Astron. Astrophys. 339, 272–277 (1998)

ASTRONOMY AND ASTROPHYSICS

Trojan collision probability: a statistical approach

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Received 2 June 1998 / Accepted 27 July 1998

Abstract. We study the long term evolution of the collision probability $< P_i >$ and of the impact velocity v_{imp} in the two Trojan asteroid swarms. The new mathematical formalism by Dell'Oro and Paolicchi (1998) has been used since, in the calculation of the collision probability, it allows to account for the dynamical links among the Trojans and Jupiter orbital angles, due to the 1:1 resonance. This statistical method permits to compute both $< P_i >$ and v_{imp} over a long timespan (we considered 1 Myr) without making use of heavy numerical integrations. Moreover, it allows to easily update the values of $< P_i >$ and v_{imp} anytime more complete samples of Trojan orbits are available.

The values of $< P_i >$ and v_{imp} over a short timescale have been compared to those of Marzari et al. (1997) and a good agreement has been observed. Over a long timescale the influence of the secular frequency $g5 - g6$ is clearly visible in $< P_i >$. The large oscillations due to the secular frequency are wider for L4 than for L5.

We have considered two different initial samples of orbits. The first is the same sample used by Marzari et al. (1997) and includes the orbits of 114 Trojans. The second, more complete, includes 223 objects. We observe an increase of $< P_i >$ in both the swarms when the more complete sample of Trojan orbits (223) is used. The v_{imp}, instead, slightly decreases compared to the v_{imp} found by Marzari et al. (1997) from the sample of 114 Trojans.

Key words: methods: numerical – celestial mechanics, stellar dynamics – minor planets, asteroids

1. Introduction

All the populations of small bodies in the Solar System are significantly affected, at different extent, by mutual collisions. During the 4.5 Byr of Solar System history the size and orbital distribution of Main Belt asteroids, Trojans and Kuiper Belt comets have been smoothly shaped by cratering and fragmentation events. A record of this impact history is provided by the numerous craters observed on the surface of Gaspra, Ida and Mathilda, and by the Hirayama families. In order to model quantitatively the collisional evolution of these populations and reconstruct their origin, sophisticated numerical models have been developed (Davis et al., 1989,1994; Davis and Farinella 1997; Marzari et al., 1997) to predict the amount of collisional grinding occurred during the Solar System evolution. These models, to be accurate, require precise estimates of the typical collisional frequencies and impact velocities of the population.

For Main Belt asteroids, Farinella and Davis (1992) and Bottke and Greenberg (1994) have adopted a statistical approach based on Wetherill's theory (1967) improved by Greenberg (1982) and Bottke and Greenberg (1993). This theory provides an analytical formulation of the collision probability and impact velocity for two bodies on independent Keplerian orbits. Critical assumptions of Wetherill's theory are that the apsides and nodes of the orbits precess uniformly and that the mean anomalies of the bodies are not correlated. To compute the collision probabilities and velocities one needs only the semimajor axis, eccentricity and inclination of the two orbits. This approach is substantially correct if applied to Main Belt asteroids and Edgeworth–Kuiper comets: secular perturbations force the perihelion argument ω and the longitude of node Ω of most bodies in the two populations to precess regularly while the differences in semimajor axis lead to a complete randomization of the mean anomalies. However in the case of Trojans this assumption fails. The libration motion of Trojan asteroids around the L4 and L5 Lagrangian points puts kinematical constraints to the values of their orbital parameters and they are all forced to occupy a limited region of the phase space, strongly correlated to the motion of Jupiter. In computing the probability of collision between two Trojans, we cannot assume that ω, Ω are not correlated and precess and that the difference between the mean anomalies of the two objects $\lambda_1 - \lambda_2$ are random, but we have to consider only those combinations of orbital angles made available by the resonance. An easy test confirming that Wetherill's theory does not work properly for Trojans is given by the collision probability between an asteroid in L4 and one in L5: Wetherill's theory predicts a non–zero collision probability while the two asteroids are always well separated in space.

