only one number, namely, the luminosity, the resulting solution cannot be expected to be unique. Because of the abovementioned reasons, we prefer to adjust the nuclear energy generation rate, which is mainly controlled by only one parameter, and solve the equations without allowing for any variation in Z. This solution is unique for a prescribed Z profile and input microphysics.

It is not obvious that such a choice will indeed produce the correct X profile since there may be some errors in opacity or the nuclear energy generation rates or in the adopted Z profile. The errors in the X profile due to those in the primary inversions can be estimated by perturbing the inferred profiles of sound speed and density. However, it turns out that these errors are fairly small as compared to those introduced by uncertainties in opacities, which can be estimated by trying out different prescribed Z profiles. We use for this purpose one of the following three basic Z profiles: (1) a homogeneous profile (denoted by HOM) without any diffusion of heavy elements, (2) a profile including diffusion (Proffitt 1994) indicated as PROF, and (3) another profile using a different treatment of diffusion including rotationally induced mixing just below the base of the convection zone (Richard et al. 1996) identified as RICH. We scale all these profiles to give a prescribed value of Z at the solar surface, $Z_{\rm surf}$ and the calculations are performed for a very large range of values for $Z_{\rm surf}$.

Instead of adjusting the X profile to match the sound speed and density, we could have adjusted the opacity (or, Z profile) to achieve the same purpose as has been suggested by Tripathy & Christensen-Dalsgaard (1996). It may be noted that they have only matched the sound speed profile to calculate the opacity modifications and hence their results cannot be compared with our results. We can easily match both sound speed and density in our approach, since if X and Z profiles are prescribed then the sound speed can be used to calculate the temperature profile. The inferred temperature profile can then be substituted in Eq. 1 to calculate the opacity, which can be converted to Z if necessary. In this case also the computed luminosity will not, in general, match the observed value. We have tried this exercise for a few different X profiles in evolutionary solar models and it turns out that in all cases the extent of modification in the nuclear reaction rates required to get the correct luminosity is of order 2%, which is similar to what we find in Sect. 4. This once again demonstrates that computed luminosity in these models is not particularly sensitive to opacity modifications and is mainly determined by the nuclear reaction rates. Thus, in either case we get similar results and the choice between adjusting X or opacities (i.e., Z) is not altogether clear, but it will not affect our conclusions about the nuclear reaction rates. However, in the region just below the base of the convection zone there are good reasons to believe that the discrepancy in sound speed in standard solar models with conventional treatment of helium diffusion, arises because of a difference in X profile rather than opacity modification (cf., Basu & Antia 1994; Gough et al. 1996; Richard et al. 1996). We therefore stick to modification of X profile in the present study.

Inside the convection zone we can determine the helium abundance independently (Gough 1984; Däppen et al. 1988a; Dziembowski et al. 1991; Kosovichev et al. 1992; Antia & Basu 1994b; Basu & Antia 1995) and the temperature can then be determined using the inverted sound speed. However, in this work we have restricted ourselves to the radiative interior only. The estimated value of X at the base of the convection zone can be compared with the independently estimated values inside the convection zone. Once the T and X profiles inside the Sun are known, it is straightforward to estimate the neutrino fluxes for the various solar neutrino experiments.

3. Inferred T and X profiles

We use data sets of p-mode frequencies from the GONG data (Hill et al. 1996) and from the Big Bear Solar Observatory (BBSO) data (Libbrecht et al. 1990) along with the low degree ($l \leq 3$) modes from BiSON data (Elsworth et al. 1994) to infer the sound speed and density profiles using the Regularized Least Squares technique (Antia 1996). It may be noted that in this inversion we have not assumed any equation of state and hence the density inversion does not depend on the equation of state as in some other techniques (Basu & Christensen-Dalsgaard 1997; Basu et al. 1997). For each data set we use all modes with frequencies in the range of 1 to 3.5 mHz. The results obtained using the GONG data for months 4–10 (23 August 1995 to 30 April 1996) are shown in Fig. 1, which displays the relative difference between inverted profiles and those in the model S of Christensen-Dalsgaard et al. (1996). It may be noted that although in this paper we have represented all differences with respect to Model S, the inversions were done using different reference models. The choice of Model S to display results is only to make the comparison with other workers simpler. The most significant difference in the sound speed between the model and the Sun occurs just below the base of the convection zone and is very likely on account of the X profile in the Sun being smoother than that in the model (Basu & Antia 1994; Gough

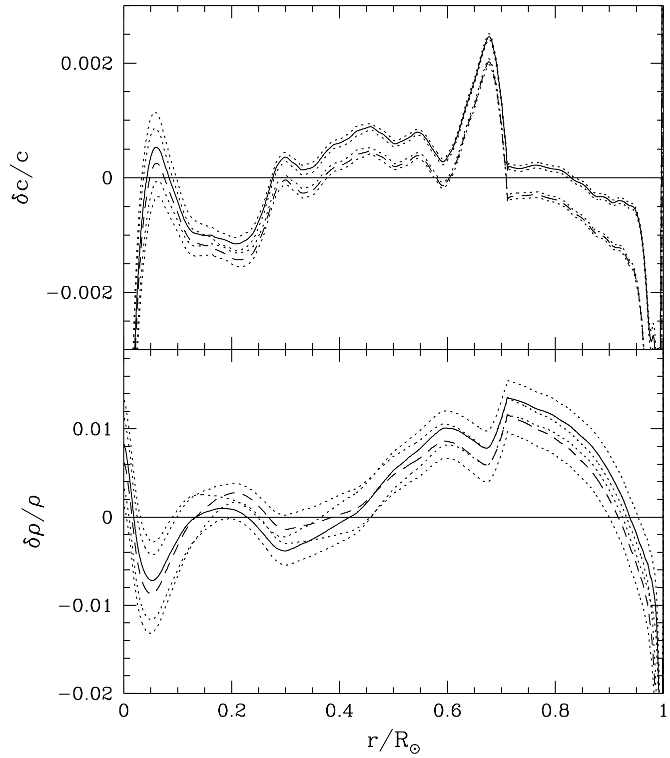

Fig. 1. Relative difference in sound speed and density between the Sun and the Model S of Christensen-Dalsgaard et al. (1996) as inferred using two different estimates for solar radius. The continuous and dashed lines represent the results obtained with estimated radius of 695.99 and 695.78 Mm respectively. The dotted lines show the 1σ error limits.

