Astron. Astrophys. 339, 610–614 (1998)

ASTRONOMY AND ASTROPHYSICS

The sunspot as a self-excited dynamo

P. Lorrain[1,2] **and O. Koutchmy**[3]

[1] Department of Earth and Planetary Sciences, McGill University, 3450 Rue Université, Montréal, Québec, H3A 2A7, Canada
[2] Institut d'Astrophysique de Paris (CNRS), 98bis Boulevard Arago, F-75014 Paris, France
[3] Analyse numérique (CNRS), Université Pierre et Marie Curie, Place Jussieu, F-75252 Paris, France

Received 23 March 1998 / Accepted 5 August 1998

Abstract. Observations show that there is downflow in and around sunspots. Following Hénoux and Somov, Heyvaerts and Hagyard, Hirayama, and Lorrain and Koutchmy, we show that sunspots are self-excited magnetic-flux-tube dynamos run by the downflowing plasma. The azimuthal current results from an azimuthal $\boldsymbol{v} \times \boldsymbol{B}$ field generated by the radial inflow of the plasma. Comparison between various observations on sunspots with the fields of various coils seems to indicate that the azimuthal current is distributed nearly uniformly over the umbra and penumbra, with a slight excess near the axis. The resulting ohmic power density is negligible. The flux tube radius, at depth, is inversely proportional to the fourth root of the ambient pressure. The Standard Solar Model of Bahcall and Pinsonneau provides the pressure as a function of depth below -36 Mm.

Key words: MHD – Sun: magnetic fields – sunspots

1. Introduction

Sunspots are exceedingly complex magnetic structures in the photosphere. See, for example, Stanchfield et al. (1997). The diameter of the central dark region, called the *umbra*, varies widely, but it is typically 5–10 Megameters, and the magnetic field at the centre is a few tenths of a tesla.

Innumerable papers have reported observations on sunspots. For example, Koutchmy & Adjabshirizadeh (1981), and later Adjabshirizadeh & Koutchmy (1983), reported careful observations on a single spot. More recently, Solanki & Schmidt (1993) summarized the observations of several groups and Stanchfield et al. (1997) reported on an approximately axisymmetric spot.

Equally innumerable theoretical papers have been written on sunspots. To our knowledge, none account for the existence of the azimuthal currents that necessarily accompany the axial magnetic field. The INSPEC compilation lists about 150 papers per year on sunspots.

The model discussed here was first proposed by Hénoux & Somov (1987), and later developed by Heyvaerts & Hagyard (1991), by Hirayama (1991), and by Lorrain & Koutchmy (1993), for magnetic elements. These latter authors later used it in their model for solar spicules (Lorrain & Koutchmy 1996). This model accounts for the existence of the azimuthal current. However, it does not account for the penumbra, nor for dots, light bridges, etc. in the umbra, nor for the Evershed flow or the Wilson depression.

This dynamo is axisymmetric, which is forbidden by the Cowling "theorem" (Cowling 1934, 1957, 1976). However, it has long been clear that the "theorem" is a misconception (Kolm & Mawardi 1961, Shercliff 1965, Fearn et al. 1988, Alexeff 1989, Lorrain, 1991, Ingraham 1995).

We assume a single, vertical, axisymmetric magnetic flux tube. This is an approximation. For example, Lites et al. (1993) studied an umbral magnetic field that comprised five maxima, all of the same polarity.

2. Plasma flows above and below sunspots

Plasma flows around sunspots have been reported by many authors. Zirin & Wang (1989, 1991, 1992) reported inward and downward flows in the umbra and penumbra of sunspots, while Wang & Zirin (1992) report that granules flow toward growing pores. Balthasar et al. (1996) measured velocities at the surface, in the moat, in the penumbra, and in the Evershed flow.

Brekke et al. (1990), and several earlier authors, for example Brueckner (1981), reported downflows above sunspots.

