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ASTRONOMY AND ASTROPHYSICS

The growth of Jupiter and Saturn and the capture of Trojans

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Abstract. We have studied the capture of planetesimals in Trojan-type orbits by a growing proto-planet. The change of the gravity field due to the mass growth causes a significant fraction of planetesimals orbiting nearby to be trapped as Trojans of the proto-planet. After a planetesimal is captured on a Trojan-type orbit, the libration amplitude of its critical argument is consistently reduced by the further mass growth of the proto-planet. The dynamical mechanism is discussed and the characteristics of the Trojan population captured by Jupiter during its growth are analysed.

We find an interesting mechanism which could explain the observed high inclination Trojans. The synergy of a Kozai secular resonance with the growth of Jupiter's mass generates high inclination Trojans from low inclination-high eccentricity planetesimals orbiting near the growing proto-planet.

The libration amplitudes of the model Trojans trapped by the mass-growth of Jupiter are higher compared to those of the observed Trojans. A possible mechanism that decreases the libration amplitudes of the model population is collisional evolution.

We also show that the simultaneous formation of Jupiter and Saturn strongly inhibits the capture of planetesimals as Saturn Trojans. The interference of the 1:1 resonance with a secular resonance and, in some cases, also with the 5:2 resonance with Jupiter (Innanen and Mikkola, 1989), generates instability and causes the ejection of most Saturn Trojans out of resonance before the end of Saturn's mass growth.

Key words: celestial mechanics, stellar dynamics – solar system: formation

1. Introduction

The trajectories of planetesimals orbiting in the proximity of a growing proto-planet in the solar nebula are usually modelled as a three-body problem with the mass of the perturber constant (Kary et al., 1993; Marzari et al., 1997). This approximation is based on the assumption that the timescale for proto-planetary growth is long compared to that of the planetesimal dynamical evolution. During the planetesimal accretion phase of terrestrial planets and of the cores of the giant planets, this approximation is generally correct since timescales of the order of $10^6 - 10^7$ are assumed for the growth of these bodies while the timescale for mean motion resonances or gas drag drift is shorter. However, when we consider the final phase of giant planet formation, the change in mass of the planet, due to the rapid gas infall, may have occurred on a timescale comparable with the dynamical timescale of nearby planetesimals. In this case we have to take into account not only the perturbations due to the disturbing function of a constant proto-planetary mass, but also the effects of the time-varying gravity field of the growing proto-planet.

In this paper we concentrate on the dynamical evolution of planetesimals which are orbiting near to a growing proto-planet. We show that the changing gravity field due to the mass growth of the proto-planet causes a large percentage ($\simeq$ 40%) of planetesimals, initially on horseshoe-type orbits, to be captured as L4 or L5 Trojans. Only a small percentage of planetesimals initially close to the resonance border ($\simeq$ 2%) are also captured. Once trapped, the libration amplitude of the critical argument of captured Trojans is damped by the further growth of the planet.

These results were partially discussed in Marzari and Scholl (1998); here we analyse in more detail the characteristics of the captured Trojan population and the sensitivity of the trapping mechanism on orbital parameters of the protoplanet which, in the numerical simulations, is assumed to be a Jupiter-size planet. We also consider different initial orbital elements for planetesimals and we find that the trapping efficiency is higher for low inclination-low eccentricity orbits. A Kozai secular resonance is present around the orbit of the proto-planet (Michel and Thomas, 1996; Michel and Froeschlé, 1997) and this resonance coupled to the mass growth of the planet can generate high inclination Trojan orbits.

We study the case of two growing proto-planets to test the influence of the gravitational perturbations induced by a second planet on the Trojan population of the first one. We consider the case of Jupiter and Saturn and show how a secular resonance and, maybe, the resonant perturbations (5:2 resonance) by the growing Jupiter strongly reduce Saturn's Trojan-trapping efficiency.

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2. Numerical modelling

The orbits of the planetesimals are numerically integrated with the Bulirsch-Stoer method within a three dimensional 3-body model including the Sun, a growing proto-planet (Jupiter size) and massless planetesimals (hereinafter, *JT-J* model). The mass growth of the proto-planet is modelled as an exponential growth:

$$M_J(t) = M_0 + (M_f - M_0)\frac{(exp(t/\tau) - 1)}{(exp(1) - 1)} \quad (1)$$

where M_0 and M_f are the initial and final masses of the proto-planet and τ is the interval of time in which the planet reaches its final mass. As shown in Marzari and Scholl (1998), a linear law for proto-planet growth does not lead to significantly different results. For M_0 we adopt 10 $M_\oplus$ which represents a reasonable value for the proto-planet mass at the end of the planetesimal accretion, while M_f is set equal to the present Jupiter mass. For τ we adopt the value of 1×10^5 yr; smaller values (1×10^4 yr) lead to a slight increase of the trapping efficiency while larger values (1×10^6 yr) appear to have no effect (see Marzari and Scholl, 1998).

In studying Jupiter and Saturn Trojans, we consider a 4-body model where both Jupiter and Saturn have growing masses. Model *JT-JS* refers to Jupiter Trojans with both Jupiter and Saturn growing while model *ST-JS* considers Saturn Trojans with growing Jupiter and Saturn. The initial orbital elements of the two planets were chosen in order to reproduce closely, at the end of the mass growth, the present secular frequencies of their orbital elements.

In order to test the effect of Jupiter perturbations on the orbits of planetesimals near Saturn, in the numerical model we set the gravitational force of Jupiter on the planetesimals equal to 0 while keeping its gravitational influence on Saturn active (hereinafter, *ST-NJS*).

3. How a growing proto-planet traps Trojans

In this section we study the effect of model parameters on the efficiency of the trapping mechanism. It is well known (Szebehely, 1967) that semimajor axis and mass of the perturbing body affects the size of the libration regions around L4 and L5. Changes in the amplitude of the libration zones due to different values of semimajor axis or mass can be accounted for by re-scaling the size of the initial ring of planetesimals. Hence, we concentrate here on the influence of the proto-planet's eccentricity on trapping. In our simulations we adopt a growing proto-planet with the final mass and semimajor axis of Jupiter and we vary its eccentricity.