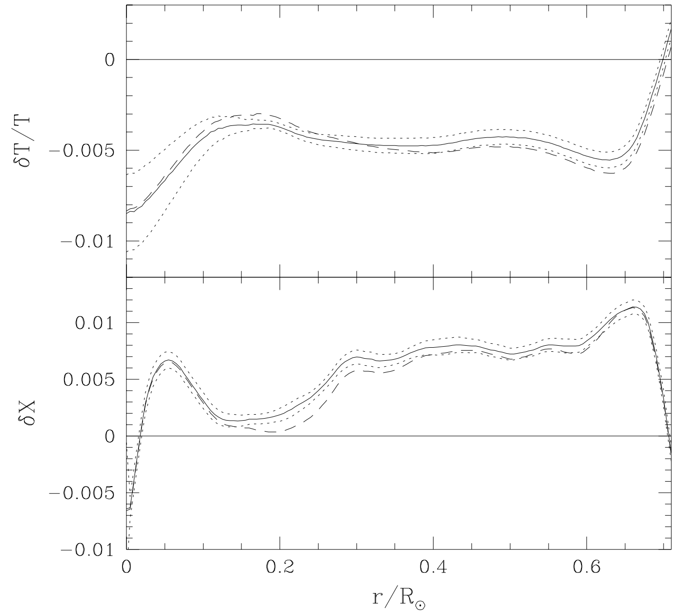

Fig. 2. Relative difference in temperature and absolute difference in the hydrogen abundance X, between the Sun and the Model S of Christensen-Dalsgaard et al. (1996) as inferred using different estimates for solar radius. The continuous and dashed lines represent the results obtained with estimated radius of 695.99 and 695.78 Mm respectively. The dotted lines show the 1σ error limits on the results with estimated radius of 695.99 Mm.

et al. 1996; Basu 1997). Apart from this, another noteworthy smaller hump occurs around $r = 0.2R_\odot$ which is opposite in sign to that below the convection zone. It is likely that in this region the X profile in the Sun is steeper than that in the model. Similarly, the most significant difference in the density profile occurs inside the convection zone and is probably due to small errors in opacity, equation of state, surface abundances and/or the depth of the convection zone (Basu & Antia 1997).

The results from primary inversions have been extensively tested by comparison between various observed data sets and using various reference models and regularization options as well as by comparison with results obtained by other workers. From all these tests it appears that the results are reliable within estimated errors, except very close to the center ($r \lesssim 0.05R_\odot$), where the results are essentially determined by the choice of reference model coupled with regularization. In fact, the sharp dip in the sound speed profile very close to the center in Fig. 1 is due to difference between the Model S and the reference model used for inversion and may not have any particular significance. As a result, in this work we do not make any attempt to interpret the results in the central region. Outside this region the inversion results have been found to be reliable by extensive comparisons. Since the integration of equations for inferring the T and X profiles is also done from the outer boundary, we do not expect the errors in primary inversion near the center to affect the results in outer regions significantly. Moreover, as will be seen later a major source of error in T and X profiles is the uncertainty in the Z profile and reasonable errors in inverted c, ρ profile are not likely to affect the results significantly.

Recently, it has been suggested that the standard value of solar radius (Allen 1973) needs to be reduced (Schou et al. 1997; Antia 1998; Brown & Christensen-Dalsgaard 1998) and it would be interesting to estimate the effect of error in the solar radius on helioseismic inversions. We have also performed inversions with a reduced radius of 695.78 Mm and Fig. 1 compares the results obtained with two different values for solar radius using the same set of observed frequencies from GONG months 4–10 data. It is clear that the small error in solar radius affects the inversion results to an extent which is much larger than the estimated errors due to those in frequencies.

Applying the procedure outlined in Sect. 2 to the inverted profiles for sound speed and density shown in Fig. 1, we obtain the T and X profiles and the results are shown in Fig. 2. All these results have been obtained with the pp reaction rate from Bahcall (1989) and a Z profile including diffusion (RICH) of heavy elements with surface value of $Z = 0.018$. The errors in these calculations arising from estimated uncertainties in the input frequencies can be calculated with a Monte-Carlo simulation. For this purpose we generate 20 sets of artificial frequency data where randomly distributed errors with standard deviation equal to the estimated errors in observed frequencies are added to every input frequency before the primary inversion. The inverted sound speed and density profiles are then used to obtain

Table 1. Properties of seismically inferred models

$R_\odot$ (Mm)	Z-profile	$Z_{\rm surf}$	$X_{\rm surf}$	T_c (10^6 K)	$L/L_\odot$	$\phi(^{37}{\rm Cl})$ (SNU)	$\phi(^{71}{\rm Ga})$ (SNU)	$\phi(^{8}{\rm B})$ (10^6 cm^{-2}s^{-1})
695.99	RICH	0.018	0.7349	15.54	0.970	6.57	124.0	4.69
695.78	RICH	0.018	0.7348	15.54	0.974	6.62	124.6	4.72
695.78	PROF	0.015	0.7612	15.40	0.964	5.64	118.8	3.92
695.78	PROF	0.018	0.7347	15.59	0.977	7.04	127.0	5.07
695.78	PROF	0.020	0.7195	15.72	0.985	8.09	132.7	5.94
695.78	HOM	0.015	0.7613	15.30	0.959	5.04	115.3	3.44
695.78	HOM	0.018	0.7348	15.48	0.971	6.21	122.4	4.39
695.78	HOM	0.020	0.7196	15.59	0.979	7.08	127.4	5.10
Error estimates			0.0008	0.03	0.010	0.30	2.3	0.24
695.78	INV	0.018	0.7378	15.56	1.000	6.67	125.5	4.77

the T and X profiles. The standard deviation in these profiles, at a fixed radius, will give an estimate of errors in inferred profiles arising from those in input frequencies. An independent estimate for errors arising from those in primary inversions can be obtained by using P instead of c in Eq. 3. This gives an increase of $0.007L_\odot$ in the computed luminosity which is comparable to the estimated error (cf., Table 1) obtained from Monte-Carlo simulations.