To overcome this limitation of the Wetherill's method, in alternative one can adopt the "brute force" numerical approach

Send offprint requests to: P. Paolicchi based on the direct integration of Trojan orbits. The actual number of collisions and corresponding impact velocities can be calculated by checking at each timestep the mutual distance between all the asteroids in the sample. This is a very demanding task from the point of view of CPU time, since the computation of the mutual distance between all the couples is more time–consuming than the orbital integration process itself. Moreover, real collisions on a small sample of asteroids occur over long timescales and this dramatically increases the length of the numerical computation. A shortcut allowing some reduction in the amount of computer–time was adopted by Marzari et al. (1997) who extrapolated from the close encounters statistics the collision probability value. They integrated the orbits of 114 Trojans for 10^4 years with a parallel computer recording all close encounters and their minimum distance and velocity. By interpolating the frequency of close encounters as a function of the minimum distance they obtained a resonable approximation of the intrinsic collision probability $< P_i >$ and the distribution of impact velocities in both the Trojan swarms. They repeated also the 10^4 years–long numerical integration using a second set of initial orbital parameters obtained propagating forward in time for 10^5 years the first set of orbital parameters. Significant variations were observed in the values of $< P_i >$ between the first and second integrations. This was ascribed to secular perturbations in the Trojan orbits, but it was impossible to investigate the real behaviour of $< P_i >$ as a function of time for the excessive amount of computer resources required.

In order to study the long term variability of $< P_i >$ and of the average impact velocity v_{imp} in the Trojan swarms without making use of long and cumbersome numerical integrations, we have adopted in this paper the new statistical method of Dell'Oro and Paolicchi (1998; hereinafter Paper I). This method is based on a mathematical formalism which allows us to overtake the restrictive dynamical hypotheses which are at the basis of the canonical version of the Wetherill's method. The collision probability is computed as an integral in the orbital element space where it is possible to define a suitable density function describing the distribution of the asteroid orbits in the orbital angles. If the perihelia and nodes process regularly, as for most Main Belt asteroids, this density function is simply constant all over the angular range. For Trojans, we have used the density function of a harmonic oscillator to simulate the libration around L4 and L5 of λ in a Jupiter–fixed reference frame. The true amplitudes of libration have been used in the computation of $< P_i >$, derived by a direct numerical integration of real Trojans orbits within a full 6–body problem with all the four giant planets included. The results we obtain show a good agreement (within 10%) with the values of $< P_i >$ and v_{imp} of Marzari et al. (1997). The behaviour of $< P_i >$ and v_{imp} has then been analyzed over a timespan of 1 Myr to cover the nodal precession periods which range from 4×10^4 to 2.5×10^5 years (Milani, 1993). The advantage of this method is related to the short computing time required, so that the values of $< P_i >$ and v_{imp} can be easily updated as the completeness limit for Trojans decreases and observative biases are compensated.

In Sect. 2 we summarize the technical details of the general statistical method of Dell'Oro and Paolicchi (Paper I) and describe in details the modifications performed to adjust the method to the Trojan case. In Sect. 3 we show the results on the long term behavior of $< P_i >$ and v_{imp} and in Sect. 4 we discuss these results.

2. The statistical approach

2.1. The general method

As it was shown in Paper I, the formula for the rate of collision between pairs of bodies in Keplerian orbits can be written as a 3–dimensional integral in the orbital elements of the target and projectile asteroid. We denote the elements of the target by subscript "0" while "p" is used for the projectile. Since no assumption is made on the orbit of the two bodies, the formula gives the same result if the target is exchanged with the projectile. The total number of collisions per unit time between two bodies with given semimajor axis, eccentricity and inclination is computed as:

$$n_T =$$

$$\pi\tau^2 \int\int\int \Delta(f_0, \omega_0, \Omega_0, f_p, \omega_p, \Omega_p) \frac{U}{|J|} df_0 d\omega_0 d\Omega_0 \qquad (1)$$

