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Cosmology with galaxy clusters

II. Projection effects on Hubble constant and gas mass fraction

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Abstract. It is well known that a combined analysis of the Sunyaev-Zel'dovich (SZ) effect and the X-ray emission observations can be used to determine the angular diameter distance to galaxy clusters, from which the Hubble constant is derived. The present values of the Hubble constant derived through the SZ/X-ray route have a broad distribution ranging from 30 to 70 km s^{-1} Mpc^{-1}. We show that this broad distribution is primarily due to the projection effect of aspherical clusters which have been modeled using spherical geometries. The projection effect is also expected to broaden the measured gas mass fraction in galaxy clusters. However, the projection effect either under- or overestimate the Hubble constant and the gas mass fraction in an opposite manner, producing an anticorrelation. Using the published data for SZ/X-ray clusters, we show that the current Hubble constant distribution is negatively correlated with the measured gas mass fraction for same clusters, suggesting that the projection effects are present in current results. If the gas mass fraction of galaxy clusters, when measured out to an outer hydrostatic radius is constant, it may be possible to account for the line of sight geometry of galaxy clusters. However, to perform such an analysis, an independent measurement of the total mass of galaxy clusters, such as from weak lensing, is needed. Using the weak lensing, optical velocity dispersion, SZ and X-ray data, we outline an alternative method to calculate the Hubble constant, which is subjected less to projection effect than the present method based on only the SZ and X-ray data. For A2163, the Hubble constant based on published SZ, X-ray and weak lensing observations is 49 ± 29 km s^{-1} Mpc^{-1}.

Key words: galaxies: clusters: general – distance scale

1. Introduction

Over the last few years, there has been a tremendous increase in the study of galaxy clusters as cosmological probes, initially through the use of X-ray emission observations, and in recent years, through the use of Sunyaev-Zel'dovich (SZ) effect. Briefly, the SZ effect is a distortion of the cosmic microwave background (CMB) radiation by inverse-Compton scattering of thermal electrons within the hot intracluster medium (Sunyaev & Zel'dovich 1980; see Birkinshaw 1998 for a recent review). By combining the SZ intensity change and the X-ray emission observations, the angular diameter distance, $D_{\rm A}$, to a cluster can be derived (e.g., Cavaliere et al. 1977). Combining the distance measurement with redshift allows a determination of the Hubble constant, H_0. On the other hand, angular diameter distances with redshift can be used to constrain cosmological world models.

The accuracy of the Hubble constant determined from a SZ and X-ray analysis depends on the assumptions. Using numerical simulations, Inagaki et al. (1995) and Roettiger et al. (1997) showed that the Hubble constant measured through the SZ effect can seriously be affected by systematic effects, which include the assumption of isothermality, cluster gas clumping, and asphericity. The Hubble constant can also be affected by statistical effects, including cluster peculiar velocities and astrophysical confusions, such as radio sources & CMB primary anisotropies. The latter statistical effects are expected to produce a broad distribution in the Hubble constant measured for a sample of galaxy clusters, while the former systematic effects are expected to offset the Hubble constant from the true value.

In recent years, several other effects have also been suggested to explain the difference between the SZ and X-ray Hubble constant and the ones derived from other techniques. These include the preferential removal of the lensed background radio sources in SZ surveys (Loeb & Refregier 1997), which would systematically lower the Hubble constant by as much as 13% for SZ observations at 15 GHz, and gravitational lensing of the arcminute scale CMB anisotropy (Cen 1998), which would broaden the Hubble constant distribution for a sample of galaxy clusters. The first effect is in opposite direction to the radio source contamination in SZ observations due to galaxy cluster member radio sources, which dominate the radio source number counts towards galaxy clusters. As discussed in Cooray et al. (1998a), the two radio source effects are likely to cancel out. The Loeb & Refregier (1997) effect is also not expected to occur for SZ observations at high frequencies. The second effect, due to gravitational lensing of CMB anisotropy through galaxy cluster potential, is not expected to be a dominant source of error in the Hubble constant, given that Cen (1998) considered the largest upper limits to arcminute scale anisotropies, which have not yet been detected.