The errors arising from those in solar radius can also be estimated from Fig. 2 and these are seen to be comparable to those due to uncertainties in frequencies. This is mainly because statistical errors in the inferred T and X profiles are much larger than those in sound speed profile. The properties of these models are summarized in Table 1, which also gives the estimated errors in each quantity due to those in the GONG months 4–10 data. In this table T_c is the central temperature, $\phi(^{37}{\rm Cl})$ the neutrino flux in the Chlorine detector, $\phi(^{71}{\rm Ga})$ the neutrino flux in the Gallium detector, while $\phi(^{8}{\rm B})$ is the flux of ^{8}B neutrinos. It can be seen that a reduction in radius by 210 km increases the computed luminosity by $0.004L_\odot$ and the neutrino fluxes are also correspondingly enhanced by similar amounts. The last line in the table gives the results for a static solar model designated as INV, which is constructed using the inferred X profile as explained in Sect. 5. In the following discussion all the results have been obtained using the GONG months 4–10 data with an estimated radius of 695.78 Mm (Antia 1998).

Apart from this there could be some errors due to uncertainties in the equation of state. It is difficult to get a proper estimate of these errors, but a reasonable estimate can be obtained by repeating the calculations with a different equation of state. Thus, while most of the results were obtained using the OPAL equation of state (Rogers et al. 1996), we have also done some calculations using the MHD equation of state (Däppen et al. 1988b; Hummer & Mihalas 1988; Mihalas et al. 1988). It turns out that the difference between these results is not significant and the computed luminosity decreases by $0.002L_\odot$ when the MHD equation of state is used instead of OPAL. Similarly, the inferred T and X profile are also well within the error limits arising from those in frequencies. It would seem that the inferred T and X profiles are not particularly sensitive to reasonable uncertainties in the equation of state as measured by the difference between the MHD and OPAL equations of state. A possibly more conservative estimate of the influence of uncertainties in the equation of state may be obtained by setting $\Gamma_1 = 5/3$ in Eq. 3, with pressure calculated from the OPAL equation of state. This gives a difference of $0.003L_\odot$ in the computed luminosity.

Of course, the inferred T and X profiles will depend on the assumed Z profile. In order to estimate this effect, we have tried several, different input Z profiles and some of the results are shown in Fig. 3. It is clear that uncertainties in these profiles arising from those in Z are much larger than those due to other effects considered earlier. Further, these uncertainties increase with r, since the effect of Z on opacities decreases as temperature increases because of increasing degree of ionization of heavy elements. There is a large uncertainty in regions immediately below the convection zone; however, in this region the value of Z is more reliably known from the measured value in the convection zone. The discordance between various profiles in Fig. 3 could give an estimate of errors expected from reasonable errors in Z profile. This is obviously much larger than errors arising from uncertainties in sound speed and density profiles.

The absolute X profiles as inferred using different Z profiles are shown in Fig. 4, which also displays the profiles in some standard solar models with different treatments of diffusion. It is evident from this figure that the X profile just below the convection zone is much smoother than that in a standard solar model with conventional treatment of diffusion (Christensen-Dalsgaard et al. 1996) suggesting that some mixing probably takes place in this region (Richard et al. 1996). The X profile in Model 5 of Richard et al. (1996) which includes rotationally induced mixing is closer to the inferred profiles, though it appears to be shifted below the inferred profiles using $Z_{\rm surf} = 0.018$, probably because it has larger $Z_{\rm surf} = 0.019$ (and correspondingly higher Z/X). The shape of the X-profile near the base of the convection zone is essentially independent of $Z_{\rm surf}$, but depends on the actual profile used. Thus, if a flat Z profile like HOM or RICH is used, the resulting X profile is also flat until about $r = 0.68R_\odot$ indicating that this region is essentially mixed. However, if a Z profile with steep gradient near the base

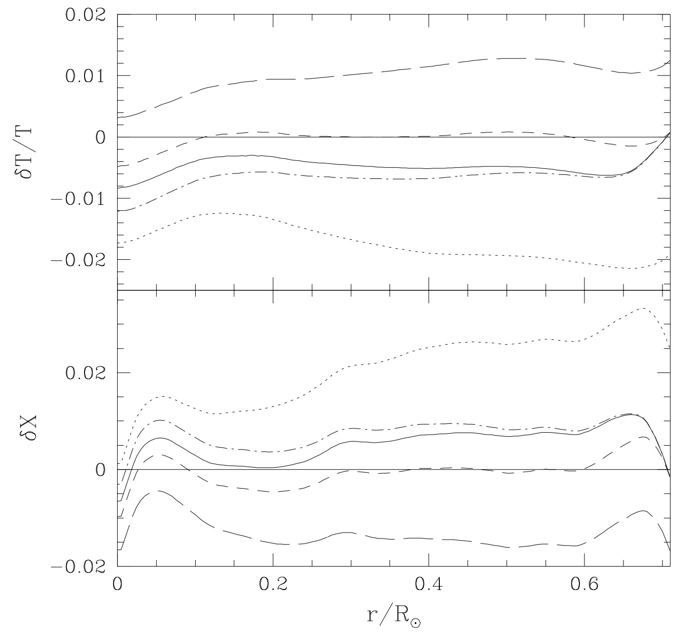

Fig. 3. Relative difference in temperature and absolute difference in the hydrogen abundance X, between the Sun and the Model S of Christensen-Dalsgaard et al. (1996) as inferred using different Z profiles. The continuous line represents the results using the profile RICH, with $Z_{\rm surf} = 0.018$. The dotted ($Z_{\rm surf} = 0.015$), short-dashed ($Z_{\rm surf} = 0.018$) and long-dashed ($Z_{\rm surf} = 0.02$) lines shows the results obtained using the profile PROF, while the dot-dashed line displays the results using a homogeneous Z profile, with $Z = 0.018$.