with f, ω and Ω the mean anomaly, perihelion argument and node longitude, respectively. τ measures the maximum distance between the centers of projectile and target and is given by $\tau = R_0 + R_p$ with R radius of the body. We omit to explicit the dependence on a, e, i in the formula since the integral is performed on the angular variables. U is the relative impact velocity while $|J|$ is the Jacobian of the transformation from the space coordinates x, y, z to the set of orbital elements f, ω, Ω (with a, e, i fixed). The function Δ defines the statistical distribution of the target–projectile pair in the space of the orbital angles. Actually, all the angular quantities in the formula are implicit functions of the keplerian elements of the target. Given f_0, ω_0 and Ω_0, the corresponding values of f_p, ω_p and Ω_p are derived by assuming that target and projectile are in the same place. In other words, the surface of integration is the locus of points where the position of the two bodies is the same while the function Δ determines if the coincidence is allowable according to the dynamics of the two bodies. There are at most four different solutions corresponding to different geometrical configurations of the target orbit with respect to the projectile orbit. All these solutions are considered in the calculation of the integral 1).

As described in Paper I, we integrate formula 1) with a Monte Carlo method in order to treat the large number of pairs formed by all the Trojan asteroids. This method allows to obtain reliable results in reasonably short computing times. For each probability calculation, an error is estimated related to the natural statistical fluctuation involved in every Monte Carlo integration.

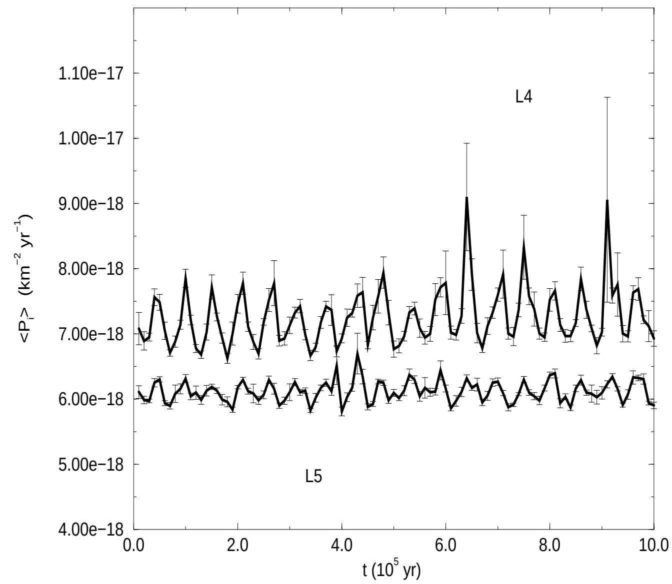

Fig. 1. Intrinsic probability of collision $< P_i >$ computed for the two Trojan swarms over a timespan of 1 Myr. The orbital elements of 114 Trojans have been used, the same sample adopted in Marzari et al. (1997). The error bars express the fluctuation in the probability value due to the use of a Montecarlo method to evaluate the probability integral. The periodic oscillations are related to the $g5 - g6$ secular frequency.

2.2. Application to Trojans

For any pair of asteroids with uncorrelated mean anomalies and with perihelia and nodes precessing at constant rates, the function Δ has an intuitive form. It is assumed to be constant with respect to the angles $\omega_0, \omega_p, \Omega_0, \Omega_p$ and is factorized respect to f_0 and f_p in order to represent the independent Keplerian motion of the bodies respect to the Sun:

$$\Delta = \delta_{kep}(f_p)\delta_{kep}(f_0) =$$

$$\frac{1}{(2\pi)^3}\frac{(1-e_p^2)^{3/2}}{(1+e_p\cos f_p)^2}\cdot\frac{1}{(2\pi)^3}\frac{(1-e_0^2)^{3/2}}{(1+e_0\cos f_0)^2} \quad (2)$$