In Fig. 1 we show the typical evolution of a planetesimal initially orbiting near a proto-planet and captured in a Trojan-type orbit during the mass increase of the proto-planet. The orbit of the body is integrated within the *JT-J* model with the proto-planet on an elliptic orbit with $e_{PP} = 0.05$ (PP refers to proto-planet). The initial horseshoe-type orbit of the planetesimal becomes, after about 2×10^4 yr, a L5 Trojan orbit.

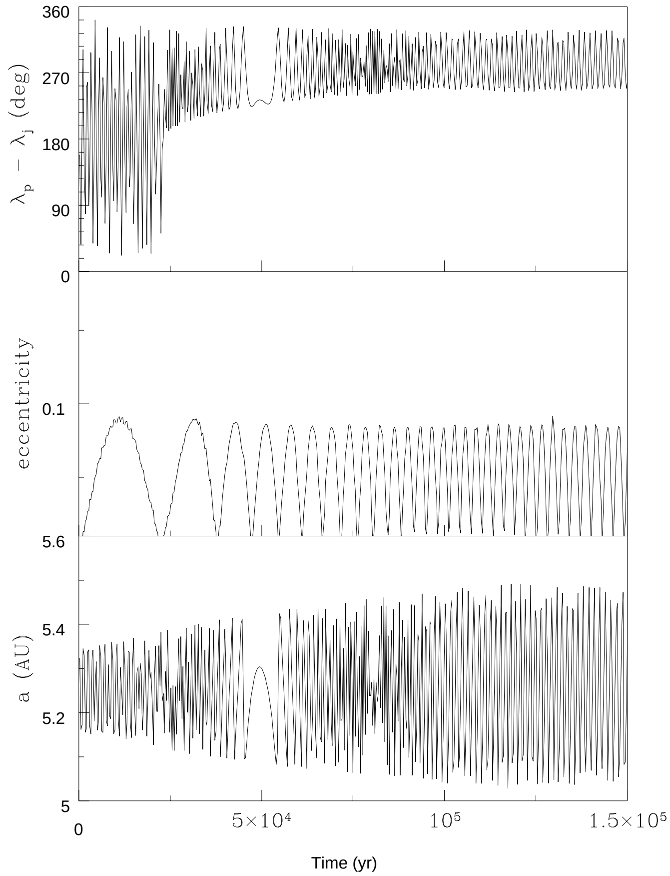

Fig. 1. Capture of a planetesimal in a Trojan type orbit (*JT-J* model). The eccentricity of the proto-planet is 0.05.

Its libration amplitude, which is large just after capture, is subsequently damped by the further increase of the proto-planet's mass; the stability of the resonance lock is increased.

In order to test whether the efficiency of capture in Trojan orbits depends on the eccentricity of the growing planet, we have run the *JT-J* model for larger values of e_{PP}. For $e_{PP} < 0.05$ the trapping rate is constant and in our initial sample 31 bodies out of 80 ($\sim$ 40 %) are trapped into Trojan-type orbits. For higher values of e_{PP}, the evolution of the planetesimal's orbit is characterized, after the trapping, by large oscillations in eccentricity which eventually cause a close encounter with the planet and ejection from the resonance (see Fig. 2). The trapping rate decreases to 16 bodies ($\sim$ 20%) when $e_{PP} = 0.1$ and to 2 bodies only ($\sim$ 3%) for $e_{PP} = 0.2$.

We also tested the dependence of Trojan trapping efficiency on the starting orbital elements of planetesimals in order to map the region in the phase space where trapping is most probable. We first studied the sensitivity of the trapping mechanism on the initial eccentricity of planetesimals. We performed a sequence of simulations with the model *JT-J* assuming different starting values for the planetesimal eccentricity ranging from 0.05 to 0.4. In each run all the planetesimals had the same initial eccentricity and inclinations randomly selected between $0°$ and $5°$. The number of trappings as a function of the initial eccentricity

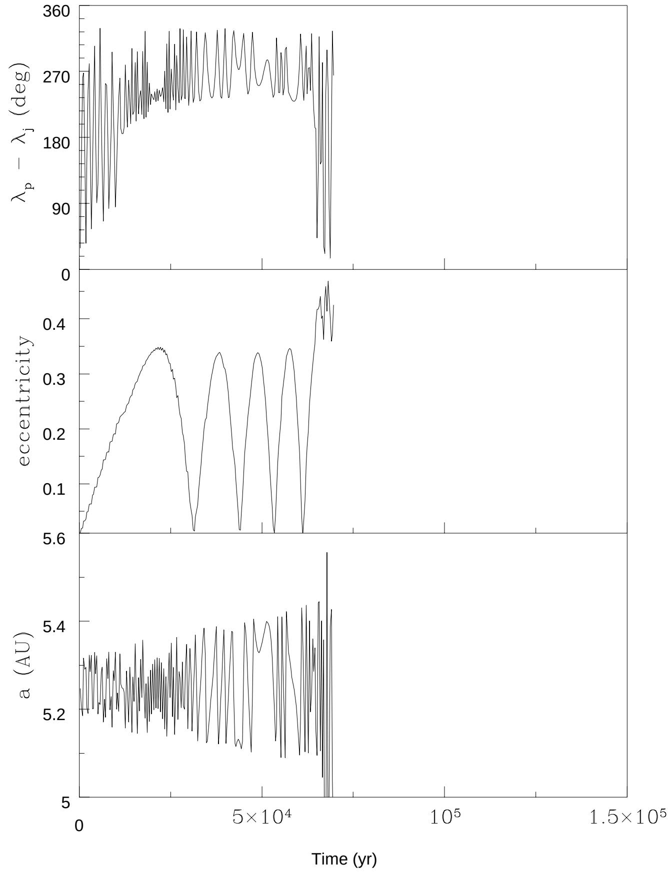

Fig. 2. Capture and subsequent ejection of a planetesimal from a Trojan-type orbit (*JT-J* model). The escape from resonance is caused by a close encounter with the proto-planet having an eccentricity of 0.2.

are shown in Fig. 3. An exponential decrease is observed for large eccentricities, as expected by the lower stability of eccentric Trojans and by the enhanced probability of close encounters with Jupiter.