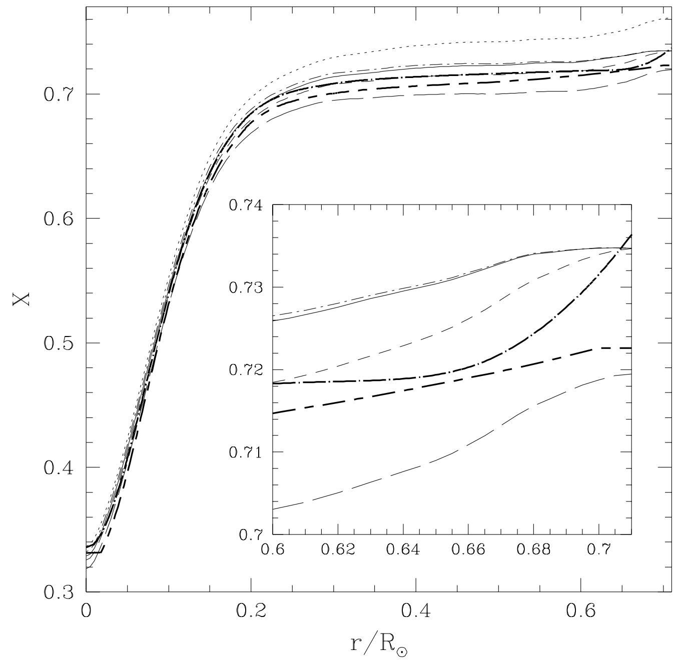

Fig. 4. The hydrogen abundance profile as inferred using different Z profiles. The various line styles have the same representation as those in Fig. 3. In addition the X profiles in the Model S of Christensen-Dalsgaard et al. (1996) is shown by heavy dot long-dashed line, while that in the model 5 of Richard et al. (1996) is shown by the heavy short-dashed long-dashed line.

of the convection zone is used, then the resulting X profile also shows some weak gradient in that region. But in order to get a gradient as steep as that in the X profile of Model S, one requires a Z gradient which is about 5 times that in the profile PROF. Hence, if the X and Z profiles from similar treatment of diffusion in a solar model are used, the resulting profiles will not be consistent with helioseismic data unless some process like turbulent diffusion or rotationally induced mixing is employed to reduce the gradients to zero at the base of the convection zone (Basu & Antia 1994; Basu 1997). We prefer to use the profiles with zero gradient in Z at the base of the convection zone for better accordance.

Notice, around $r = 0.25R_\odot$ the X profile in the Sun is steeper than that in the solar model. This can be seen more clearly from Fig. 2 which shows the difference in X profile between the Sun and a solar model. The steep positive gradient around $r = 0.25R_\odot$ indicates that the X profile in the Sun is steeper than that in the model.

4. Helioseismic estimate for the pp reaction rate

The nuclear energy generation in the solar interior is mainly controlled by the pp reaction rate. The theoretically estimated cross-section for this reaction varies from 3.89×10^{-25} MeV barns (BP95) to 4.21×10^{-25} MeV barns (Turck-Chiéze & Lopes 1993). For convenience we denote the usually accepted value (Bahcall 1989) by $S_0 = 4.07 \times 10^{-25}$ MeV barns, and express the cross-section in terms of S_0. Recently, there has been a claim that the pp nuclear reaction rate should be revised upwards by a factor of 2.9 (Ivanov et al. 1997). Although this claim has been contested (Bahcall & Kamionkowski 1997) on the nuclear physics grounds, it would be nice to have an independent check from helioseismic data (Degl'Innocenti et al. 1997). A comparison of the computed luminosity using prescribed nuclear reaction rates with observed luminosity can impose some constraint on the nuclear reaction rates. However, the inferred profiles depend on the assumed profile for heavy element abundance. We therefore, investigate the effect of an assumed Z profile on integrated luminosity to constrain the nuclear reaction rate.

Apart from the Z profile, there could also be uncertainties in the theoretically calculated values of opacities. In order to obtain constraints which are independent of errors in opacity we can consider arbitrary X profiles. These may not satisfy the equations of thermal equilibrium with any reasonable estimate for opacities. In order to estimate an upper limit on the pp nuclear reaction rate we can try to construct a profile which generates the minimum energy for the given sound speed and density profiles. Since the sound speed essentially constrains the value of T/μ, where μ is the mean molecular weight of the solar material, it seems in order to cut down the energy generation one should reduce T as well as μ to keep the ratio constant. It is clear that the minimum value of μ is achieved when $X = 1$ and $Z = 0$, i.e., when there is no helium or heavy elements present in the central region. From more detailed calculation of energy generation rate, we have verified that this is indeed true, although strictly speaking, since the temperature is not high enough for helium burning reactions, the minimum energy generation occurs when $X=0$, when there is no fuel to burn! But even a value of $X=0.005$ gives much higher energy generation rate as compared to $X=1$ in the core, because the temperature has to be increased when X decreases to keep the sound speed constant.

For the case of a profile with $X=1$ and $Z=0$ we can easily demonstrate that the computed luminosity in the resulting model is about $0.617L_\odot$ when the usual nuclear reaction rates are adopted. Now if we increase the pp nuclear reaction rate for obtaining the correct solar luminosity with this profile, it turns out that the cross-section needs to be increased to about $1.62S_0$. It is clear that if the cross-section is increased beyond this value it is not possible to find any X profile (apart from the one where hydrogen is almost totally exhausted throughout the solar core), which will simultaneously yield the correct sound speed, density and luminosity in these models. The exact limiting value of the cross-section will depend on the inverted sound speed and density profiles, but as we have seen in the previous section these uncertainties are very small. We can therefore, conclude that any value higher than $1.65S_0$ is inadmissible even if arbitrary errors in opacities are allowed and the Sun is assumed to generate the observed luminosity. An increase in the pp nuclear reaction rate by a factor of 2.9 (Ivanov et al. 1997) is certainly ruled out by the helioseismic data. In fact, in actual practice even the profile with $Y=0$ considered in obtaining this limit is unacceptable since one would expect significant amount of helium to be present in the solar core. If we consider a profile with $Y=0.2$, which is still lower than the expected helium abundance, the limiting cross-section for the pp reaction drops to $1.27S_0$. It is therefore evident that, any significant increase in the pp cross-section is demonstrably inconsistent with helioseismic constraints.