For Trojan asteroids this form of Δ cannot be used since the motion of the target cannot be disjointed from that of the projectile. Moreover, the longitudes λ_0 and λ_p of target and projectile cannot assume any value regardless of the longitude of Jupiter λ_J: they librate around an equilibrium value λ_L which is $\lambda_J + 60°$ for L4 and $\lambda_J - 60°$ for L5. Few Trojans are also $\tilde{\omega}$ librators (Erdi, 1979): their perihelion longitudes librates respect to Jupiter's perihelion longitude $\tilde{\omega}_J$. To derive a form for Δ that takes into proper account this libration motion, we multiply Δ by four functions: $\Phi(\Delta\lambda_0)$, $\Psi(\Delta\tilde{\omega_0})$ $\Phi(\Delta\lambda_p)$, $\Psi(\Delta\tilde{\omega_p})$ where $\Delta\lambda = \lambda - \lambda_L$ and $\Delta\tilde{\omega} = \tilde{\omega} - \tilde{\omega}_L$. Both Φ and Ψ are periodical functions between $[0, 2\pi]$ with mean equal to 1 and they are density functions of an harmonic oscillator. If we indicate with σ_λ and $\sigma_{\tilde{\omega}}$ the half–amplitude of libration of λ and $\tilde{\omega}$, the functions Φ and Ψ have the following form:

$$\Phi(\Delta\lambda) = \begin{cases} 0 & |\Delta\lambda| > \sigma_\lambda \\ (2/\sigma_\lambda)\left[1-(\frac{\Delta\lambda}{\sigma_\lambda})^2\right]^{-1/2} & |\Delta\lambda| < \sigma_\lambda \end{cases}$$

$$\Psi(\Delta\tilde{\omega}) = \begin{cases} 0 & |\Delta\tilde{\omega}| > \sigma_{\tilde{\omega}} \\ (2/\sigma_{\tilde{\omega}})\left[1-(\frac{\Delta\tilde{\omega}}{\sigma_{\tilde{\omega}}})^2\right]^{-1/2} & |\Delta\tilde{\omega}| < \sigma_{\tilde{\omega}} \end{cases}$$

If the target or the projectile is not a librator in $\tilde{\omega}$ then $\Psi(\Delta\tilde{\omega})$ is simply equal to 2π. The appropriate form of the function Δ, that accounts for the Trojan libration motion, is then the following:

$$\Delta \propto$$

$$\delta_{kep}(f_p)\delta_{kep}(f_0)\Phi_0(\Delta\lambda_0)\Phi_p(\Delta\lambda_p)\Psi_0(\Delta\tilde{\omega}_0)\Psi_p(\Delta\tilde{\omega_p}) \quad (3)$$

In the integral 1) giving the frequency of collisions we have to include also the two new variables λ_J and ω_J:

$$n_T = \pi\tau^2 \int\int\int\int\int \Delta\frac{U}{|J|}df_0 d\omega_0 d\Omega_0 d\lambda_J d\tilde{\omega}_J \quad (4)$$

The values of σ_λ and $\sigma_{\tilde{\omega}}$, used in the definition of the harmonic density functions Φ and Ψ, are computed for each pair of Trojans in the two swarms via numerical integration. The Everhart's (1985) numerical integrator is used to calculate the orbits of Trojans for a time span of 10^6 yr in the context of a six–body problem (Sun, the four Jovian planets and the assumedly massless asteroid). Averaged values of $a, e, sin(i)$ and of σ_λ are computed every 10^4 years while the libration amplitude ($\sigma_{\tilde{\omega}}$) is given every 10^5 years. The mean orbital elements and the libration amplitudes are used to compute the integral 4) for each Trojan pair and then all the probabilities are added up to give the $< P_i >$ of the whole Trojan populations in L4 and L5. To estimate the statistical fluctuation in the calculation of the collision probability due to the use of the Monte Carlo method, we repeat each computation 10^2 times with different $N = 10^6$ random numbers. From these 10^2 values we derive the mean and the standard deviation of the mean σ_M.

We have used the Marzari et al. (1997) sample of 114 Trojans in our computations and also a more complete sample of 223 objects derived from the updated asteroid data file originaly presented by Bowell et al. (1993).