A strong dependence on the starting inclinations of planetesimals has also been observed in the trapping process. In Fig. 4 we show the number of planetesimals captured in L4-L5 Trojan orbits of an initial set of 100 bodies which all started with the same inclination, i. For i larger than 20° there is a significant decrease in the number of trapped cases which is reduced to zero for i larger than 40°. The same behaviour is not observed for planetesimals starting in Trojan orbits. This means that there is not an intrinsic instability in the Lagrangian points at high i, but that the trapping mechanism becomes inefficient at high inclinations.

The results obtained with model *ST-JS* are not significantly different from those of model *JT-J* and the perturbation of Saturn on both Jupiter and the planetesimal do not affect the capture probability. In Fig. 5 we show the evolution of a planetesimal with the same initial conditions as the case shown in Fig. 1. The planetesimal is trapped in L4 instead of L5. This depends on the different angular distance from Jupiter at trapping.

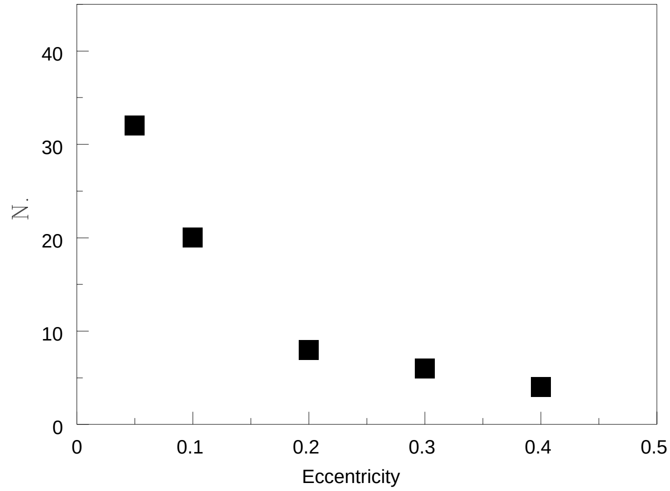

Fig. 3. Number of planetesimals trapped in Trojan-type orbits as a function of their initial eccentricity (model *JT-J*).

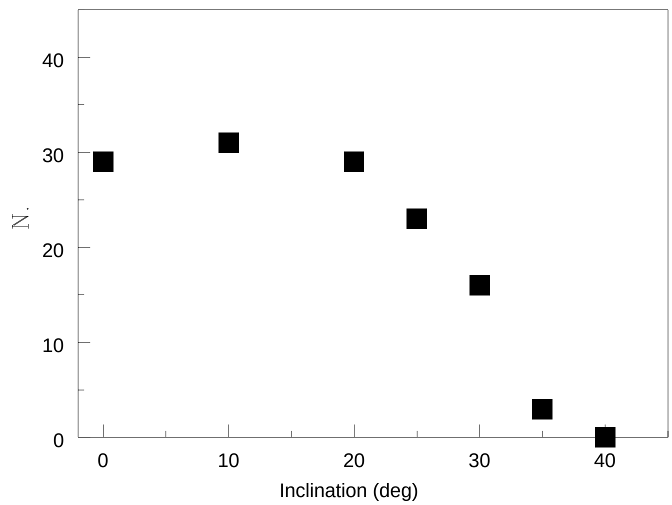

Fig. 4. Starting inclination vs. number of trapped planetesimals in model *JT-J*.

4. Kozai secular resonance and capture of high inclination Trojans

While analysing the results of the simulations with planetesimals started on high eccentricity orbits, we observed an interesting phenomenon related to the Kozai secular resonance. At the beginning of the numerical integration, some planetesimals are trapped in a Kozai resonance with the proto-Jupiter. As shown in Michel and Thomas (1996) and Michel and Froeschlé (1997), the libration region for this resonance surrounds the orbit of the planet. In Fig. 6 we show an example of a planetesimal in the resonance with the argument of perihelium ω librating around 180° (we also found cases librating around 0°) as predicted in Michel and Froeschlé (1997). The libration about 270°, for orbits close to that of the planet, occurs only in the presence of an external perturbing body such as Jupiter for the Kozai resonances around Venus or the Earth. In our model Saturn is too far

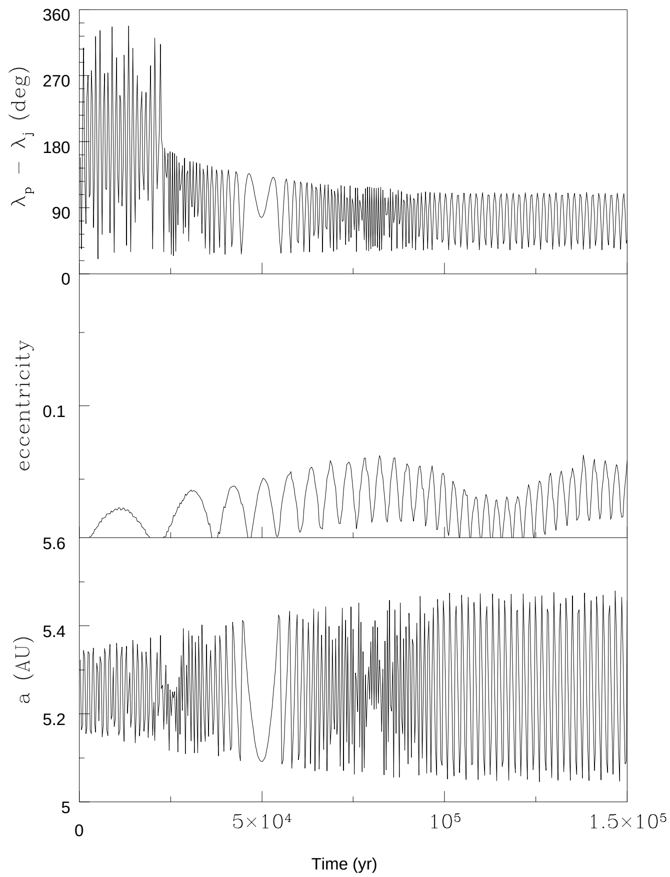

Fig. 5. Planetesimal orbit started with the orbital parameters of Fig. 1 but integrated within the *JT-JS* model (Jupiter and Saturn are both growing).