In the foregoing discussion we have allowed for arbitrary errors in standard opacity tables. Even though such an analysis helps in illustrating that the helioseismic data are able to put severe constraints on nuclear reaction rates, the resulting bounds on cross-section are highly conservative and are unlikely to be achieved in realistic situations. Further, it is clear that for some reaction rates even an arbitrary variation in opacities does not yield the correct computed luminosity. It would be possible to obtain more meaningful bounds on nuclear reaction rates if one allows only reasonable errors in opacities. There are two problems with this approach; first, it is difficult to define what is a reasonable error in opacity and second, the error in opacities may have arbitrary variation with temperature and density, thus making it difficult to consider all possible variations even within the assumed limits. In order to get some idea of the effect of opacity errors we calculate the models using different Z profiles with a large range of $Z_{\rm surf}$ to see how the computed luminosity varies with Z. In this process, the opacity changes are accounted through changes in Z profiles.

Using the Z profile with diffusion (Proffitt 1994) scaled to different values of $Z_{\rm surf}$ we can calculate the X profiles following the procedure outlined in Sect. 2. The total luminosity and neutrino fluxes in the resulting models are shown in Fig. 5.

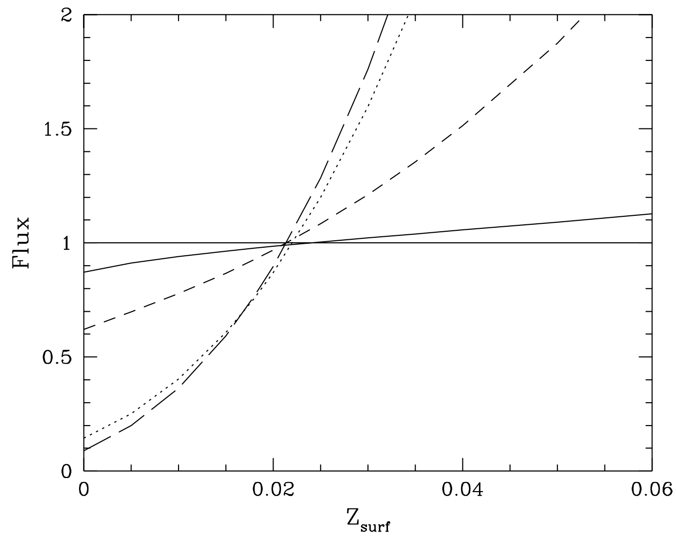

Fig. 5. The integrated luminosity and neutrino fluxes for seismically inferred models as a function of $Z_{\rm surf}$. All these quantities are scaled with respect to the values in the standard solar model of BP95. The continuous line shows the computed luminosity, dotted line shows the neutrino flux in the chlorine experiment, short-dashed line shows the neutrino flux in the Gallium experiment while the long-dashed line shows the ^{8}B neutrino flux.

It is clear that the integrated luminosity goes up with $Z_{\rm surf}$ as a result of increase in opacities, but not very significantly – a variation of $Z_{\rm surf}$ from 0 to 0.06, results in an increase in the luminosity from $0.87L_\odot$ to $1.13L_\odot$. The range of Z values covered by these models is in all probability much larger than the expected uncertainties in the Z profile. With the allowance of an error of 50% in Z ($0.01<Z<0.03$), one gets an error of about 4% in computed luminosity. Although we have only considered profiles where Z is scaled uniformly by the same factor, it would appear that if Z is changed by a varying factor within a specified maximum variation, the effect would be smaller than what is obtained by changing Z everywhere by the same factor. It may also be noted that Z profiles of the type considered by Levy & Ruzmaikina (1994) where Z is much lower than the surface value, will result in very low computed luminosity and the nuclear reaction rates may need to be enhanced well beyond the normally accepted range. It may be noted that Degl'Innocenti et al. (1997) have used a much lower uncertainty of 10% each in Z and opacities to estimate the cross-section of pp reaction.

Apart from opacities (or Z profile) there are other uncertainties in computing the luminosity arising from errors in primary inversion (2%), solar radius (0.4%), equation of state (0.3%) and those in other nuclear reaction rates (1%). The last contribution can be estimated by varying each of these reaction rates by their estimated errors (BP95) and recomputing the model. The individual contributions can then be added to estimate the total effect of all reaction rates. The first contribution arising from primary inversion has been increased by a factor of 2 to include systematic errors. These are found to be comparable to or smaller than the statistical errors, when we compare different results obtained using independent data sets, regularization options, reference models and inversion techniques. With all these independent uncertainties included, we believe that a reasonable estimate for error in computed luminosity is 5%. Thus the integrated luminosity is consistent with the observed value within these uncertainties for a reasonable Z profile. It should be noted that all these results are obtained using the pp reaction cross-section to be S_0. If the recent value adopted by BP95 ($0.9558S_0$) were to be used, the computed luminosity would be about 4% lower, while for the normal value of Z the computed luminosity would be significantly lower than the observed value. This leads us to surmise that the cross-section for the pp-reaction rate needs to be increased to its earlier value given by Bahcall (1989). Similar conclusions were also reached earlier by Antia & Chitre (1995).

In order to obtain a proper estimate for the cross-section of pp reaction, we try to compute the luminosity using different values for the cross-section of the pp reaction, with Z profile PROF and the normal value of $Z_{\rm surf} = 0.018$. From these results we can identify the range of cross-section values which yield the computed luminosity within 5% of the observed value. This can be treated as the helioseismic estimate for the cross-section of pp reaction, which turns out to be $(4.15 \pm 0.25) \times 10^{-25}$ MeV barns, where the quoted errors correspond to an uncertainty of 5% in the computed luminosity. This range is consistent with the value adopted by Bahcall (1989), but slightly larger than the more recent value adopted by BP95. Our estimate of cross-section for pp nuclear reaction is consistent with that obtained by Degl'Innocenti et al. (1997).

