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Cosmological Malmquist bias in the Hubble diagram at high redshifts

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Abstract. The Malmquist bias in luminosity distances for gaussian standard candles is discussed within cosmological models where the Euclidean r^3-law for volumes and r^{-2}-law for fluxes is not valid. Furthermore, the influence of K-corrections and luminosity evolution are analyzed. It is noted that the usual way of comparing theoretical predictions and data points in the Hubble diagram ($\log z$ vs. m) should be modified in view of the cosmological Malmquist bias. When the space distribution of galaxies is uniform, the classical Malmquist bias is constant at all apparent magnitudes, which is no more generally true within uniform cosmological models.

Especially, calculations are made in Friedmann models for standard candles with different gaussian dispersions σ around average absolute magnitude M_0. The usual $\log z$ vs. m (or Mattig) relations are deformed by amounts depending on the Friedmann model itself, on σ, and on the apparent magnitude of the standard candle. The implications on estimations of q are shown to be significant when $\sigma \geq 0.3$ mag.

It is concluded that the cosmological Malmquist bias is a necessary part of the theory of gaussian standard candles at high redshifts. It is also emphasized that one should always consider two complementary aspects of the Hubble diagram as a cosmological test, i.e. the $\log z$ vs. m and m vs. $\log z$ approaches, the first one influenced by the bias here discussed, while the second one is plagued by the magnitude limit (Malmquist bias of the 2nd kind).

For example, with $\sigma = 0.5$ mag, in the case of bolometric magnitudes, the traditional $\log z$ vs. m procedure in the brighter part ($\langle z \rangle$ less than about 1.5) of the Hubble diagram, would make one believe that $q_0 = 0.25$ when it actually is 0.5. Without evolution, but in the presence of K-effect typical for V-band and E-galaxies, one would derive $q_0 \approx 0.1$ in the case of $q_0 = 1.0$ when the K-effect is simply put into the zero-dispersion theoretical curve. With a good standard candle having $\sigma = 0.3$, these results would change to $q_0 = 0.4$ (instead of 0.5) and = 0.5 (instead of 1.0).

We also discuss the bias in angular size distance, which is shown to work in a different sense than the bias in luminosity distance, and the deviation from the classical bias is large already well below the distance maximum in Friedmann models.

Key words: galaxies: distances and redshifts – distance scale

1. Introduction

In the treatment of the classical Malmquist (1920) bias, the space is assumed to be Euclidean and transparent. Furthermore, the standard candles (with a gaussian distribution of absolute magnitudes) do not change with lookback time nor have any K-effect. In modern cosmological models these assumptions are not valid, when e.g. Supernovae of type Ia are detected up to $z > 0.5$ and radio galaxies and quasars well beyond $z = 1$. As the Hubble diagram is usually looked at as $\log z$ vs. m and the theoretical predictions are compared with $\langle \log z \rangle$ vs. m, it is important to study how the non-classical situation influences the run of the standard candle data points in a Hubble diagram and how one should compare observations and model curves.

1.1. Malmquist biases of the first and second kind

Actually, there appears in the literature two ways of looking at the Hubble diagram – both at small redshifts where one studies the Hubble constant and large scale galaxy streams, and also at high redshifts where one has in mind applications on the value of q_0 and on evolution of galaxies. These two ways, either considering $\langle \log z \rangle$ vs. m or $\langle m \rangle$ vs. $\log z$ for a gaussian standard candle, are respectively accompanied by two distortion effects, Malmquist bias of the 1st kind and Malmquist bias of the 2nd kind (see Teerikorpi 1997 where this terminology is suggested in order to unify the rather varied practice). The 1st bias is closely related to the classical Malmquist bias, while the 2nd bias arises due to a magnitude limit cutting away parts of the fainter wing of the luminosity function (LF) of the standard candle.