and too small to really perturb the system Sun - proto-Jupiter - planetesimal.

The orbit of Fig. 6 is calculated within the model *JT-JS* with the masses of both Jupiter and Saturn constant and equal to 10 $M_{\oplus}$. The Kozai secular resonance causes oscillations of the planetesimal eccentricity in anti-phase to oscillations of the inclination (see Fig. 6): when eccentricity e is minimum, inclination i is maximum, and vice versa. This behaviour, coupled to the mass growth of Jupiter, can lead to the trapping of high inclination-low eccentricity Trojans. When we have a planetesimal initially on a high eccentricity-low inclination orbit but librating in a Kozai resonance, it can be trapped in a Trojan orbit by the mass growth of the proto-Jupiter with i at its maximum and, consequently, with e at its minimum. Once in a Trojan orbit, the planetesimal will forget about its past history and stay in a high inclination-low eccentricity orbit with only small oscillations about these values. In Fig. 7 we show an example of this behaviour. The planetesimal, started with an eccentricity of 0.3 and in a Kozai resonance, is trapped in a horseshoe orbit after about $1.5{\times}10^4$ years when its inclination is 16° and the eccentricity has decreased to 0.1. Once in the 1:1 resonance, the body is no longer in the Kozai resonance, which however could still be effective inside horseshoe orbits as shown in Michel (1997). When it is finally trapped as an L4 Trojan after 4.0 $\times 10^4$ the

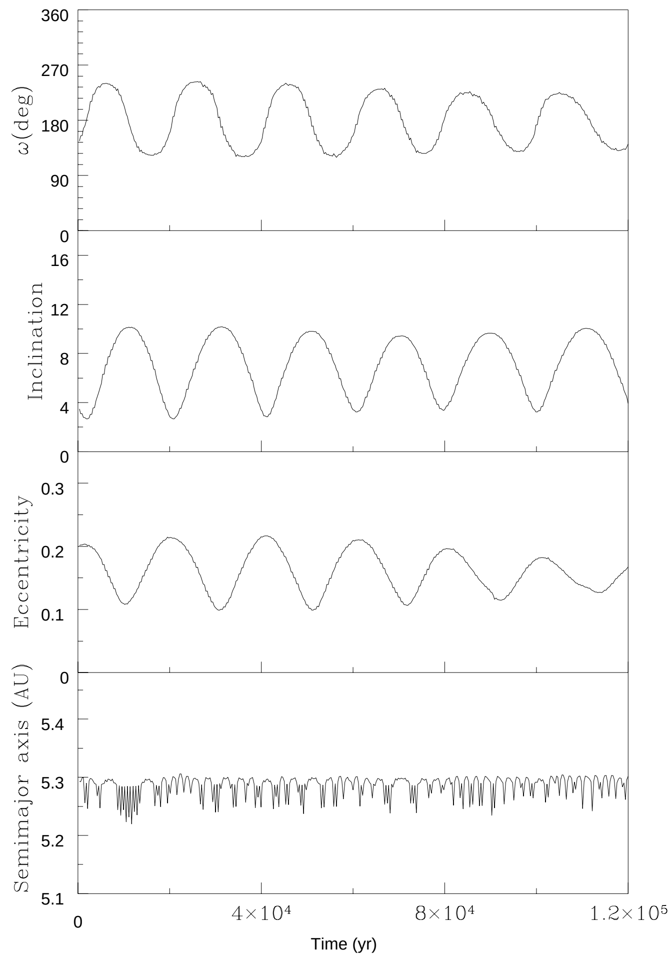

Fig. 6. Planetesimal trapped in a Kozai secular resonance. The perihelium argument ω librates around 180°. The masses of Jupiter and Saturn are kept constant and set equal to $10 M_{\oplus}$.

Kozai resonance can no longer occur since the frequency of the critical argument 2ω is far from 0 (Milani, 1993). As a Trojan, the body maintains the inclination and eccentricity acquired when trapped in the horseshoe orbit. A similar case is observed in Fig. 8 where the starting eccentricity is 0.4 and the final inclination, once trapped as a Trojan, is 23°. A close encounter with the proto-Jupiter, occurring after about $2.0{\times}10^4$ years, is responsible in this case for the transition from the Kozai resonance to a horseshoe orbit. After $5.0{\times}10^4$ years it is trapped by the mass growth mechanism as an L4 Trojan.

This mechanism, given by the synergy between the Kozai secular resonance and the growth in mass of the proto-Jupiter, might explain some of the observed high inclination Trojans. Close encounters with the proto-Jupiter or mutual collisions among planetesimals orbiting nearby proto-Jupiter could inject some planetesimals into high eccentricity-low inclination orbits. The Kozai resonance transforms high eccentric orbits into high inclination orbits; the growth in mass of Jupiter finally traps them as Trojans.