5. The direct technique

A possible drawback of the technique described in Sect. 2 to obtain the T and X profiles is that the computed luminosity does not match the observed value, unless the nuclear energy generation rate is slightly adjusted. It may be argued that modification of the nuclear energy generation rate may introduce some non-uniqueness in the solution depending on how the free parameter is introduced in calculating ϵ. However, since the adjustment is generally small (of order 2%) for usually accepted Z profiles, the results are not expected to be significantly affected by the modification. In order to test this we attempt a direct approach, similar to that followed by Tripathy & Christensen-Dalsgaard (1996) to estimate the opacity correction. In this method we construct a static solar model with the standard equations of stellar structure, but adjust the X profile to match the inverted sound speed and density profiles. Following the usual procedure, the mixing length and surface value of X are adjusted to obtain the correct radius and luminosity, using a prescribed Z profile and standard microphysics. Thus, we can express the X profile in terms of cubic B-spline basis functions:

$$X(r) = X_0(r) \left(1 + \sum_{i=1}^{n_s} a_i \phi_i(r) \right), \qquad (5)$$

where $X_0(r)$ is the X profile in some standard solar model and $\phi_i(r)$ are the cubic B-spline basis functions based on suitably

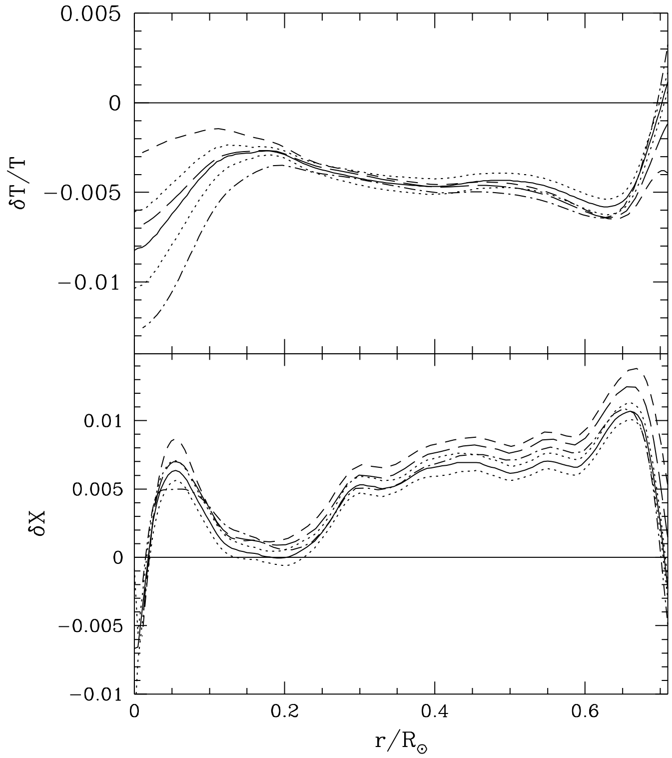

Fig. 6. Relative difference in temperature and absolute difference in the hydrogen abundance X, between the Sun and the Model S of Christensen-Dalsgaard et al. (1996) as inferred using the direct technique with different values of pp reaction cross-section, S_{11}. The short dashed, long dashed and dot-dashed line respectively, represents the results using $S_{11} = S_0, 1.02S_0$ and $1.05S_0$. The continuous line represents the results using the technique described in Sect. 2 with the same Z profile, while the dotted lines represent the 1σ error limits.

chosen knots and a_i are the unknown coefficients of expansion and n_s is the number of basis functions. We use knots with a spacing of 0.02–0.04$R_\odot$, with smaller spacing near the center. This X profile is scaled by multiplication with an adjustable constant to obtain the correct luminosity while constructing the solar model. The coefficients a_i are then determined by minimizing the function

$$F = \int_0^{r_b} \left(\frac{c - c_{\rm inv}}{\sigma_c} \right)^2 \frac{dr}{R_\odot} + \int_0^{r_b} \left(\frac{\rho - \rho_{\rm inv}}{\sigma_\rho} \right)^2 \frac{dr}{R_\odot}, \qquad (6)$$

where $c_{\rm inv}$ and $\rho_{\rm inv}$ are the inverted sound speed and density profiles, and σ_c and σ_ρ are the estimated errors in primary inversions. This is a non-linear least squares problem and we have tried the method of simulated annealing (Vanderbilt & Louie 1984; Press et al. 1993) or the direction set method (cf., Antia 1995) to determine the coefficients. Of course, this technique requires much more computation as compared to the method described in Sect. 2.

Fig. 6 shows the resulting T and X profiles obtained using the Z profile RICH with $Z/X = 0.0245$ at the surface and different estimates for the pp nuclear reaction rate, S_{11}. This figure also shows the profiles inferred in Sect. 3 (which is more or less