At least four groups have devised methods for observing flows in and around sunspots *below* the photosphere.

a) Duvall (1995) and Duvall et al. (1993, 1996) observed downflows under sunspots and plages.
b) Kosovichev (1996) observed downflows under sunspots and *up*flows in *decaying* active regions. We return to this upflow near the end of the next section.
c) Lindsey et al. (1996) detected sub-photospheric outflows around sunspots.
d) Finally, Braun (1995), Bogdan & Braun (1995), and Braun et al. (1996), observed outflows that increased linearly with depth.

It is the inflow and downflow in sunspots, and the resulting magnetic flux tube, that we are concerned with here.

Send offprint requests to: P. Lorrain

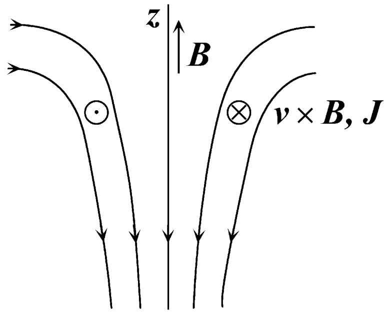

Fig. 1. The self-excited magnetic-flux-tube dynamo of a sunspot is similar to that of a magnetic element. For references, see the Introduction and Sect. 3. Schematic diagram of *streamlines* for the down-flowing plasma in a sunspot: the plasma flows downward and inward, and the density increases with depth. Below the region shown here, the plasma flow diverges and turns upward. We use cylindrical coordinates $\rho,\ \phi,\ z$, with the unit vector $\hat{\boldsymbol{z}}$ pointing up. The radial component v_ρ of the plasma velocity $\boldsymbol{v}$ is *negative*. Assuming a seed magnetic field $\boldsymbol{B}$ that points up, the $\boldsymbol{v} \times \boldsymbol{B}$ field and the resulting induced current, of density $-\sigma v_\rho B_z \hat{\boldsymbol{\phi}}$, point in the positive azimuthal direction shown. The magnetic field of this induced current points up, *in the same direction as the seed field*. The induced current thus *amplifies* the axial magnetic field. This is positive feedback. If, on the contrary, the seed magnetic field points down, then $\boldsymbol{v} \times \boldsymbol{B}$, $\boldsymbol{J}$, and its magnetic field all change sign. The induced current again amplifies the axial magnetic field. We thus have a self-excited dynamo that amplifies a seed axial magnetic field, whatever its polarity. The asymptotic value of B is a function of the power that drives the convection (Lorrain 1995).

3. The self-excited dynamo

Sunspots exhibit axial magnetic fields and require a *local* azimuthal current distribution, like a solenoid. Many authors have discussed current distributions in sunspots, but without considering the generation of the currents. See, for example, Osherovich & Garcia (1990), and Molodenskii & Solov'ev (1993).

Sunspots, like magnetic elements, are magnetic flux tubes (Lorrain & Salingaros 1993), and they are associated with down-flow. Both are self-excited dynamos of a type that was suggested several years ago by the authors listed in the Introduction.

Fig. 1 shows a plausible set of streamlines in a region of the Sun where there is downflow. The velocity vector of the down-welling plasma has a negative radial component, in cylindrical coordinates. The plasma density increases with depth. Below the region shown in the figure, the plasma diverges and returns to the surface. It is presumably this outflow that was observed under c) and d) above.

Assume a seed axial magnetic field $\boldsymbol{B}$ that points up, as in the figure. The electric current density induced in a plasma flowing at a velocity $\boldsymbol{v}$ in a field $\boldsymbol{E},\ \boldsymbol{B}$ is given by Ohm's law for moving conductors,

$$\boldsymbol{J} = \sigma[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})] = \sigma \left[-\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} + (\boldsymbol{v} \times \boldsymbol{B}) \right], \quad (1)$$

where σ is the conductivity. The term ∇V comes from electric volume charges, if any, in the convecting plasma, while $-\partial \boldsymbol{A}/\partial t$ is the electric field induced by a time-dependent magnetic field, according to the law of Faraday.