These two different biases are often called in the contemporary literature as ”Malmquist bias” and ”selection bias”, respectively, with the understanding that the Malmquist bias is basically caused by the geometry or space distribution of the standard candles, while the selection bias refers to the influence of observational (e.g. magnitude) limits. Concrete examples of how these two types of biases are discussed and treated in the study of the local galaxy universe, one may find e.g. in Willick (1994), Freundlich et al. (1995), Gonzales & Faber (1997), and Theureau et al. (1997). Sandage (1995) gives a very illuminating discussion of how these biases are related to each other in the classical space, uniformly filled by galaxies.

It is useful to state short definitions of these two types of biases, analogous to Sect. 3.1 in Teerikorpi (1997), but here in terms of $\log z$ and m, directly applicable to a Hubble diagram, instead of true and derived distance moduli. Consider a standard candle with (bolometric) average absolute magnitude $\langle M \rangle = M_0$. Within the cosmological model in question, there is an exact relation between apparent magnitude m of the candle having $M = M_0$ and redshift z:

$$m = M_0 + f(z)$$

This may be converted to give

$$\log z = F(m, M_0).$$

Assume that the candles have been selected from the sky only on the basis of apparent magnitude so that there is no selection, say, discriminating against some values of z. For the sample galaxies within $m \pm \frac{1}{2}dm$ there is the average $\langle \log z \rangle_m$.

Then the Malmquist bias of the first kind is

$$\langle \log z \rangle_m - F(m, M_0)$$

and the Malmquist bias of the second kind is

$$\langle m \rangle_z - (M_0 + f(z))$$

Note that the first bias is concerned with the average $\log z$ at a fixed value of apparent magnitude m, as compared with the prediction from the exact relation $\log z = F(m, M_0)$. The second bias is concerned with the average apparent magnitude at some constant redshift. At small redshifts, where $z \to (H/c)r$ and $f(z) \to 5 \log r$+const, these definitions go over to the usual classical ones.

These two biases have been rather extensively discussed in connection with the Hubble diagram at small redshifts, in the local galaxy universe (see the above references), while they have received less attention at high redshifts. Perhaps one reason is that the classical Hubble magnitude-redshift cosmological test has lost much of its attraction when one has encountered formidable problems trying to use evolving galaxies as standard candles, the uncertain evolution apparently dominating over other effects. When the Hubble diagram is presented, either as $\log z$ vs. m or m vs. $\log z$, it has been common to display, in the frame of Friedmann models, the classical Mattig (1958) curves. Such curves give the connection between redshift and apparent (bolometric) magnitude for ideal standard candles with exactly the same absolute magnitude (see Sandage (1995) for an interesting discussion of the Mattig equation). Use of the Mattig curves is justified in the m vs. $\log z$ approach, if there is no Malmquist bias of the 2nd kind or the selection bias, because then $\langle m \rangle$ for a gaussian standard candle really follows the exact Mattig curve (if one can ignore any scatter in z). Sometimes, but not at all always, the selection bias has been discussed in the m vs. $\log z$ approach, while the bias of the 1st kind has been generally ignored in the $\log z$ vs. m approach where it inevitably appears. Its omission actually is equivalent to the assumption that the bias is constant at all m, as is true classically (when the space distribution of standard candles is uniform).

1.2. $\log z$ *vs.* m *and* m *vs.* $\log z$ *approaches*

The traditional $\langle \log z \rangle$ vs. m approach, instead of $\langle m \rangle$ vs. $\log z$, is often preferable, because in the latter case one encounters problems with the magnitude limit, or more generally, the progressively increasing incompleteness (Malmquist bias of the 2nd kind) which may be difficult to model. This happens just there, i.e. at high redshifts, where one also expects test information about the world model. In the $\langle \log z \rangle$ vs. m approach, the magnitude limit or observational incompleteness in m does not matter, and the relevant effects are directly related to either the world model or to intrinsic properties of the standard candles. Of course, there may be cases when the magnitude limit problem is not serious, as has been stated by Perlmutter et al. (1997) in their study of the Hubble diagram of Supernovae. Then the $\log z$ vs. m approach should give a consistent result.