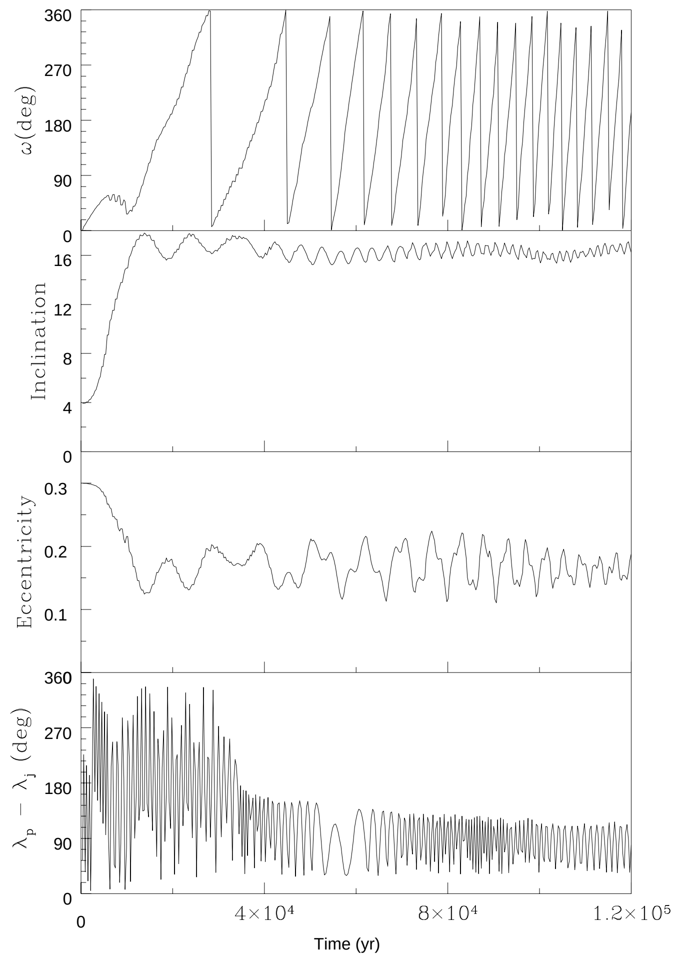

Fig. 7. Planetesimal evolving from a Kozai secular resonance to a horseshoe orbit and then into a L5 Trojan type orbit. The starting eccentricity was 0.3 and the initial inclination 4°. The numerical integration was performed within the *JT-JS* model (Jupiter and Saturn are both growing).

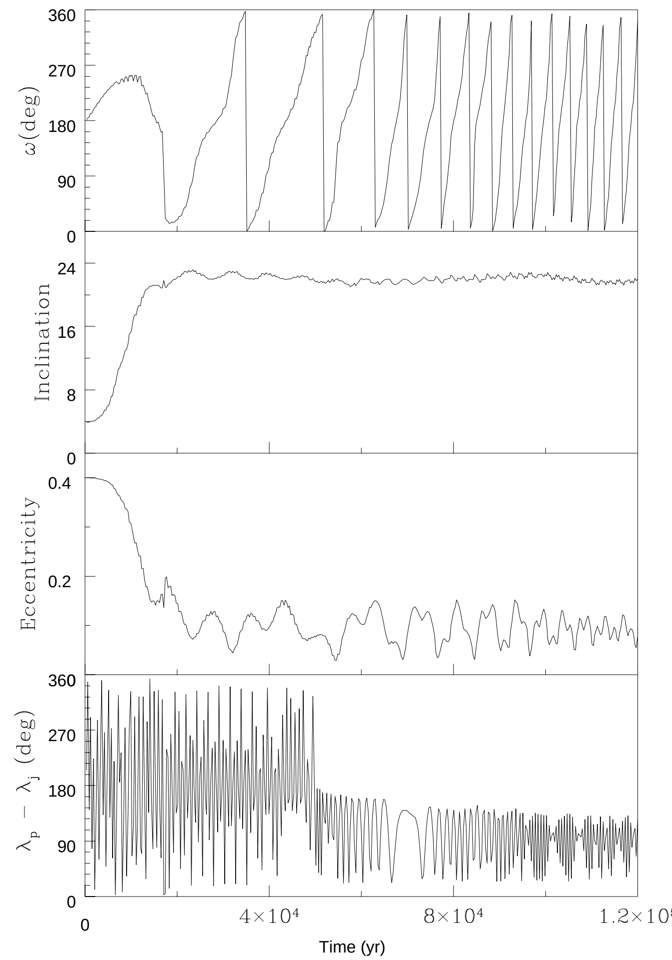

Fig. 8. As in Fig. 7 but with a higher initial eccentricity equal to 0.4. The final inclination of the planetesimal trapped in the L4 tadpole orbit is about 23°.

5. Comparison between model and observed populations of Jupiter Trojans

The decrease in the libration amplitude during mass growth of the planet is an important mechanism for reinforcing the stability of Trojan orbits. In the simulation *JT-JS*, 23 bodies were started on Trojan orbits. For each orbit we compute the libration amplitude intended as the difference between the maximum and minimum values of the critical argument over a period of 5×10^4 years. We call $dl0$ the libration amplitude at the beginning of the numerical integration when Jupiter has only 10 $M_\oplus$ (the mass of the proto-planets was kept constant for 5×10^4 years) and dlf the final libration amplitude when both the planets are fully formed. In Fig. 9 we show $dl0$ vs. dlf; an average reduction to 40% of the original value is observed for all cases. This mechanism might also be relevant in stabilizing small planetesimals trapped in the early stages of the giant planet formation either by gas drag (Kary et al, 1995) or collisional diffusion (Shoemaker, 1989).

In order to compare the libration amplitude of planetesimals trapped as Trojans by the mass growth of Jupiter with the amplitudes of the observed Trojans, we integrated for 1 Myr the orbits of 223 Trojans derived from the asteroid orbit database of Bowell et al. (1993). We selected only those orbits with reliable orbital elements, i.e. with arcs longer than 60 days. The libration amplitude was then computed for each asteroid in the sample every 5×10^4 years and these values were then averaged to derive a single mean amplitude. We have compared in a histogram (Fig. 10a) the libration amplitudes of the observed Trojans with the amplitudes of 80 Trojans derived from a *JT-JS* simulation. The amplitudes of the Trojans captured with the mass growth mechanism are shifted towards high libration amplitudes.