3. Evolution of $< P_i >$ and V_{imp} on a long timescale

As pointed out in the introduction, the utility of a statistical method for calculating both $< P_i >$ and V_{imp} is mainly related to the small amount of computing time required. The variation of $< P_i >$ and V_{imp} over a long timespan can be analyzed and the calculation can be repeated easily as soon as more complete samples of the asteroid population are available. In this section

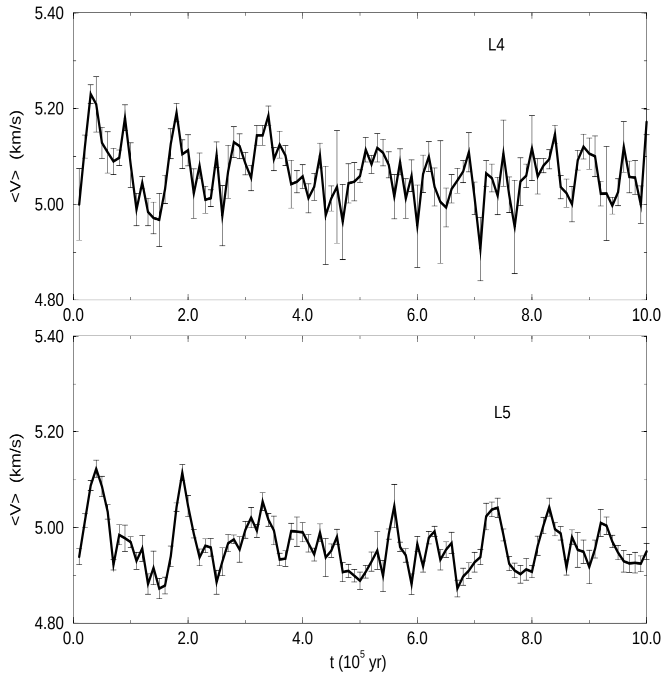

Fig. 2. Impact velocity in the two Trojan swarms.

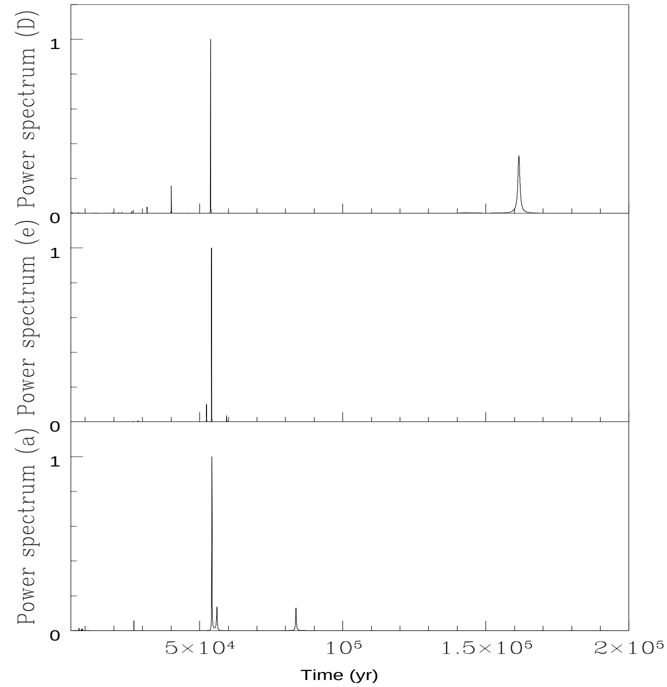

Fig. 3. Power spectra of the semimajor axis, eccentricity and libration amplitude of asteroid 588 Achilles. The dominating spectral line is around T=$\sim$ 5.5×10^4 years corresponding to the $g5 - g6$ secular frequency.

we show the evolution of $< P_i >$ and V_{imp} for both the Trojan swarms over a timescale of 10^6 years. We have used the same set of orbital elements (114 asteroids) used by Marzari et al. (1997) in order to compare with their results. We have also considered an updated sample of 223 Trojans to test the stability of the values of $< P_i >$ and V_{imp} when a more complete sample of asteroids is adopted.