Apart from the SZ and X-ray Hubble constant, the gas mass fraction, $f_{\rm gas}$, measurements from X-ray (also using SZ, gravitational lensing and optical velocity dispersion measurements), can also be used to constrain the cosmological parameters. The primary assumption in such an analysis is that the gas mass fraction, when measured out to a standard (hydrostatic) radius is constant. Evrard (1997) applied these arguments to a sample of galaxy clusters using X-ray data, and put constraints on the cosmological mass density of the universe, Ω_m, with some dependence on the Hubble constant. Under the assumption that the cluster gas mass fraction is constant in a sample of galaxy clusters, the apparent redshift evolution of the baryonic fraction can also be used to constrain the cosmological parameters (e.g., Pen 1997). Cooray (1998) and Danos & Pen (1998) used the present X-ray gas mass fraction data to derive $\Omega_m < 0.6$ in a flat universe ($\Omega_m + \Omega_\Lambda = 1$) and $\Omega_m < 0.7$ in an open universe ($\Omega_\Lambda = 0$; 90% C.I.). In Shimasaku (1997), the assumption of constant gas mass fraction in galaxy clusters was used to put constrains on σ_8, the rms linear fluctuations on scales of 8 h^{-1} Mpc, and on n, the slope of the fluctuation spectrum.

Given the importance of SZ and X-ray emission observations in cosmological studies, we initiated a program to study the systematic effects in the present SZ and X-ray Hubble constant measurements and gas mass fraction measurements. As part of this study, we found a negative correlation between the broad distribution of the Hubble constant and the gas mass fraction measurements. We explain this observation as due to a projection effect of aspherical clusters modeled with a spherical geometry. In Sect. 2, we present the effects of projection on the Hubble constant and the gas mass fraction by projecting triaxial ellipsoidal clusters and extending the work of Fabricant et al. (1984). The observational evidence for projection effects in the present Hubble constant values based on SZ and X-ray route are presented in Sect. 3. In Sect. 4, we outline an alternative method to calculate the Hubble constant, by combining SZ, X-ray, gravitational lensing, and velocity dispersion measurements of clusters, and which is subjected to less projection effects than current method involving only the SZ and X-ray observations. We apply this technique to A2163 based on the published observational data, and derive a new Hubble constant. A summary and conclusions are presented in Sect. 5.

2. Projection effect of aspherical clusters

In order to study the effect of aspherical clusters in present SZ and X-ray Hubble constant, we extend the work of Fabricant et al. (1984) to calculate the X-ray surface brightness and the SZ temperature change produced by clusters with ellipsoidal geometries. Independent of the cluster shape, the X-ray surface brightness towards a clusters is given by:

$$S_X = \frac{1}{4\pi(1+z)^3} \int n_e^2 \Lambda_e dl, \tag{1}$$

where $\Lambda_e \propto T_e^{1/2}$. In order to model the electron number density profile within clusters, we consider the β-model, which can be written as:

$$n_{\rm e}(x_1, y_1, z_1) = n_{\rm e0} \left[1 + \frac{x_1^2 + y_1^2}{r_1^2} + \frac{z_1^2}{r_2^2}\right]^{-\frac{3\beta}{2}}, \tag{2}$$

where x_1, y_1 and z_1 are coordinates of the ellipsoid axes, while r_1 and r_2 are the observed semi-major and semi-minor axes. To simplify the calculations, we assume that the symmetry axis z_1 is at an inclination angle θ to the line of sight along the observer, which we take to be the z-axis. Following Fabricant et al. (1984, Appendix A), we integrate along the z-axis to derive:

$$S_X(x,y) = \frac{\sqrt{\pi} n_{\rm e0}^2 \Lambda_{\rm e0}}{4\pi(1+z)^3} \frac{\Gamma(3\beta - \frac{1}{2})}{\Gamma(3\beta)} \frac{r_1 r_2}{\sqrt{r_1^2 \cos^2\theta + r_2^2 \sin^2\theta}} \times \left[1 + \frac{x^2}{r_1^2\cos^2\theta + r_2^2 \sin^2\theta} + \frac{y^2}{r_1^2}\right]^{\frac{1}{2} - 3\beta}. \tag{3}$$