We use cylindrical coordinates $\rho,\ \phi,\ z$, and assume a steady state and axisymmetry:

$$\partial/\partial t = 0, \quad \partial/\partial \phi = 0. \quad (2)$$

We also set

$$v_\phi = 0, \quad B_\phi = 0, \quad B_\rho = 0. \quad (3)$$

We have set $B_\rho = 0$, despite the fact that, in Sect. 5, that condition is not quite satisfied. For a more general discussion, see Lorrain & Salingaros (1993).

The electrostatic space charge density Q' inside a conductor that moves at a velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$ is given by (van Bladel 1984; Lorrain et al. 1988; Lorrain 1990; Lorrain & Koutchmy 1995, 1996)

$$Q' = -\epsilon_r \epsilon_0 \boldsymbol{\nabla}(\boldsymbol{v} \times \boldsymbol{B}). \quad (4)$$

With the above assumptions, $Q' = 0$ and $V = 0$. Thus, for a steady state,

$$\boldsymbol{J} = \sigma(\boldsymbol{v} \times \boldsymbol{B}) = -\sigma v_\rho B_z \hat{\boldsymbol{\phi}}. \quad (5)$$

In Fig. 1, the radial component v_ρ of the plasma velocity $\boldsymbol{v}$ is negative, while B_z is positive, so that $\boldsymbol{v} \times \boldsymbol{B}$ and $\boldsymbol{J}$ are both azimuthal, in the direction shown. The increase of the plasma density with depth maintains the negative radial velocity along the flux tube.

The magnetic field of the induced azimuthal electric current $\boldsymbol{J}$ points up in Fig. 1, *in the same direction as the assumed seed magnetic field*: the induced azimuthal current *amplifies* any existing axial magnetic field. This is characteristic of a self-excited dynamo (Lorrain 1996).

If, instead, the seed magnetic field points down then $\boldsymbol{v} \times \boldsymbol{B}$ and $\boldsymbol{J}$ change direction and the magnetic field of the induced azimuthal current points down, again in the same direction as the seed field. So self-excitation applies for either polarit y of $\boldsymbol{B}$. The axial magnetic field B tends to an asymptotic value that is a function of the power that drives the convection (Lorrain 1996).

In Fig. 1, if $\boldsymbol{B}$ flares slightly, as it undoubtedly does (Sect. 5), the magnetic force density $\boldsymbol{J} \times \boldsymbol{B}$, which is normal to $\boldsymbol{B}$, points somewhat downward and anchors the magnetic flux tube, canceling the upward buoyancy force on the tube.

Now suppose that the magnetic field is established with $\boldsymbol{B}$ pointing up, and that the flow reverses sign: the downwelling becomes upwelling. As the upflowing plasma rises, it expands because of the decreasing ambient pressure, its radial velocity v_ρ is positive, and it generates a $\boldsymbol{v} \times \boldsymbol{B}$ field that points in the $-\hat{\boldsymbol{\phi}}$ direction and a corresponding current whose magnetic field opposes the existing field. Since the flux tube has an inherent time constant (Lorrain & Salingaros 1993), its magnetic field decreases more or less slowly. This is just what Kosovichev (1996) observed: there is an upflow in a decaying active region.

The induction equation for this dynamo reduces to the identity $0 \equiv 0$.