In Fig. 1 we plot the values of $< P_i >$ for both the swarms; the mean values over the whole timespan are $7.26 \pm 0.44 \times 10^{-18}$ for L4 and $6.12 \pm 0.16 \times 10^{-18}$ for L5. The impact velocities are plotted in Fig. 2 and the corresponding average values are 5.06 ± 0.06 km/s for L_4 and 4.96 ± 0.05 km/s for L_5. To test if our statistical results are in agreement with the numerical estimates reported in Marzari et al. (1997), we have to compare our values of $< P_i >$ and V_{imp} computed at t=10^4 yr (hereinafter STA1) and at t=1.1×10^5 yr (hereinafter STA2) with the corresponding values from the CAD1 and CAD2 simulations, respectively, in Marzari et al. (1997). The CAD1 and CAD2 simulations give in fact the $< P_i >$ and V_{imp} as outcome of a numerical integration spanning 10^4 yr starting from time 0 and time 10^5. In Table 1 we compare the values of $< P_i >$ and V_{imp} for STA1 vs. CAD1 and for STA2 vs. CAD2. A difference of about 10% is observed between the values of $< P_i >$ in L4 while for L5 there is a difference of up to 18% for CAD1 vs. STA1, which reduces to only 7% for CAD2 vs. STA2. The impact velocity in the two swarms differ by only 2%. These differences may be ascribed also to the fact that the results of Marzari et al. (1997) are computed with a numerical integration over 10^4 yr while our statistical results are obtained using average orbital elements over 10^4 yr.

The interesting aspect of the $< P_i >$ time evolution is its oscillation with an approximate period of 5.4×10^4 years. This oscillation is related to the secular frequency $g5 - g6$ (period $T = 54655.4$ yrs; Laskar, 1988) which strongly affects the semimajor axis a, the eccentricity e and the libration amplitude D of all Trojans. In Fig. 3 we plot the spectra of the changes in a, e and D of asteroid 588 Achilles whose orbit has been numerically integrated for 4×10^5 years. In the interval of periods between 5×10^3 and 2×10^5 yr, the dominating spectral line is due to $g5 - g6$, as already noted in Milani (1993). It is not easy, instead, to interpret the long term evolution of the impact velocity. The scenario is more complex since the inclination of Trojans contributes significantly in modulating the values of V_{imp}, and the inclination is perturbed by longer period secular terms. We repeated the computation of the $< P_i >$ and V_{imp} with a more complete sample of 223 Trojan asteroids derived from the data base of Bowell et al. (1993). In Table 2 we report the average values of $< P_i >$ and V_{imp} over 10^6 years compared to those derived by the 114 asteroid sample.

We observe an increase in $< P_i >$ and a decrease in V_{imp}. The larger standard deviation in the (223) sample depends on the wider oscillations in $< P_i >$ as shown in Fig. 4.

For both the sample of asteroids we observe larger values of $< P_i >$ and, to less extent, of V_{imp} in L4 with respect to L5. Even the oscillations due to the secular frequency $g5 - g6$ are larger in L4. We do not have a clear explanation of this phenomenon which depends on a different distribution of the orbital parameters in the two swarms. It could be someway related to a different evolutionary history (collisional?) in the two swarms. In Fig. 5 we show the evolution with time of the impact ve-

Table 1. Comparison with Marzari et al (1997).

	$< P_i >_{L4}$ $\times 10^{-18} km^{-2} yr^{-1}$	$< P_i >_{L5}$ $\times 10^{-18} km^{-2} yr^{-1}$	V_{imp} (L4) (km/s)	V_{imp} (L5) (km/s)
STA1	7.08 ± 0.25	6.11 ± 0.10	4.99 ± 0.07	4.94 ± 0.02
CAD1	6.36 ± 0.09	4.98 ± 0.10	4.88 ± 0.07	4.82 ± 0.10
STA2	7.21 ± 0.08	6.04 ± 0.05	4.99 ± 0.06	4.93 ± 0.02
CAD2	6.56 ± 0.09	5.62 ± 0.11	4.91 ± 0.07	4.96 ± 0.10

Table 2. Average values of $< P_i >$ and V_{imp} for the two swarms. (114) refers to the sample of Trojan orbits used in Marzari et al. (1997), (223) refers to a recent sample obtained by Bowell et al . (1998).