A process which can account for this discrepancy between observed and model populations is the collisional evolution. We can identify two different phases of collisional evolution. During the mass growth of Jupiter planetesimals orbiting near the proto-planet, including those which will be trapped later as Trojans, can collide with one another. The mass of Jupiter is still low and small changes in the orbital velocity due to a collision can lead to significant changes in the libration amplitudes of planetesimals being trapped as Trojans during the growth of the

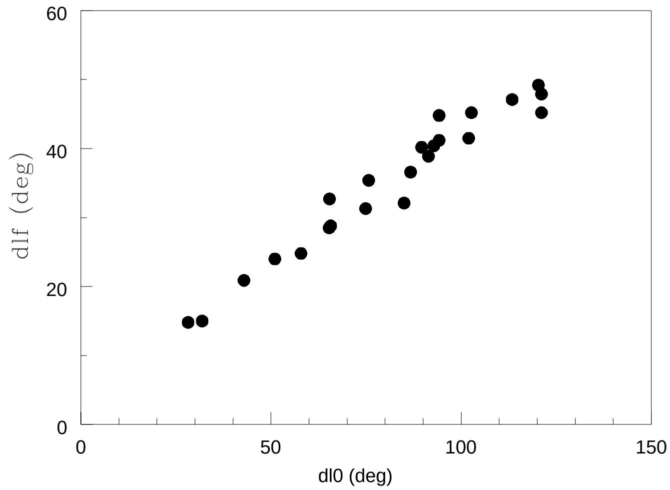

Fig. 9. Damping of the libration amplitude due to the proto-planet mass growth. $dl0$ is the libration amplitude of the critical argument of the Trojan orbit at the beginning of the numerical integration when Jupiter is only 10 $M_{\oplus}$, while dlf is the final libration amplitude with the fully formed planets.

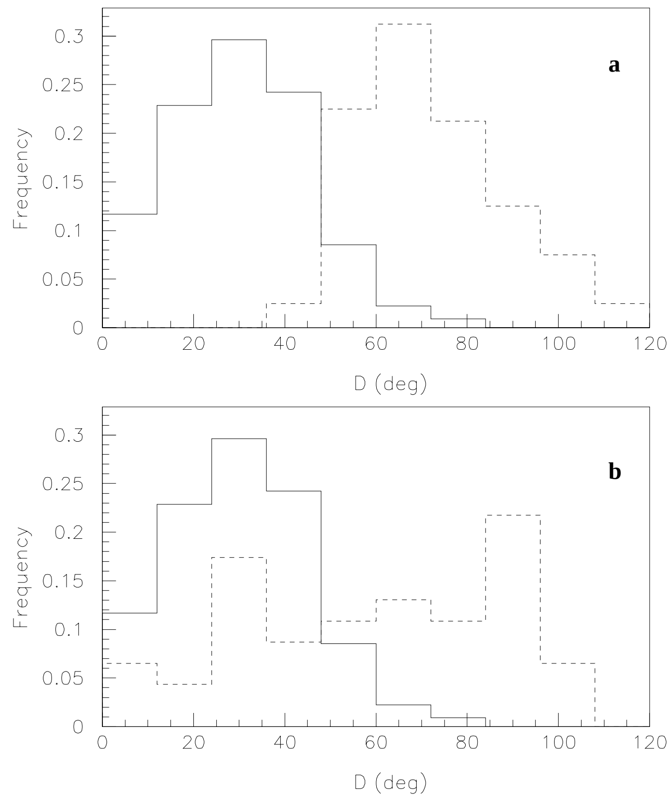

Fig. 10. **a** Histogram showing the number density (normalized to 1) of observed Trojans as a function of their libration amplitude (continuous line), compared with the number density of model Trojans (dotted line) trapped by the growing Jupiter (model *JT-JS*). **b** As in **a** but in the numerical model *JT-JS* the effects of collisions were simulated as random changes in the orbital velocity of the planetesimal every 5×10^3 years. The value of the velocity increment was 80 m/s.

proto-planet. Some Trojans will be ejected out of resonance, but some others will be injected more deeply into the resonance.

When Jupiter is fully formed, planetesimals not yet captured in Trojan-type orbits are scattered away by gravitational encounters with Jupiter and the surviving Trojans can interact only with the members of their swarm. Marzari et al (1997) have shown that both the size and orbital distribution of Trojans are considerably affected by the post-capture collisional evolution. In this phase some Trojans can still be injected in orbits with lower libration amplitudes. However, larger variation in the orbital velocity is needed to produce significant changes in the orbital elements since in this phase Jupiter has its full mass.

To simulate the effects of collisions during the growth of proto-Jupiter, we have added every 5×10^3 years a small velocity increment Δv, randomly directed in space, to the orbital velocity of each planetesimal. The histogram in Fig. 10b shows the libration amplitude distribution obtained by adopting a Δv of 80 m/s in the simulation. The libration amplitudes of Trojans trapped by the mass growth are spread between 0° and 120°. The low amplitude Trojans overlap the observed ones, while those with large amplitude could become unstable on long timescales as shown in Levison et al. (1997).

6. Resonance interference: Saturn Trojans

While the perturbations by Saturn on Jupiter's capture of planetesimals in Trojan-type orbits is negligible, we expect *a priori* the Saturn trapping efficiency to be decreased by the perturbations of Jupiter. As shown by Innanen and Mikkola (1989) and Holman and Wisdom (1993), the stability regions around the Saturn Lagrangian points are reduced by Jupiter's perturbations. In order to test the effects of Jupiter on the capture of Trojans by Saturn, we performed additional simulations (*ST-NJS*) where Jupiter's gravitational force is applied to Saturn but *not* to the planetesimals. In this way it was possible to separate the direct perturbations of Jupiter on planetesimals from the indirect ones related to the Jupiter induced variations of Saturn's orbital elements.