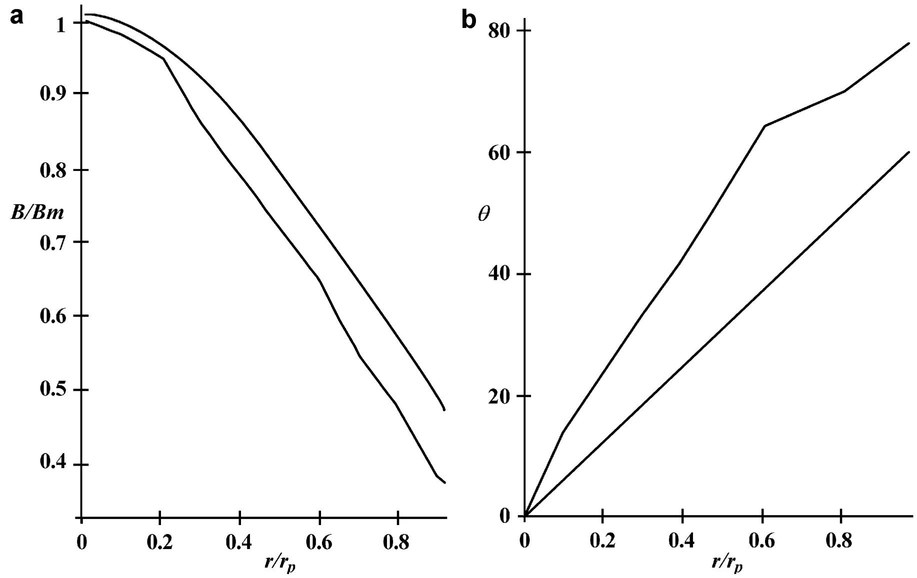

Fig. 2. **a** The ratios B/B_m as functions of r/r_p for the sunspot data compiled by Solanki & Schmidt (1993) (broken line) and for a coil of zero inside radius and whose length is equal to 20 times its radius (smooth line). The radius r_p is either the outer radius of the penumbra or the outer radius of the coil. **b** The angle θ between a magnetic field line and the axis of symmetry, for the same data. According to both sets of curves, J is somewhat larger than average near the axis.

4. The radial distribution of the azimuthal current density in the flux tube

Weiss (1990), Jahn (1989), and others proposed that there exists a sharp transition between the magnetic flux tube and the external plasma. Hirayama (1992) proposed a thin current sheet at the periphery of magnetic elements, and Solanki & Schmidt (1993), Jahn (1989), and Pizzo (1990) also proposed thin current sheets for sunspots. If J, and hence $v_\rho B$ in Eq. (5), are significant only near the periphery of the tube then, a short distance inside the flux tube, the plasma flows down along the lines of $\boldsymbol{B}$.

On the contrary, observations seem to indicate that J is uniform, as shown below.

Many authors have reported observations on the value of $\boldsymbol{B}$ as a function of the radius in the umbra and penumbra. Solanki & Schmidt (1993) compiled observations by Beckers & Schröter (1969), Kawakami (1983), Lites & Skumanich (1990), Solanki et al. (1992), and Wittman (1971). See also Koutchmy & Adjabshirizadeh (1981), Abjabshirizadeh & Koutchmy (1983), Arena et al. (1990), Title et al. (1993), Howard (1996), Sutterlin et al. (1996), Abramov-Maksimov et al. (1996), Keppens & Martinez-Pillet (1996), and Stanchfield et al. (1997).

One would hope to gain information on a sunspot's magnetic flux tube by comparing the field of a sunspot with that of coils of various geometries.

We averaged the Solanki & Schmidt (1993) curves for the ratio B/B_m, where B is the field at the radius r and B_m the field at the center, and for the angle θ between magnetic field lines and the axis of symmetry, at seven values of r/r_p, where r_p is the outer radius of the penumbra. See also Solanki (1990).

We then compared with the corresponding variables for six coils of circular cross-section, including a single loop and a cone with its larger radius on top. The outer boundary of the penumbra corresponds to the outer radius of the coil.

The field that agrees best with the S&S data is the one in a plane at the end of a coil of zero inside radius, whose length is 20 times its radius. Figs. 2a and b show plots of the fields.

Our reference sunspot is the one observed by Stanchfield et al. (1997). Its radius was 3 Mm, and B at the center was 0.25 T. Using an average B of 0.125 T, the magnetic flux Φ was about 3.5×10^{12} Wb.