	$< P_i >_{L4}$ $\times 10^{-18} km^{-2} yr^{-1}$	$< P_i >_{L5}$ $\times 10^{-18} km^{-2} yr^{-1}$	V_{imp} (L4) (km/s)	V_{imp} (L5) (km/s)
(114)	7.26 ± 0.44	6.12 ± 0.16	5.06 ± 0.06	4.96 ± 0.05
(223)	7.79 ± 0.67	6.68 ± 0.18	4.66 ± 0.08	4.51 ± 0.05

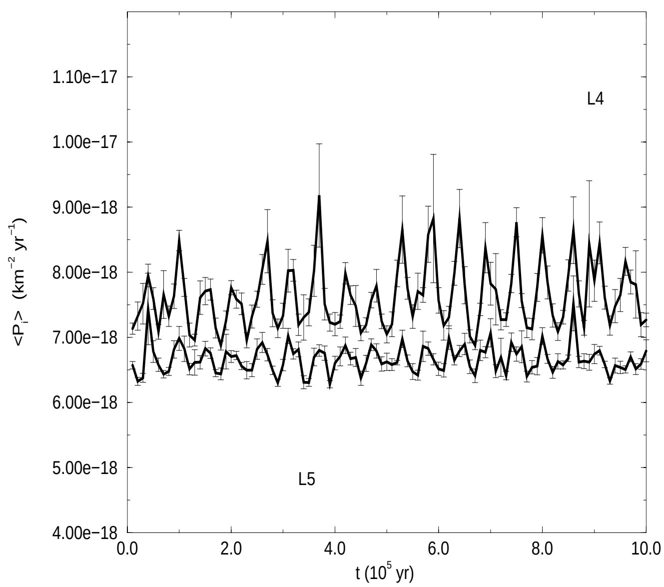

Fig. 4. Intrinsic probability of collision $< P_i >$ computed with a more complete sample of Trojan orbits (223). Larger oscillations around the average values are observed respect to Fig. 1.

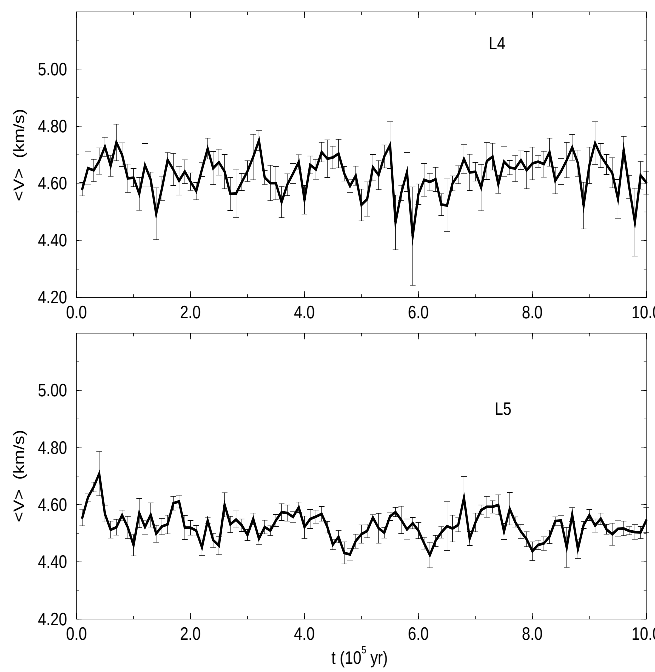

Fig. 5. Impact velocity for the sample of 223 Trojan orbits. The behaviour has to be compared with the one in Fig. 2.

locities for the 223 sample. There are no significant differences between Fig. 2 and Fig. 5, apart from a reduction in the average value of V_{imp} of about 8–9%.

4. Discussion and conclusions

We have analysed the long term behaviour of the collision probability and impact velocity in the Trojan swarms using the statistical method described in Dell'Oro and Paolicchi (1998). This method has been adjusted to the libration motion of Trojans around the Lagrangian points L4 and L5 introducing a density function of a harmonic oscillator in the phase space of the orbital elements. The reduction of the available volume in the phase space increases the probability of collision by about a factor ten respect to an estimate based on a Wetherill's approach which does not include the libration of the critical angle. This shows as any estimate of the probability of collision simply based on an asteroid density derived from observational surveys (Chen et al. 1996) is not meaningful if the libration in the orbital angles is not properly taken into account.