The other important observable towards clusters is the SZ effect, which is given by:

$$\frac{\Delta T}{T_{\rm CMB}} = f(x) \int \left(\frac{k_B T_e}{m_e c^2}\right) n_e \sigma_T dl, \tag{4}$$

where

$$f(x) = \left[\frac{x(e^x+1)}{e^x - 1} - 4\right] \tag{5}$$

is the frequency dependence with $x = h\nu/k_B T_{\rm CMB}$, $T_{\rm CMB} = 2.728 \pm 0.002$ (Fixsen et al. 1994) and σ_T is the cross section for Thomson scattering. The integral is performed along the line of sight through the cluster. As with the X-ray surface brightness, we consider the same ellipsoidal shape to evaluate the observed SZ temperature change. Again by integrating along the line of sight, z-axis, we derive:

$$\frac{\Delta T(x,y)}{T_{\rm CMB}} = f(x)\sqrt{\pi} n_{\rm e0} T_{\rm e0} \frac{\Gamma(\frac{3\beta}{2} - \frac{1}{2})}{\Gamma(\frac{3\beta}{2})} \frac{r_1 r_2}{\sqrt{r_1^2\cos^2\theta + r_2^2\sin^2\theta}} \times \left[1 + \frac{x^2}{r_1^2\cos^2\theta + r_2^2\sin^2\theta} + \frac{y^2}{r_1^2}\right]^{\frac{1}{2} - \frac{3\beta}{2}}. \tag{6}$$

The Hubble constant is usually derived by combining the X-ray brightness and the SZ temperature change to eliminate the central number density $n_{\rm e0}$. By this combination, one can derive the observed length of one of the axis, e.g.:

$$r_2 = \left[\left(\frac{\Delta T_{\rm SZ}(x,y)^2}{S_{\rm X}(x,y)}\right)\left(\frac{m_e c^2}{k_{\rm B} T_{\rm e0}}\right)^2 \frac{\Lambda_{\rm e0}}{4\pi f(x)^2 T_{\rm CMB}^2 \sigma_{\rm T}^2 (1+z)^3}\right] \times Z, \tag{7}$$

where Z is the scale factor first introduced in Birkinshaw et al. (1991), which can now be written as:

$$Z = \frac{\left(r_1^2\cos^2\theta + r_2^2\sin^2\theta\right)^{\frac{1}{2}}}{r_2}. \tag{8}$$

When the symmetry axis of the cluster is along the line of sight ($\theta = 0$), then $Z = r_1/r_2$ which is directly related to the observed cluster ellipticity, while when the cluster is spherical ($r_1 = r_2$), $Z = 1$, and no effects due to projection is present in the data. In Eq. 7, we know from SZ and X-ray observations all the quantities except the scale factor Z. Therefore, the length of the cluster along the line of sight can be known up to a multiplicative factor. The Hubble constant is derived based on the angular diameter distance to the cluster, D_A, using an assumed cosmological model, and the observed size of the axis,

$\theta_{r_2} = r_2/D_A$, used to calculate the distance in Eq. 7. The derived Hubble constant can be written as:

$$H_0 \propto \frac{\theta_{r_2}}{Z}. \quad (9)$$

Based on observed ellipticities of galaxy clusters, we can estimate the expected error in the Hubble constant. Using X-ray emission from a sample of clusters, Mohr et al. (1995) showed that the median ellipticity is $\sim$ 0.25. This suggest that the ratio r_2/r_1 is $\sim$ 0.7 if clusters are intrinsically prolate or $\sim$ 1.5 if clusters oblate. Therefore, ignoring the effects due to inclination, the Hubble constant as measured from SZ and X-ray observations of an individual cluster can be offseted as much as 30% to 50%, based on a spherical model of clusters where asphericity is ignored. Here, we have assumed that clusters are ellipsoids. The derived scale factor in Eq. 8, as well as the numerical values, are likely to be different if clusters are biaxial or triaxial. Recently, Zaroubi et al. (1998) studied the projection effects of biaxial clusters and determined $h \propto \sin\theta$, where θ is the inclination angle. The observational evidence which suggest clusters are biaxial is limited. For ellipsoidal clusters, we have determined that h varies with both the inclination angle and the sizes of semi-major and semi-minor axes. For triaxial clusters, it is likely that h will vary with all three rotation angles and the length scales of the three axes that define the cluster. In a future paper, we plan to study the projection effects of triaxial clusters; for the purpose of this paper, we will only consider ellipsoids.