The ohmic power density resulting from the presence of an electric current is J^2/σ. For the reference spot, assuming a uniform J, $B_m = \mu_0 J b$ and $J = 66$ mA/m^2. Here, $\boldsymbol{J}$ is orthogonal to $\boldsymbol{B}$. According to one source (Lorrain & Koutchmy, 1993), $\sigma_\perp \approx 0.07$ S/m in the photosphere. Then the ohmic power density at the surface is only about 0.1 W/m^3, which is less than would be required to make the umbra bright, by many orders of magnitude.

5. The magnetic flux tube radius b below -36 Mm

Call z the vertical coordinate normal to the photosphere, with the unit vector $\hat{\boldsymbol{z}}$ pointing up, and the origin at the photosphere. We now calculate the flux tube radius b as a function of the external *pressure* p_{ext}, and then use the Standard Solar Model (Bahcall & Pinsonneau 1992) to deduce the flux tube radius b as a function of the *depth* z, where z is negative.

We saw above that, judging by the S&S data, the electric current density J seems to be about uniform. However, let us be more general and set

$$J = J_b(\rho/b)^n, \quad J_{\rho>b} = 0, \tag{6}$$

where J_b is the azimuthal current density at the periphery. Assume that the exponent n is independent of z. If $n = 0$, then J is uniform.

We have no information on the conductivity σ, except for the value of $\sigma_\perp$ at the surface, as in Sect. 4. If we did know the conductivity at depth, we could estimate the value of v_ρ from the values of J and of B.

Below -36 Mm, b varies slowly with z and we can use the analysis of Lorrain & Salingaros (1993). At the radius ρ,

$$B = \mu_0 \int_\rho^b J d\rho = \frac{\mu_0 J_b b}{n+1} \left[1 - \left(\frac{\rho}{b} \right)^{n+1} \right]. \quad (7)$$

As with a long solenoid, B outside the flux tube is negligible because the return flux extends over a large region.

The magnetic flux in the tube is

$$\Phi = \int_0^b B 2\pi\rho d\rho = \frac{\pi \mu_0 J_b b^3}{n+3}. \quad (8)$$

If Φ and n are independent of z, then $J_b b^3$ is also independent of z.

Call B on the axis of the flux tube B_0, and set the magnetic pressure on the axis equal to a fraction K of the external pressure p_{ext}, where $K \leq 1$. We do not assign a value to K for the moment. Then

$$\frac{B_0^2}{2\mu_0} = \frac{1}{2\mu_0} \left(\frac{\mu_0 J_b b}{n+1} \right)^2 = K p_{ext}. \quad (9)$$

Eliminating J_b from Eqs. (8) and (9),

$$b = \left[\frac{(n+3)\Phi}{\pi(n+1)} \right]^{1/2} \frac{1}{(2\mu_0 K p_{ext})^{1/4}} = \frac{U}{p_{ext}^{1/4}}. \quad (10)$$

The magnetic flux tube radius is not very sensitive to the value of either n or K: if n increases from 0 to 5, b decreases by a factor of 2/3 while, if K decreases from 1 to 0.1, b increases by a factor of 1.8. In view of the coarseness of our model, setting $n = 0$ and $K = 1$ is a fair approximation at this stage.

Also,

$$\frac{db}{dz} = \frac{db}{dp_{ext}} \frac{dp_{ext}}{dz} = -\frac{U}{4p_{ext}^{5/4}} \frac{dp_{ext}}{dz}. \quad (11)$$

With the z-axis pointing up as above, the last derivative is negative, and the flux tube radius b increases with increasing z, because U, defined in Eq. (10), is positive.

For the Stanchfield et al. (1997) sunspot, if $n = 0$ and $K = 1$, then $U = 4.6 \times 10^1$.