It is important to note that the alternative approaches give complementary cosmological test information. As a simple example, one may have obtained the "classical" relation $m = 5 \log z+$ const. from m versus $\log z$ for a standard candle. Then, without any knowledge of Friedmann models or physics of redshift, one might conclude that redshift z is just proportional to Euclidean distance in a static universe. In a homogeneous universe this conclusion may be tested from $\log z$ versus m. In the Euclidean case, one would thus recover the expected relation $\langle \log z \rangle = 0.2m$+const', where const' includes a constant Malmquist term. However, in a Friedmann universe, $\log z$ versus m would not give directly $q_0 = 1$, but only if one takes into account the non-classical Malmquist bias.

When we speak in this paper about the Hubble diagram, we have in mind especially the traditional $\log z$ vs. m approach. Consequently, *we are concerned with the Malmquist bias of the first kind, which we call the cosmological Malmquist bias, in distinction from the classical bias*.

1.3. Aim and contents of the paper

We describe in the present paper the basic ingredients of the cosmological Malmquist bias (of the 1st kind) and how its amount is calculated in a given situation. Primarily, the intention is to show how much the omission of the cosmological Malmquist bias influences analysis of the Hubble diagram and encourage its consideration in future studies of observational data.

We study the cosmological generalization of the Malmquist bias step by step, starting from the pure (non-classical) volume and flux effects in the case of bolometric magnitudes, in order to see clearly the difference to the classical bias (Sects. 2 – 4).

Then it is emphasized in Sect. 5 that one cannot simply "shift the curve" in order to include the K-effect (or the redshift dependent luminosity evolution). In Sect. 6, the analogous problem with angular size is shortly discussed, and Sect. 7 illustrates how inclusion of the cosmological constant influences the situation. The present discussion is concerned with the Friedmann model, and one should note that any other model needs its own detailed treatment. However, the basic ideas are general.

2. Basic concepts: distances, redshifts, and co-moving volumes

In Friedmann models (in the general case including the cosmological constant), absolute bolometric magnitude M, apparent bolometric magnitude m, and redshift z are connected by

$$m = M + 5\log d_L(z) \tag{1}$$

where $d_L(z)$ is the luminosity distance (in Mpc). It is useful to remind of the connection of d_L to two other distance measures, angular size distance d_A and proper motion distance d_M (Weinberg 1972; Ch. 14):

$$d_L = (1+z)d_M \tag{2a}$$

$$d_L = (1+z)^2 d_A \tag{2b}$$

The distance d_M, and hence d_L and d_A, may be given in terms of the present day Hubble constant H_0, together with the curvature parameter k, the mass density parameter Ω_m, and the cosmological constant parameter Ω_l, connected by

$$k = 1 - \Omega_m - \Omega_l \tag{3}$$

Here Ω_m refers to matter and Ω_l gives the contribution of the cosmological constant Λ to the total density parameter $\Omega_{\rm tot}$ (see the review by Carroll et al. 1992 for discussion, relevant formulae, and useful diagrams). Then

$$d_M = (c/H_0)|k|^{-1/2}\mathrm{sinn}\left(|k|^{1/2}\int_0^z \left[(1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_l\right]^{-1/2} dz'\right) \tag{4}$$

In this formula sinn = sin when $k < 0$ (closed) and = sinh when $k > 0$ (open). In the special flat case ($k = 0$), the formula adopts the form

$$d_M = (c/H_0)\int_0^z \left[(1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_l\right]^{-1/2} dz' \tag{5}$$

In the present paper, we mostly adopt $\Lambda = 0$. Then the luminosity distance becomes, in terms of H_0 and deceleration parameter q_0 (Mattig 1958):

$$d_L = (c/H_0 q_0^2)\left(q_0 z + (q_0 - 1)\left[(1+2q_0 z)^{1/2} - 1\right]\right) \tag{6}$$

and the Mattig equation for the distance modulus $m - M$ is

$$m = M - 5\log q_0^2 + 5\log\left(q_0 z + (q_0-1)\left[-1 + (1+2q_0 z)^{1/2}\right]\right) + 43.89 \tag{7}$$

Here the numerical term 43.89 comes from $25 + 5\log c/H_0$, with $H_0 = 50$ km s^{-1} Mpc^{-1}. Below we use the short notation $f(z)$ for $m - M$ from the above equation.