Apart from the Hubble constant, the projection effects are also present in the total gas mass derived from the X-ray emission observations with $M_{gas} \propto D_A^{5/2} Z^{1/2}$, and the total mass based on the virial theorem using X-ray temperature as $M_{\rm total} \propto D_A Z^{-1}$. Then, the gas mass fraction can be written as $f_{\rm gas} \propto D_A^{3/2} Z^{3/2}$. Since $H_0 \propto Z^{-1}$ and the $f_{\rm gas} \propto Z^{3/2}$, we expect the H_0 and $f_{\rm gas}$ to exhibit a negative correlation, if both measurements are affected by the projection effect.

3. Hubble constant and gas fraction

Table 1 lists the Hubble constant values that have so far been obtained from SZ observations (Cooray et al. 1998b, see also Hughes 1997). These values have been calculated under the assumption of a spherical gas distribution with a β profile for the electron number density and an isothermal atmosphere. For the same clusters, we compiled a list of gas mass fraction measurements using X-ray, SZ, and gravitational lensing observations. Most of the clusters in Table 1 have been analyzed by Allen & Fabian (1998), where they included cooling flow corrections to the X-ray luminosity and the gas temperature. For the two clusters (A2256 & Cl0016+16) for which H_0 measurements are available, but not analyzed in Allen & Fabian (1998), we used the results from Buote & Canizares (1996) and Neumann & Böhringer (1996), respectively. The gas mass fractions in Allen & Fabian (1998) have been calculated to a radius of 500 kpc, while for the A2256 and Cl0016+16, they have been calculated to different radii, and also under different cosmological models. Using the angular diameter distance dependence on the gas

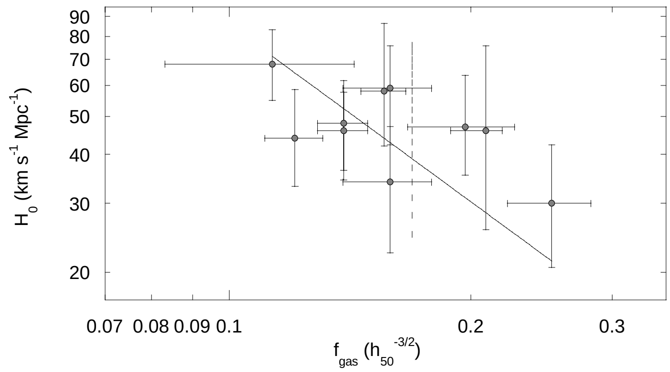

Fig. 1. The observed gas mass fraction of galaxy clusters and the SZ/X-ray Hubble constant. The vertical dashed line is the mean value of the gas mass fraction. The solid line is the best-fit relation between the H_0 values and the $f_{\rm gas}$ values, assuming $h \propto f_{\rm gas}^{-2/3}$. This line is favored at $\sim 2\,\sigma$ confidence over a constant H_0.

mass fraction measurements with redshift (Cooray 1998), we converted all the gas mass fraction measurements to a cosmology of $\Omega_m = 0.2$, $\Omega_\Lambda = 0$, and $H_0 = 50\, h_{50}^{-1}$ km s^{-1} Mpc^{-1}. In order to facilitate comparison between the gas mass fractions measured at various radii, we scaled them to the r_{500} radius based on relations presented by Evrard (1997). The r_{500} radius has been shown to be a good approximation to the outer hydrostatic boundary of galaxy clusters (Evrard, Metzler, Navarro 1996). We list the derived cluster gas mass fraction at the r_{500} radius in Table 1.