From Eq. (9), for any n,

$$J_b = \frac{B_0(n+1)}{\mu_0 b} \quad (12)$$

and, from Eq. (6), the azimuthal current per meter of length is

$$I' = \int_0^b J d\rho = \frac{J_b b}{n+1} = \frac{B_0}{\mu_0}. \quad (13)$$

So, with increasing pressure p_{ext}, or with increasing depth,

a) B_0 increases, $B_0 \sim p_{ext}^{1/2}$, from Eq. (9),
b) b decreases, $b \sim 1/p_{ext}^{1/4}$, from Eq. (10),
c) J_b increases, $J_b \sim 1/b^3 \sim p_{ext}^{3/4}$, from Eqs. (8) and (10),
d) I' increases, $I' \sim B_0 \sim p_{ext}^{1/2}$, from Eqs. (9) and (13), and
e) $\mid v_\rho \mid$ decreases, from Fig. 1.

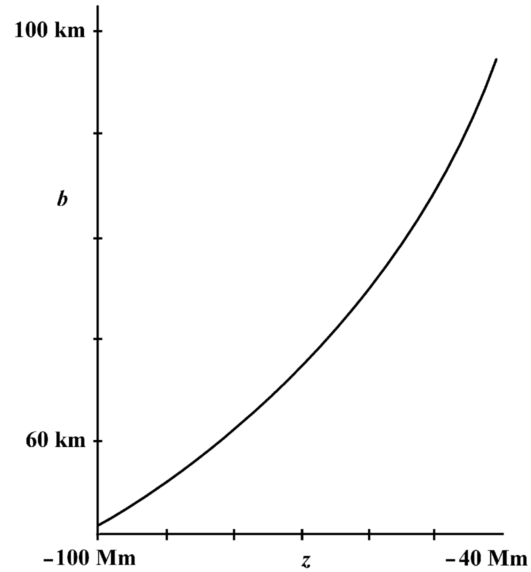

Fig. 3. Magnetic flux tube radius b as a function of depth for the Stanchfield et al. (1997) sunspot. The z-axis points up. Over this range, the flux tube radius decreases slowly with depth. That sunspot had a radius of 3 Mm at the level of the photosphere.

To calculate b as a function of z, we now require the external pressure p_{ext} as a function of depth below the photosphere. See Bahcall & Ulrich (1988), Bahcall & Pinsonneau (1992), and Hendry (1993).

The Bahcall & Pinsonneau (1992) table extends from the center of the Sun up to -36 Mm, but the range -100 Mm to -36 Mm is sufficient for our purposes. A third-order regression analysis for this range yields

$$p_{ext} = -2.72 \times 10^{10} - 1470z - 2.22 \times 10^{-5} z^2 - 7.13 \times 10^{-13} z^3 \text{ Pa}. \quad (14)$$

The fit is near-perfect but, clearly, the Bahcall-Pinsonneau table cannot be extrapolated upward to the photosphere, where $z = 0$. The table given in Bahcall & Ulrich (1988) does not extend above -56 Mm.

Fig. 3 shows the magnetic flux tube radius b for the Stanchfield et al. (1997) sunspot as a function of z, using Eq. (10) with $\Phi = 3.5 \times 10^{12}$ Wb, $K = 1$, and $n = 0$, for the range $z = -100$ Mm to $z = -40$ Mm. Note that the radius scale covers about 50 km, while the depth scale covers 60 Mm. So b varies slowly with z at these depths. Indeed, at -50 Mm, $db/dz = 0.006$.

According to the Bahcall & Pinsonneau (1992) table, the pressure at a depth of 50 Mm is 8×10^{10} Pa. Then, at that depth, if $n = 0$ and $K = 1$, from Fig. 4 and from Eqs. (10), (9), (12), and (13), $b = 86$ km, $B_0 \approx 450$ T, $J_b \approx 4$ kA/m^2, and $I' \approx 4 \times 10^5$ kA/m.