By comparing the results of simulations *ST-NJS* and *ST-JS* we observe a reduction of about 50% in Saturn's trapping efficiency due to Jupiter perturbations: 35 Trojans are captured at the end of simulation *ST-NJS* while only 17 in simulation *ST-JS* (over a sample of 80 planetesimals). In Fig. 11 we show an example of the trapping and subsequent escape of a planetesimal from a L4 Saturn Trojan orbit. The instability begins to build up towards the end of Jupiter's mass growth and the escape occurs a few thousand years after Jupiter has reached its final mass.

The mass growth of Jupiter also perturbs planetesimals which are Saturn Trojans from the beginning of the simulation. Out of 39 Saturn Trojans, only 8 survived to the end of the simulation (less than 20%). In Fig. 12 we present an example of a Saturn Trojan which is ejected out of resonance by Jupiter's mass growth. This phenomenon strongly inhibits the survival of those small planetesimals which could be trapped as Saturn Trojans in the initial stages of planet formation (Peale, 1993, Kary and Lissauer, 1995). Most of them would be ejected out

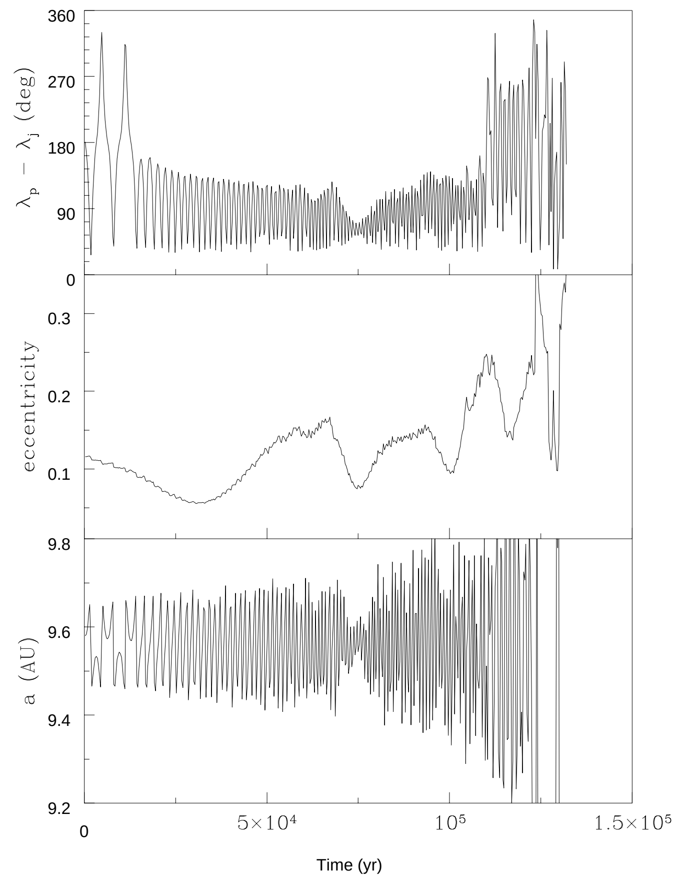

Fig. 11. Example of planetesimals trapped in a Saturn Trojan orbit (model *JT-JS*) and ejected out of the Trojan region by Jupiter's perturbations.

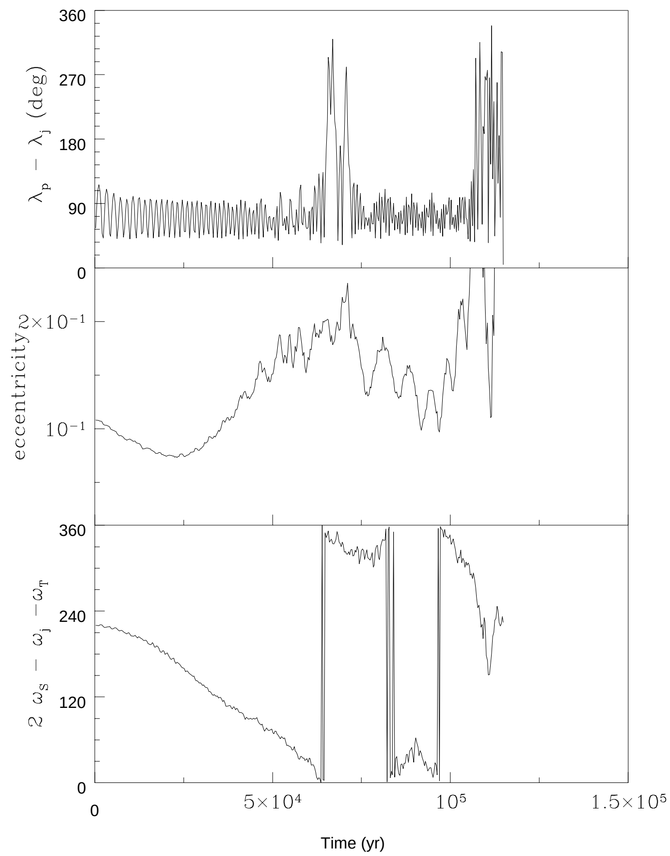

Fig. 12. A Saturn Trojan initially trapped in L4 is perturbed by the growing Jupiter and forced to escape from the tadpole orbit. The critical argument of the secular resonance $2\tilde{\omega}_S - \tilde{\omega}_j - \tilde{\omega}_T$ librates for a short period of time around 0° (model *JT-JS*).

of the resonance by the growth of Jupiter and would not build up into larger objects.

To investigate in more detail how the gravitational influence by Jupiter contributes to reduce Saturn Trojan trapping efficiency and stability, we have repeated simulation *ST-JS* but with the semimajor axis of Saturn shifted outwards and inwards by a small amount ($\Delta a = 0.3$ AU). In both cases the number of trapped Trojans is not reduced and is the same as in simulation *ST-NJS* where the perturbations by Jupiter on planetesimals were switched off. Moreover, no initial Trojan is thrown out from its tadpole orbit before the end of the simulation. This confirms that non-resonant secular perturbations by Jupiter are not responsible for the reduction of the stability region of Saturn Trojans.