Large oscillations are observed in the $< P_i >$ due to the secular frequency $g5 - g6$ while the impact velocity has a more complex behaviour influenced by the variations in inclination of Trojans. Oscillations of $< P_i >$ in L4 are systematically wider than in L5 and this could be related to a different history of the two swarms from their capture in resonant orbits.

If, according to Levison et al. (1997), there has been a slow leak of large librators from the Trojan swarms, this may have affected the $< P_i >$. A population with an average larger libration amplitude of the critical angle would span a wider volume in the phase space and have a lower impact probability respect to the present one. On the other hand, instability in the Trojan swarms occur also at high e and, as a consequence, the present population might have an average eccentricity lower than in the past. Higher values of e in the past history of Trojans could compensate the possible larger libration amplitudes and keep almost constant the $< P_i >$.

The method we have developed here can be applied to study the collisional evolution of other clustered populations of minor bodies like the Hildas. Trapped in the 3:2 mean motion resonance with Jupiter, the Hildas have a peculiar distribution of the orbital elements caused by the resonance effects. The computation of the collision probability and impact velocity of planetesimals captured in resonances with a proto–Jupiter represents another possible field of application for the method we have developed.

Appendix A: normalization of the Δ function

One major technical problem in applying our statistical formalism concerns the normalization of the Δ function. However, we can show how from a normalized Δ function of f and f_0 it is possible to derive a normalized function in f_0, ω_0, Ω_0, f, ω, Ω simply by multiplying the former for a function of a new variable w, linear combination, with integer coefficients, of $f_0, \omega_0, \Omega_0, f, \omega, \Omega$. Moreover, the new function will be periodical in 2π. The original Δ describing the keplerian motion of the objects $\Delta_{kep} = \delta_{kep}(f)\delta_{kep}(f_0)$, can be easily normalized. The more complex function for Trojans $\Delta \propto \delta_{kep}(f)\delta_{kep}(f_0)\Phi_0(\Delta\lambda_0)\Phi_c(\Delta\lambda_c)\Psi_0(\Delta\tilde{\omega}_0)\Psi_c(\Delta\tilde{\omega})$ must be normalized to 1 with the technique described in this appendix.

We show here the 3–D case for simplicity, but our demonstration applies also to a 6–dimensional space. Let us consider the 3-D space of the variables x, y, z, and inside it, the region R cartesian product of the three intervals $x \in [0, 2\pi]$, $y \in [0, 2\pi]$, $z \in [0, 2\pi]$. $f(x)$ has the property that

$$\int_R f(x)dxdydz = 1$$

Now let be $w = x + y + z$, and $g(w)$ a function periodic of 2π with the property

$$\int_R g(w)dw = 2\pi$$

i.e. its mean value is equal to 1. We show that

$$\int_R f(x)g(w)dxdydz = 1$$

Being f dependent only on x, then

$$\int f(x)dxdy = \frac{1}{2\pi}$$

We consider now the following transformation of variables:

$$x = x \qquad y = y \qquad w = x + y + z$$

Its Jacobian is equal to 1, so:

$$dxdydz = dxdydw$$

The region of integration R is transformed into a region R' in the space x, y, w. It is easy to show, by exploiting the periodicity of the defined functions f and g, that our integral written in the coordinates x, y, w has the following property:

$$\int_{R'} f(x)g(w)dxdydw = \int_R f(x)g(w)dxdydw =$$

$$\frac{1}{2\pi}\int_0^{2\pi} g(w)dw = 1$$

The same result can be obtained if w is any linear combination of the variables x, y and z with integer coefficients.

Acknowledgements. We are grateful to an anonymous referee and to P. Farinella for useful comments and illuminating suggestions. One of us (ADO) has been funded by MURST with a grant. PP has used funds by MURST, Pisa University and ASI.

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