6. Conclusions

Considering a sunspot as a self-excited magnetic-flux-tube dynamo proves to be a fruitful approach because it provides a) an explanation for the existence and the nature of the azimuthal electric current distribution that is required by the axial magnetic field, and b) information on the geometry of the flux tube at depth.

Clearly, much work remains to be done in this direction.

Acknowledgements. We thank Serge Koutchmy and the referee for their comments. This work was subsidized in part by NATO through the Cooperative Research Grant 940291.

References

Abramov-Maksimov V.E., Vyalshin G.F., Gelfreikh G.B., and Shatilov V.T., 1996, Solar Physics 164, 333

Adjabshirizadeh A. and Koutchmy S., 1983, A&A 122, 1

Alexeff I., 1989, IEEE Trans. Plasma Sci. 17, 282

Arena P., Landi Degl'Innocenti E., and Noci G., 1990, Solar Physics 129, 259

Bahcall J.N. and Pinsonneau M.H., 1992, Rev. Mod. Phys. 64, 885

Balthasar H., Schleicher H., Bendlin C., and Volkmer R., 1996, A&A 315, 603

Beckers J.M. and Schröter E.H., 1969, Sol. Phys. 125, 37

van Bladel J., 1984, Relativity and Engineering, Springer, Berlin, p. 287

Bogdan T.J. and Braun D.C., 1995, Fourth SoHO Workshop on Helioseismology, vol. 1 ESA SP-376

Braun D.C., 1995, Astronomical Society of the Pacific Conference Series, vol. 76

Braun D.C., Fan Y., Lindsey C., and Jefferies S.M., 1996, Preprint, National Solar Observatory, Tucson, Ariz, USA

Bray R.J., Cram L.E., Durrant C.J., and Loughhead R.E., 1991, Plasma Loops in the Solar Corona, Cambridge University Press, Cambridge, G.B.

Brekke P., Kjeldseth-Moe O., Bartoe J.-D., and Brueckner G.E., 1987, ESA SP-270, p. 341

Brekke P., Kjeldseth-Moe O., and Brueckner G.E., 1990, Astrophysics and Space Science 170, 135

Brueckner G.E., 1981, in Solar Active Regions, Orall, F., (ed.), Colorado Associated University Press, p. 113

Cambel A.B., 1963, Plasma Physics and Magnetofluidmechanics, McGraw-Hill, New York

Cargill P.J., Chen J., and Garren D.A., 1994, ApJ 423, 854

Cowling T.G., 1934, Mon. Notic. Roy. Astron. Soc. 94, 39

Cowling T.G., 1957, Quart. Jour. Mech. Appl. Math. 10, 129

Cowling T.G. 1976, Magnetohydrodynamics, p. 87, Adam Hilger, Bristol, U.K.

Dere K.P., 1982, Solar Phys. 77, 77

Duvall T.L., 1995, Astronomical Society of the Pacific Conference Series, vol 76

Duvall T.L., Jefferies S.M., Harvey J.W., and Pomerantz M.A., 1993, Nature 362, 430

Duvall T.L., Jr, D'Silva S., Jefferies S.M., Harvey J.W., and Schou J., 1996, Nature 379, 235

Fearn D.R., Roberts P.H., and Soward A.M., 1988, Convection, stability, and the dynamo, in Galdi G.P. and Straughan B., editors, Energy Stability and Convection, Longman, New York