In Fig. 1, we show the calculated $f_{\rm gas}$ against H_0 values for each of the clusters. As shown, the gas fraction measurements have a broad distribution with a scatter of $\sim$ 40% from the mean value. A similar broadening of the Hubble constant, from 30 to 70 km s^{-1} Mpc^{-1} with a mean of $\sim$ 50 km s^{-1} Mpc^{-1} is observed. The correlation is negative, and suggest that clusters with high gas mass fraction measurements produces Hubble constant values at the low end of the distribution, while the opposite is seen for clusters with high gas mass fraction. The solid line in Fig. 1 is the best-fit relation between h and $f_{\rm gas}$ assuming $h \propto f_{\rm gas}^{-0.66}$. For values in Table 1, the best-fit line, when the slope between h and $f_{\rm gas}$ is allowed to vary, scales as $h \propto f_{\rm gas}^{-0.8\pm0.4}$, which is fully consistent with the expected relation. Since the current SZ cluster sample is small, a careful study of a complete sample of galaxy clusters are need to fully justify the projection effects between SZ and X-ray derived Hubble constant and gas mass fractions values. We derived a similar negative correlation between h and $f_{\rm gas}$ when the cluster gas mass fraction is measured from SZ. For example, Myers et al. (1997) derived a gas mass fraction of $(0.120 \pm 0.022)\, h_{50}^{-1}$ for A2256, which is at the low end of the gas mass fraction values, while a gas mass fraction of $(0.33 \pm 0.028)\, h_{50}^{-1}$ was derived for A478, which is the cluster at the high end. We note here that, as we discuss later, the SZ derived gas mass fractions scale with h as only h^{-1}, while X-ray derived gas mass fractions, which are presented in Table 1, scale with h as $h^{-3/2}$. In comparison, the gas mass fractions derived from SZ and X-ray observations may

Table 1. SZ Effect/X-ray H_0 Measurements and X-ray Gas Mass Fractions.

Cluster	Redshift	H_0 (km s^{-1} Mpc^{-1})	H_0 Reference	$f_{\rm gas}$ (r_{500}) (h$_{50}^{-3/2}$)
A2256	0.0581	68^{+21}_{-18}	Myers et al. 1997	$0.11^{+0.03}_{-0.03}$
A478	0.0881	30^{+17}_{-13}	Myers et al. 1997	$0.25^{+0.03}_{-0.03}$
A2142	0.0899	46^{+41}_{-28}	Myers et al. 1997	$0.21^{+0.01}_{-0.02}$
A1413	0.143	44^{+20}_{-15}	Saunders 1996	$0.12^{+0.01}_{-0.01}$
A2218	0.171	59 ± 23	Birkinshaw & Hughes 1994	$0.16^{+0.02}_{-0.02}$
A2218	0.171	34^{+18}_{-16}	Jones 1995	$0.16^{+0.02}_{-0.02}$
A665	0.182	46 ± 16	Hughes & Birkinshaw 1998b	$0.14^{+0.02}_{-0.02}$
A665	0.182	48^{+19}_{-16}	Cooray et al. 1998c	$0.14^{+0.02}_{-0.02}$
A2163	0.201	58^{+39}_{-22}	Holzapfel et al. 1997	$0.15^{+0.01}_{-0.01}$
Cl0016+16	0.5455	47^{+23}_{-15}	Hughes & Birkinshaw 1998a	$0.17^{+0.03}_{-0.03}$

H_0 & $f_{\rm gas}$ is calculated assuming $\Omega_m = 0.2$ and $\Omega_\Lambda = 0$.

be affected similar to the measurements based on only the X-ray data. Additional probes of the total mass are the gravitational lensing measurements and the optical virial analysis of internal galaxy velocity dispersion measurements. In the present SZ/X-ray sample, A2218 (Kneib et al. 1995) and A2163 (Squires et al. 1997) have lensing mass measurements. In both these clusters total virial masses when measured using X-ray gas temperature, agrees with the weak lensing mass measurements at large radii, and since these two clusters are not the ones which are primarily responsible for the observed negative correlation, we cannot state the effect of lensing mass measurements on the above data. Also, in the present SZ cluster sample, A2256 and A2142 (Girardi et al. 1998), and Cl0016+16 (Carlberg et al. 1997) have measured total masses from optical virial analysis. These virial masses are in good agreement with X-ray masses, allowing an independent robust measurement of the total mass (Girardi et al. 1998).