With Saturn in its present position, secular resonances could be present inside the Trojan swarms and generate instability. We checked all the secular arguments and we found that in most cases the starting of instability for Saturn Trojans coincides with a slowing down or a short libration or an inversion of the direction of motion of the critical argument $2\tilde{\omega}_S - \tilde{\omega}_j - \tilde{\omega}_T$, where $\tilde{\omega}_S, \tilde{\omega}_j$ and $\tilde{\omega}_T$ are the perihelium longitudes of Saturn, Jupiter and the planetesimal in a Trojan orbit, respectively. This occurs on average after 5×10^4 years from the beginning of the numerical integration. Since the mass of both Jupiter and Saturn are growing with time, the secular frequencies changes and the instability builds up when the secular resonance enters the Saturn Trojan region. In Fig. 12 we plot as an example the critical argument of the 1:1 resonance, the eccentricity and the critical argument of the secular resonance $2\tilde{\omega}_S - \tilde{\omega}_j - \tilde{\omega}_T$ of a Saturn Trojan in the model *JT-JS*. The eccentricity starts to oscillate and the locking in the L4 Trojan orbit becomes unstable when the critical argument $2\tilde{\omega}_S - \tilde{\omega}_j - \tilde{\omega}_T$ oscillates slowly around 0°.

In a reduced number of cases we do not observe a peculiar behaviour of critical arguments related to secular frequencies. The instability in these cases could be due to the closeness to the 5:2 resonance, as argued by Innanen and Mikkola (1989).

7. Discussion and conclusions

To reconcile the present population of Trojans with planetesimals trapped by the mass growth mechanism it is necessary to assume that their libration amplitude was damped after their capture. From Fig. 10a we see that the libration amplitudes of trapped Trojans, even if reduced after capture by a growing proto-planet, are significantly higher than the amplitudes of the observed Trojans of Jupiter. An efficient mechanism which can modify the orbital parameters of Trojans is collisional evolution.

Collisions are effective during the mass growth of Jupiter since the planetesimal density is still high, but an intense collisional activity is also expected during the whole history of the solar system, as shown in Marzari et al. (1997), when Trojans interact among themselves within the L4 and L5 swarm.

We can imagine a reasonable scenario for Trojan origin: during the mass growth of Jupiter, planetesimals are trapped in Trojan-type orbits by the change in the gravitational potential of the planet. Collisions modify the libration amplitudes during the capture process when the mass of the planets are still growing, and in the subsequent post-capture history. Some planetesimals are injected in the low libration amplitude region of the present Trojans, while others are ejected out of the stable islands surrounding L4 and L5.

Within this scenario we also found a mechanism that produces highly inclined Trojan orbits starting from low inclination-high eccentricity orbits. The Kozai secular resonance surrounding the proto-Jupiter orbit alternates high values of eccentricity with high values of inclination for each period of libration of the perihelium argument. A planetesimal trapped in the Kozai resonance with an initially high eccentricity orbit is pumped up periodically to high inclinations. If it is trapped in a Trojan orbit by the mass growth mechanism when it is in the high inclination status, then it becomes a highly inclined Trojan. In the solar nebula high eccentric orbits can be explained more easily than high inclination orbits. A high eccentricity can be the result of repeated close encounters with the proto-planet. Before these encounters become deep enough to eject the planetesimal far away from the planet, it can be trapped in the Kozai resonance which can pump up its inclination; the mass growth can then trap it as a Trojan. Mutual collisions with nearby planetesimals can also help in producing highly eccentric planetesimals orbiting close to a growing proto-planet.

We have studied also the trapping mechanism of Saturn Trojans. We find as the resonance interference between the 1:1 commensurability with a secular resonance or, eventually, with the 5:2 resonance with Jupiter makes it difficult to trap planetesimals as Saturn Trojans. There is a drastic reduction in the trapping efficiency of Saturn when Jupiter is growing on the same, or a shorter, timescale as Saturn. As a consequence, on the basis of this model, we predict that the number of Saturn Trojans, if any, should be rather low.

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References

Bowell, E., K. Muinonen, L.H. Wasserman, 1993, IAU Symp. 160, Asteroids, Comets, Meteors 1993 (A. Milani, M. DiMartino and A. Cellino, Eds.) 477.
Holman, M.J., and J. Wisdom, 1993, Astron. J. 105, 1987.
Innanen, K.A., and S. Mikkola, 1989, Astron. J. 97, 900.
Kary, D.M., and J.J. Lissauer, 1995, Icarus, 117, 1.
Kary, D.M., J.J. Lissauer, and Y. Greenzweig, 1993, Icarus 106, 288.
Marzari,F., P. Farinella, D.R. Davis, H. Scholl, and A. Campo Bagatin 1997, Icarus, 125, 39.
Marzari,F., and H. Scholl, 1998, Icarus, 131, 41.
Michel, P., and F. Thomas, 1996, Astron. and Astrophys. 307, 310.
Michel, P., and Ch. Froeschlé, 1997, Icarus, 128, 230.
Michel, P., 1997, Astron. and Astrophys. 328, L5.
Milani, A. 1993b, IAU Symp. 160, Asteroids, Comets, Meteors 1993 (A. Milani, M. DiMartino and A. Cellino, Eds.) 159.
Peale, S., 1993, Icarus 106, 308.
Shoemaker, E.M., C.S. Shoemaker, and R.F. Wolfe 1989, in Asteroids II (R.P. Binzel, T. Gehrels, and M.S. Matthews, eds.), 921, Univ. of Arizona Press, Tucson.
Szebehely, V., Theory of Orbits: The Restricted Problem of the Three Bodies (Academic, London, 1967).