Foukal P., 1989, Solar Astrophysics, John Wiley, New York

Hendry, A.W., 1993, Am J. Phys 61, 906

Hirayama T., 1991, Solar Physics 137, 33

Hénoux J.C., Somov B.V., 1987, A&A 185, 306

Howard R.F., 1996, Solar Physics 167, 95

Ingraham R.L., 1995, Phys. Scripta 52, 60

Jahn K., 1989, A&A 222, 264

Kawakami H., 1983, PASJ 35, 459

Keppens R. and Martinez Pillet V.,1996, A&A 316, 229

olm H.H. and Mawardi O.K., 1961, J. Appl. Phys. 32, 1296

Kopecky M. and Kuklin G.V., 1966, Bull. Astron. Inst. Czech. 17, 45

Kosovichev A., 1996, Ap. J. 461, L55

Koutchmy S. and Adjabshirizadeh A., 1981, A&A, 99, 111

Larmor J., 1919, Brit. Ass. Rep. p. 159

Larmor J., 1928, Math. and Phys. Papers 2, 612 and 615

Larmor J., 1934, Monthly Notices 94, 469

Lindsey C., Braun D.C., Jefferies S.M., Woodward M., Fan Y., Gu Y., and Redfield S., 1996, preprint, Solar Physics Research Corporation, Tucson, Ariz, USA

Lites B.W., Elmore D.F., Seagraves P., and Skumanich A.P., 1993, ApJ 418, 928

Lites B.W. and Skumanich A., 1990, ApJ 348, 747

Livingston W., 1991, Nature 350, 45

Lorrain P., 1990, Eur. Jour. Phys. 11, 94

Lorrain P., 1991, IEEE Trans. Plasma Sci. 19, 862

Lorrain P., 1995, Physica Scripta 52, 349

Lorrain P., Corson D.R., and Lorrain F., 1988, Electromagnetic Fields and Waves, 3rd edition, Freeman, New York

Lorrain P. and Koutchmy S., 1993, A&A 269, 518

Lorrain P. and Koutchmy S., 1996, Solar Physics 165, 115

Lorrain P., Koutchmy S., and Koutchmy O., 1998, A model for solar coronal loops, in preparation

Lorrain P. and Salingaros N., 1993, Amer. Jour. Phys. 61, 811

Molodenskii M.M. and Solov'ev L.S., 1993, Astron. Rep. 37, 83

Nicolas K.P., Kjeldseth-Moe O., Bartoe J.-D., and Brueckner G.E., 1982, Solar Physics 81, 253

Osherovich V.A. and Garcia H.A., 1990, Geophys. Res. Lett. 17, 2273

Oster L., 1968, Solar Physics 3, 543

Pizzo V.J., 1990, ApJ 365, 764

Priest E.R., 1982, Solar Magneto-Hydrodynamics, Geophysics and Astrophysics Monographs vol. 21, Kluwer, Dordrecht

Ruzmaikin A., 1997, A&A 319, L13

Shercliff J.A., 1965, Textbook of Magnetohydrodynamics, p.48, Pergamon, London, U.K.

Solanki S.K., Empirical models of flux tubes in Solar Photosphere, Structure, Convection, and Magnetic Fields, J.O. Stenflo, ed., 1990, IAU

Solanki S.K., Rüedi I, and Livingston W., 1992, A&A 263, 339

Solanki S.K. and Schmidt H.U., 1993, A&A 267, 287

Solov'ev A.A., 1997, Astronomy Reports 41, 121

Solov'ev L.S., 1993, Astronomy Reports, Letters, 19, 1101

Stanchfield D.C.H., Thomas J.H., and Lites B.W., 1997, ApJ 477, 485

Sutterlin P., Schroter E.H., and Muglach K., 1996, Solar Physics 164, 311

Title A.M., Frank Z.A., Shine R.A. Tarbell T.D., Topka K.P., Scharmer G., and Schmidt W., 1993b, ApJ 403, 780

Vernazza J.E., Avrett E.H., and Loeser R., 1981, ApJ 45, 635

Wang H. and Zirin H., 1992, Solar Phys. 140, 41

Weiss N.O., 1990, in Basic Plasma Processes in the Sun, Priest E.R. and Krishan V., eds.

Wittmann A.D., 1974, Sol. Phys. 36, 29

Zirin H., 1987, Astrophysics of the Sun, Cambridge University Press, Cambridge G.B.

Zirin H. and Wang H., 1989, Solar Physics 119, 245

Zirin H. and Wang H., 1991, Space Rev. 11, 225