We also need to know the co-moving volume in the Friedmann model. Consider the galaxies with redshift z_0. In a homogeneous universe the number of galaxies with $z \le z_0$ is proportional to the co-moving volume $V(r(z_0))$ within the co-moving distance r corresponding to the galaxies which presently are observed to have $z = z_0$. The volume $V(z)$ may be calculated for any z and any Friedmann model (e.g. Sandage 1988). E.g., in the Einstein-de Sitter flat model, the co-moving volume $V(z)$ has the especially simple expression

$$V(z) = A\left[1 - 1/(1+z)^{1/2}\right]^3 \tag{8}$$

where $A = (32\pi/3)(c/H_0)^3$. For other $q_0 > 0$ one may numerically calculate $V(z)$ by integrating the derivate of $V(z)$:

$$dV(z)/dz = (4\pi c/H_0)d_L^2/\left[(1+z)^3(1+2q_0 z)^{1/2}\right] \tag{9}$$

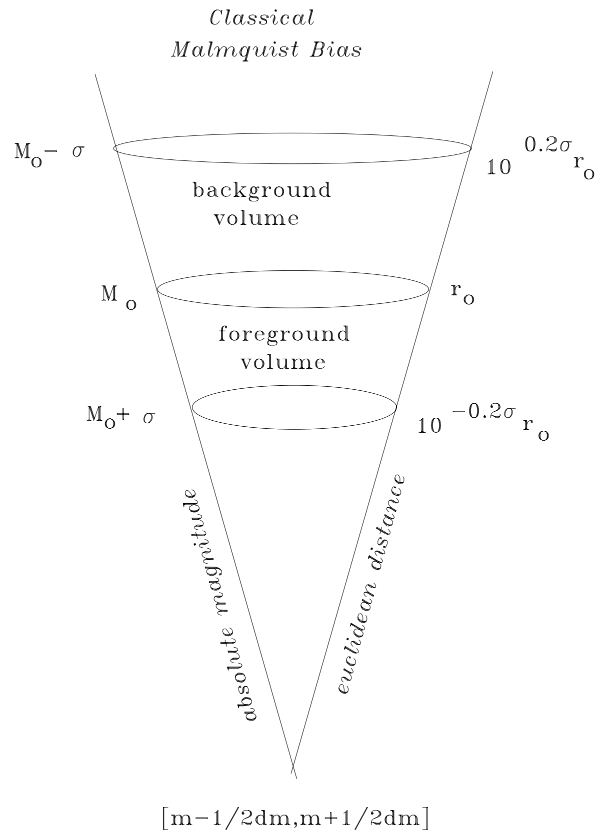

Fig. 1. Classical Malmquist Bias: Symmetrical parts of the gaussian luminosity function, observed through the magnitude window [m-1/2dm, m+1/2dm], originate from different foreground and background volumes as determined by the r^3-law of Euclidean volumes and the r^{-2}-law of fluxes.

3. Volumes probed by galaxies at $m \pm 1/2dm$

If M for a standard candle has a gaussian distribution around M_0, galaxies with $M = M_0$ and observed to have apparent magnitude m, must lie at z given by $m = M_0 + f(z_0)$. Galaxies from the luminous wing of the LF and also having apparent magnitude m, originate from the volume beyond z_0, while those from the fainter wing come from the foreground volume.