Finally, there is a slight possibility that the observed broad distribution and negative correlation in H_0 and $f_{\rm gas}$ is not really present. The negative correlation is only present at a level of $\sim 2\,\sigma$, assuming that the errors in h and $f_{\rm gas}$ are independent. The removal of either one of the clusters at high or low end reduces the negative correlation, decreasing the significance of the observed correlation. However, both the Hubble constant and, possibly, the gas mass fraction is expected to be constant, suggesting that a point, or a region when considering errors in H_0 and $f_{\rm gas}$, is preferred. We rule out the possibility that both H_0 and $f_{\rm gas}$ are constants in the present data with a confidence greater than 95%.

3.1. Evidence for a projection effect?

Usually, the broad distribution of the SZ and X-ray Hubble constants has been explained in literature based on the expected systematic effects. The systematic effects in the gas mass fraction measurements are reviewed in Evrard (1997) and Cooray (1998). We briefly discuss these systematic uncertainties in the context of their combined effects on H_0 and $f_{\rm gas}$.

It has been suggested that cluster gas clumping may overestimate H_0 from the true value. As reviewed in Evrard (1997), cluster gas clumping also overestimates $f_{\rm gas}$, suggesting that if gas clumping is responsible for the observed trend, a positive correlation should be present. The nonisothermality underestimates H_0 by as much as 25% (e.g. Roettiger et al. 1997). To explain the distribution of H_0 values, the cluster temperature profile from one cluster to another is expected to be different. However, Markevitch et al. (1997) showed the similarity between temperature profiles of 30 clusters based on ASCA data (including A478, A2142 & A2256 in present sample). Since SZ and X-ray structural fits weigh the gas distribution differently, even a similar temperature profile between clusters can be expected to cause the change in the Hubble constant from one cluster to another. Another result from the Markevitch et al. (1997) study is that the $f_{\rm gas}$ measurements as measured using β-models and standard isothermal assumption is underestimated. The similarity of cluster temperature profiles also suggests that the gas mass fractions are affected by changes in temperature from one cluster to another. It is likely that the present isothermal assumption has underestimated both H_0 and $f_{\rm gas}$, and that temperature profiles are responsible for the observed behavior. A large sample of clusters, perhaps the same cluster sample studied by Markevitch et al. (1997), should be studied in SZ to determine the exact effect of radial temperature profiles on H_0, and its distribution.

The third possibility is the cluster asphericity. The effect of cluster projection on H_0 was first suggested by Birkinshaw et al. (1991), who showed that the derived values for H_0 can be offset by as much as a factor of 2 if the line of sight along the cluster is different by the same amount. The present cluster isophotal ellipticities suggest that H_0 may be offset as much as $\pm$ 27% (e.g., Holzapfel et al. 1997). The present $f_{\rm gas}$ distribution is suggestive of this behavior. Cen (1997), using numerical simulations, studied the effects of cluster projection on gas mass fraction measurements, and suggested differences of the order $\sim$ 40%. The $f_{\rm gas}$ distribution is similar to what has been seen in Cen (1997). It is more likely that the projection effects are causing the distribution of H_0 and $f_{\rm gas}$ values, unless a systematic effect still not seen in numerical simulations is physically present in galaxy clusters. Such effects could come from effects due to variations in the temperature profiles from one cluster to another. For the rest of the discussion, we assume that the present values are affected by projection effects, rather than temperature profiles.

4. Hubble constant without projection effects

Here, we consider the possibility of deriving the Hubble constant in a meaningful manner without any biases due to cluster projections. It has been suggested in literature that observations of a large sample of galaxy clusters can be used to average out the dependence on the scale factor Z and to produce the true value of the Hubble constant, which we define as $H_0^{\rm true}$ from individual Hubble constant measurements, $H_0^{\rm i}$, in a large sample of clusters. We investigate the possibility of such an averaging by considering the different projections of clusters at different inclination angles. Assuming the previously described ellipsoidal shape and the effect of the scale factor Z in the Hubble constant, we can over the all possible inclination angles θ and the ratio r_2/r_1 to derive the expected average value of the Hubble constant $< H_0^{\rm i} >$:

$$< H_0^i >= \frac{1}{2}\left[x + \frac{\sin^{-1}\sqrt{1-x^2}}{\sqrt{1-x^2}}\right] \times H_0^{\rm true}, \tag{10}$$

if all clusters are prolate, and

$$< H_0^i >= \frac{1}{2}\left[x + \frac{\sinh^{-1}\sqrt{x^2-1}}{\sqrt{x^2-1}}\right] \times H_0^{\rm true}, \tag{11}$$

if all clusters are oblate. Here $x = r_2/r_1$. When all clusters are prolate and that the semi-major axis used to calculate the Hubble constant, then the distribution has a mean of $H_0^{\rm true}$. However, if the semi-minor axis is used, then the average Hubble constant is underestimated from the true value by about $\sim$ 10%, assuming that the mean r_2/r_1 is 0.7 for prolate clusters. If all clusters are oblate, and the semi-major axis is used to derive the Hubble constant, then the mean of the distribution overestimates the true value of the Hubble constant by as much as $\sim$ 20%, if the mean r_2/r_1 is 1.5 for oblate clusters. For oblate clusters, the true value of the Hubble constant can be obtained when the semi-minor axis is used. However, in both oblate and prolate cases, the distribution has a large scatter requiring a large sample of galaxy clusters to derive a reliable value of the Hubble constant. A similar calculation can also be performed for the gas mass fraction to estimate the nature of the value derived by averaging out a gas mass fraction measurements for a large sample of clusters. Here again, a similar offset as in the Hubble constant is present, and measurements of gas mass fraction in a large sample of clusters are needed to put reliable limits on the cosmological parameters, especially the mass density of the universe based on cosmological baryon density (e.g., Evrard 1997).

So far, we have only considered the SZ and X-ray observations of galaxy clusters. By combining weak lensing observations towards galaxy clusters, we show that it may be possible to derive a reliable value of the Hubble constant based on observations of a single cluster. The gravitational lensing observations of galaxy clusters measure the total mass along the line of sight through the cluster. The SZ effect measures the gas mass along the line of sight, and thus, the ratio of SZ gas mass to gravitational lensing total mass should yield a measurement of the gas mass fraction independent of cluster shape assumptions and asphericity. Here, we assume that the cluster gas distribution exactly traces the cluster gravitational potential due to dark matter, and that these two measurements are affected equally by cluster shape. This is a reasonable assumption, but however, it is likely that gas distribution does not follow the dark matter potential, and that there may be some dependence on the cluster shape between the two quantities. For now, assuming that the gas mass fraction from SZ and gravitational lensing is not affected by cluster projection, we outline a method to estimate the Hubble constant independent of the scale factor Z. The gas mass fraction based on SZ and lensing is $f_{\rm SZ}^{\rm lens} \propto h^{-1}$, while the gas mass fraction based on X-ray emission gas mass and the total mass based on X-ray temperature is $f_{\rm X-ray}^{\rm temp} \propto h^{-3/2} Z^{3/2}$. Since the two gas mass fraction measurements are expected to be the same, then one can solve for a combination of h and Z. However to break the degeneracy between h and Z an additional observation or an assumption is needed. In general, there are large number of clusters with X-ray measurements and X-ray based gas mass fraction measurements. By averaging out the gas mass fraction for such a large sample of clusters, we can estimate the universal gas mass fraction value for clusters, e.g. $(0.060 \pm 0.002)h^{-3/2}$ (Evrard 1997; Cooray 1998). If assumed that this gas fraction is valid for the cluster for which SZ and weak lensing observations are available, we can then calculate the Hubble constant.