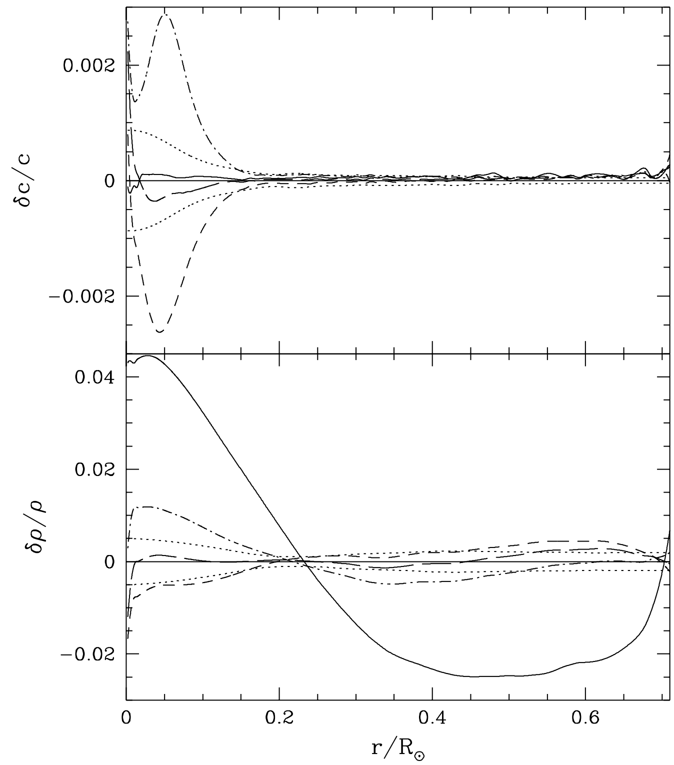

Fig. 7. Relative difference in sound speed and density between the Sun and the solar models obtained by using the direct technique with different values of pp reaction cross-section, S_{11}. The short dashed, long dashed and dot-dashed line respectively, represents the results using $S_{11} = S_0, 1.02S_0$ and $1.05S_0$. The dotted lines represent the 1σ error limits in primary inversions. The continuous lines represent the results obtained using $S_{11} = 1.1S_0$ but when only the sound speed profile is fitted to the inverted profile.

independent of nuclear reaction rate since that is adjusted to obtain the correct luminosity), with the same Z profile. It can be seen that the profiles obtained using the direct technique with $S_{11} = 1.02S_0$ (which is the estimated cross-section) agrees very well with those estimated in Sect. 3. For other values of pp reaction rate there is some noticeable difference in the central region. The estimated X at the base of the convection zone also depends to some extent on the nuclear reaction rates, since the profile is scaled to obtain the correct luminosity, and the scaling depends on the nuclear reaction rates. The good agreement between the two inferred profiles demonstrates the reliability of the technique described in Sect. 2, which is far more efficient as compared to the direct method.

Although the profiles obtained using the direct technique satisfy the global constraint on the luminosity accurately, the resulting sound speed and density profiles may not necessarily match the inverted profiles. Fig. 7 shows the difference in the sound speed and density profiles in these models and those in the Sun as inferred by primary inversions for different values of S_{11}. It is clear that although for $S_{11} = 1.02S_0$ the profiles match very well with the inverted profiles, the agreement is not particularly good when the nuclear reaction rate is different from this value. This result again supports our estimate for the cross-section of pp reaction rate. Thus it is clear that only for certain nuclear energy generation rates it is possible to fit both the inverted sound speed and density profiles simultaneously. We also find that if instead, we try to fit only the sound speed profile, then it is possible to obtain good fit over a wide range of nuclear reaction rates and in that case the density profile departs from the inverted profile when S_{11} differs from the estimated value. The results obtained using $S_{11} = 1.1S_0$ are displayed in Fig. 7, which clearly shows that although the sound speed agrees very well with the inverted profile, the density is significantly at variance. This clearly suggests that density profile is not uniquely determined by the sound speed profile. Thus in order to match the seismic models obtained from primary inversions it is necessary to match both the sound speed and density profiles.

The resulting solar model constructed adopting $S_{11} = 1.02S_0$ has sound speed and density very close to the inverted profiles and also satisfies all the global constraints accurately. We refer to this model as Model INV and the properties of this model have been listed in Table 1. It should be realized that this model satisfying the seismic constraints is probably not unique, as it may be possible to construct different solar models satisfying the helioseismic constraints by modifying the opacities or nuclear reaction rates or the Z profiles suitably.

6. Discussion and conclusions

In this work we have shown that the use of sound speed and density profiles obtained from primary inversions enables us to infer the temperature and hydrogen abundance profiles, provided the heavy element abundance profile as well as the microphysics like the equation of state, opacities and nuclear energy generation rates are known. From these profiles we can compute the integrated luminosity, which may not necessarily match the observed value. The difference may arise due to uncertainties in primary inversions, and/or the assumed Z profile, and/or the microphysics. In principle, it is possible to adjust the Z profile to yield the observed luminosity, but it is not clear if the discrepancy may not arise due to other reasons. Moreover, adjusting the Z profile to match only one number, namely, the computed luminosity may result in a non-unique solution. In any case, a very large change in Z profile is required to adjust the computed luminosity even by a few percent and thus changing Z profile within reasonable limits does not necessarily remove the discrepancy between the computed and observed luminosity for all admissible values of nuclear reaction rates. We have therefore, decided to consider other alternatives. We have attempted to estimate the extent to which various uncertainties, including those in Z profile, can influence the luminosity to find that the effect of equation of state or primary inversions on computed luminosity turns out to be fairly small. The dominant uncertainty arises from the nuclear reaction rates and opacities (or equivalently the Z profile). It is difficult to separate out the influence of these two factors, but if we assume a reasonable error in one of these the other effect can be quantified. The computed luminosity is very sensitive to the nuclear reaction rate, while it depends only weakly on Z and we are able to limit the nuclear reaction rates using the luminosity constraint.

It turns out that if we use the nuclear reaction rates adopted by BP95, except for the pp reaction for which the older reaction rate from Bahcall (1989) is used, then the integrated luminosity with the normal Z profile is close to the observed value. It is thus tempting to conclude that these nuclear reaction rates, together with the current opacity tables and a Z profile including diffusion are consistent with helioseismic data. Similarly, from a detailed study of the base of the convection zone it appears that uncertainties in the current opacities at the base of the convection zone as well as the estimated Z/X values (Grevesse & Noels 1993) are fairly small (Basu & Antia 1997). One expects opacities to be more reliably determined in the solar core where temperatures are upwards of several million degrees. It is therefore reasonable to assume that there are no significant uncertainties in current OPAL opacity tables in the solar core. Of course, there could be some error on account of an inappropriate Z diffusion profile, but that is not expected to be too large. Even when we account for an uncertainty of 50% in Z, the resulting uncertainty in the computed luminosity is only 4%.