We applied this to SZ, X-ray and weak lensing observations of galaxy cluster A2163. The SZ observations of A2163 are presented in Holzapfel et al. (1997), while weak lensing and X-ray observations are presented in Squires et al. (1996). The SZ effect towards A2163 can be described with a y ($\Delta T_{\rm SZ}/T_{\rm CMB}$) parameter of $3.07^{+0.54}_{-0.60} \times 10^{-4}$, which includes various uncertainties described in Holzapfel et al. (1997). The weak lensing observations of A2163 has been used to derive the total cluster mass in Squires et al. (1996), and the lensing observations are most sensitive out to a radius of $\sim 200''$ (0.423 $\rm h^{-1}$ Mpc) from the cluster center, where the total mass is $(5 \pm 2) \times 10^{14}$ $h^{-1}\ M_\odot$. Using the cluster model (β and r_c) in Holzapfel et al. (1997), we integrated the SZ temperature change to this radius from cluster center along the line of sight to derive a gas mass of $(4.3 \pm 2.5) \times 10^{13}\ h^{-2}\ M_\odot$. This represents the gas mass within the cylindrical cut across the cluster, and effectively probes the same region as the weak lensing observations. The gas mass fraction based on the SZ effect and the weak lensing total mass is $(0.086 \pm 0.060)h^{-1}$. When this gas mass is compared to the effective gas mass fraction of clusters, $(0.060 \pm 0.020)h^{-3/2}$ (Evrard 1997; Cooray 1998), we obtain $h = 0.49 \pm 0.29$. We have slightly overestimated the error in the average gas mass fraction to take into account the fact that this fraction is measured at the outer hydrostatic radius ($\sim$ 1 Mpc), and may not correspond to the value at the observed radius of A2163. In Squires et al. (1997), the gas mass fraction was measured to be $(0.07 \pm 0.03)h^{-3/2}$ for A2163, which is in agreement with our universal value, but the value in Squires et al. (1997) may be subjected to a scaling factor. The combined SZ/lensing gas mass fraction and the average gas mass fraction for clusters result in a Hubble constant of $H_0 = 49 \pm 29$ km s^{-1} Mpc^{-1}. Given that we used data from 2 different papers in deriving this Hubble constant, it is likely that this value may be subjected to unknown systematic effects between the two studies. We strongly recommend that a careful analysis of cluster data be carried out to derive the Hubble constant based on SZ, X-ray and weak lensing observations. In addition, total virial masses from velocity dispersion analysis should also be considered in such an analysis to constrain the cluster shape. It is likely that much stronger and reliable result may be obtained through this method, instead of just SZ and X-ray observations. In Holzapfel et al. (1997), the Hubble constant was derived to be ~ 60 km s^{-1} Mpc^{-1} for an isothermal temperature model and ~ 78 km s^{-1} Mpc^{-1} for a hybrid temperature model. Our value is lower than these two values, but is in good agreement with the average value of H_0 based on SZ and X-ray as tabulated in Table 1, which is in agreement with the average gas mass fraction value.

5. Conclusions

Using the Hubble constant measurements based on SZ and X-ray, and the gas mass fraction measurements, we have suggested a possible systematic effect due to cluster projection. Even though, cluster projection had been suggested as a possible systematic bias in H_0 measurements, more attention has recently been given to various *exotic* effects as a way to explain the broad distribution of Hubble constant values. We have shown here the presence of projection effects in the present H_0 and $f_{\rm gas}$ measurements and have analytically calculated the effect of cluster projection in deriving the Hubble constant. It is also assumed in literature that for a large sample of clusters, the average of the individual Hubble constants, after making various corrections, can be used to determine the true Hubble constant. We have shown here that this may not be easily possible, and that when a random and large sample is available with a mix of prolate and oblate clusters, the best that one could expect to obtain is a Hubble constant value within 10% of the true value, unless the distribution of ellipticities for cluster sample is carefully taken into account. Thus, we strongly recommend that more attention be given to the cluster asphericity in deriving cosmologically important measurements such as Hubble constant and the cluster gas mass fraction. For individual clusters, for which SZ observations are available, we have shown that a combined study of SZ, X-ray, velocity dispersion measurements and weak lensing observations can be used in a more physical manner to derive the Hubble constant. Thus, we have demonstrated the usefulness of gravitational lensing observations of galaxy clusters for cosmologically important studies, and when combined, more meaningful results are expected to be produced instead of just combining SZ and X-ray observations. We strongly recommend that weak lensing observations and velocity dispersion measurements be carried out to test the reliability of Hubble constant values in Table 1, and to complement SZ observations of clusters.

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