It should be noticed that different values of cross-section for the pp-reaction have been adopted by various workers (Bahcall 1989; BP95; Turck-Chiéze & Lopes 1993; Dar & Shaviv 1996) and recently, Ivanov et al. (1997) have even suggested an increase in the pp reaction rate by a factor of 2.9. Since there is no experimental measurement of this cross-section, it would be interesting to indulge in an exercise to estimate this cross-section helioseismically. From our results in Sect. 4 it is clear that an increase in this reaction rate by a factor of 2.9 is essentially ruled out, even when arbitrary variations in opacities are allowed. In fact, even an increase by a factor of 1.3 in the pp reaction rate is inconsistent with helioseismic data, with no restriction on opacity when the helium abundance is constrained to a minimum of 0.2. However, these bounds are too conservative since unrestricted errors in opacity are permitted.

On the other hand, if we should make the assumption that opacities are known to reasonable accuracy and that error is quantified by an uncertainty of up to 50% in Z, then including all other error estimates we find an uncertainty of 5% in computed luminosity and the estimated value of the cross-section for the pp reaction turns out to be $(4.15\pm0.25)\times10^{-25}$ MeV barns. This value is consistent with the estimate of 4.07×10^{-25} MeV barns (Bahcall 1989; Dar & Shaviv 1996) or 4.21×10^{-25} MeV barns (Turck-Chiéze & Lopes 1993), but slightly larger than the value of 3.89×10^{-25} MeV barns adopted by BP95. It thus appears that the estimate of the pp reaction cross-section adopted by BP95, needs to be increased by a few percent. With the adoption of the recent estimate of this cross-section ($0.9558S_0$), the Z profile will need to be modified by about a factor of two to obtain the correct computed luminosity. Thus if we adopt the Z profile PROF, then $Z_{\rm surf}=0.036$ yields the correct computed luminosity. We cannot, of course, strictly rule out such Z profiles, but they appear unlikely to be realized in practice. The amount of heavy elements required to increase the value of Z in the core, where most of the solar mass is located, would be much more than what may be expected at the time of formation of the solar system. A factor of two increase in Z is also beyond the limits obtained by Fukugita & Hata (1998).

The reliability of the inferred seismic profiles from the observed frequencies has been demonstrated by a direct approach, where the X profile in a standard static solar model is adjusted to match the inverted sound speed and density profiles. From the direct approach also it appears that the sound speed and density profiles can be matched satisfactorily only for some nuclear reaction rates and this further supports the estimated cross-section for the pp reaction.

The inferred T and X profiles are found to be close to those in the Model S of Christensen-Dalsgaard et al. (1996). The major noticeable difference arises just below the base of the convection zone, where the inferred X profile is smoother than that in the standard model. The X profile is in fact, sensibly flat in the region $r>0.68R_\odot$. This is probably owing to some process involving turbulent diffusion or rotationally induced mixing just below the base of the convection zone, which is not accounted for in the usual treatment of diffusion (Richard et al. 1996). The temperature at $r=0.68R_\odot$ is about 2.5×10^6 K, which should be enough to burn lithium. Such a mixing could smoothen the composition gradient and also explain the anomalously low lithium abundance in the solar photosphere. It should be stressed that the estimated uncertainties due to errors in the Z profile are fairly large and consequently, significance of the flatness of the profile may not be obvious. However, a mere increase or decrease in the opacity by a constant factor will not change the nature of the profile as similar results can be obtained for different values of $Z_{\rm surf}$. Only if there is a sharp gradient in modified opacity over this narrow region (or equivalently a sharp gradient in the Z profile) it will be possible to obtain composition profiles which are not flat just below the convection zone. If the gradient in Z profile were to be increased by a factor of five over that in Proffitt (1994), it would be possible to get an X profile with gradient similar to that in Model S at the base of the convection zone. Thus, composition profiles obtained using similar treatment of diffusion for both helium and heavy elements are not consistent with inverted profiles unless the gradient vanishes as in the case of rotationally induced mixing (Richard et al. 1996). These results are consistent with conclusions drawn from the oscillatory signal in the frequencies (Basu & Antia 1994; Basu 1997), which also supports the presence of mixing in this region. Similar evidence is also suggested by the inversion of sound speed (Gough et al. 1996). All this seems to indicate that the region just below the convection zone is probably mixed (Richard et al. 1996) by some process.

In contrast, in the central region around $r=0.25R_\odot$ the composition profile in the Sun appears to be steeper than that in the solar model, perhaps suggesting that mixing is unlikely to have occurred in this region of solar interior. From Fig. 2 it can be seen that δX has a negative gradient in the inner core around $r=0.1R_\odot$, which would imply that the X profile in the Sun is smoother than that in the model. This difference has presumably been considered as a hint of mixing in the core (Gough et al. 1996). However, considering the fact that the X gradient is very steep in this region, the difference is extremely small and mixing if any, could only have taken place in the very early history of solar evolution or the mixing process is extremely slow. A more likely cause of this difference is the errors in nuclear reaction rates. It is also possible that this difference could arise from uncertainties in the primary inversion in the core.

Using the inferred T and X profiles it is possible to estimate the neutrino fluxes. From the results shown in Table 1, it appears that these neutrino fluxes are significantly lower than those in the standard solar model of BP95, with diffusion of helium and heavy elements. Some of the difference could be due to somewhat lower cross-section for pp reaction used by BP95. A part of the difference will also arise from the diffusion of heavy elements. As argued earlier there are good reasons to believe that the region immediately below the convection zone is mixed and hence the heavy element abundance will not increase as steeply as in the model of BP95. A reduction in Z value inside the core will reduce the opacities and hence the temperature and the corresponding neutrino fluxes. However, the computed neutrino fluxes in these seismically inferred models are significantly larger than the observed values. In fact, it has been found (Antia & Chitre 1997) that even if arbitrary variations in opacities are allowed it is not possible to reduce the neutrino fluxes in any two solar neutrino experiments simultaneously to the observed values. Similar conclusions have also been reached by arguments that are essentially independent of any solar model (Heeger & Robertson 1996). Thus, it appears that the solution of solar neutrino problem should be sought in terms of neutrino properties, though the seismically inferred models can be used to constrain the particle physics solutions. Since the neutrino fluxes in the standard solar model of BP95 are somewhat larger than those in the seismically inferred models, the constraints on the particle physics solution (e.g., Hata & Langacker 1997) could change when seismic models are used instead of the standard solar model.

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