In the classical case, one has instead of redshift the usual Euclidean distance r and the distance modulus $m - M$ has the familiar connection with r. Then in the case of a uniform space distribution, the background and foreground volumes defined by galaxies having, say, $M = M_0 - \sigma$ and $M = M_0 + \sigma$ and

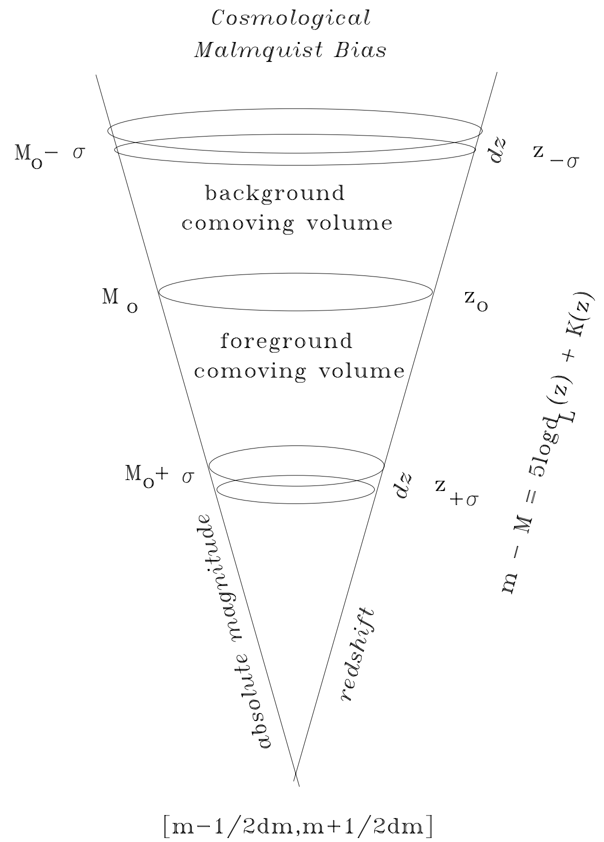

Fig. 2. Cosmological Malmquist Bias in Friedmann models: Symmetrical parts of the gaussian luminosity function, observed through the magnitude window $[m - 1/2dm, m + 1/2dm]$, originate from different comoving volumes. Fundamental theory gives the change of M with redshift z (together with the K-term K(z)) and the value of comoving volume, allowing one to calculate the average value of logz for standard candles with apparent magnitude m.

falling on the magnitude interval $m \pm 1/2dm$, are different (see Fig. 1). Their ratio $R_{1\sigma}$ is a useful indicator of the Malmquist bias. Classically, it has the constant value:

$$R_{1\sigma} = \frac{10^{0.6\sigma} - 1}{1 - 10^{-0.6\sigma}} \quad (10)$$

In Friedmann models, such foreground and background volumes must be defined as co-moving volumes.

In order to get at once a feeling of how the classical case is deformed when one considers a Friedmann model, we calculate the mentioned "1σ volume ratios" $R_{1\sigma}$ for a few z_0 in the Einstein-de Sitter model and with two other models ($q_0 =$ 0.05 and 1.0), and compare these with the classical ratio (that depends only on σ).

Suppose that the dispersion of the standard candle is σ. Then galaxies symmetrically around M_0, at $M_0 + \sigma$ and $M_0 - \sigma$, are found at redshifts $z(+\sigma)$ and $z(-\sigma)$ ($z(+\sigma) < z(-\sigma)$) (see Fig. 2). These redshifts are determined by the requirements:

$$f(z(+\sigma)) - f(z_0) = -\sigma \quad (11a)$$

$$f(z(-\sigma)) - f(z_0) = +\sigma \quad (11b)$$

Table 1. 1σ volume ratios

z/q_0	0.05	0.5	1.0	0.5 & K$(z)^*$	$\Omega_l = \Omega_m = 0.5$
0.001	1.99	1.99	1.99	1.99	1.99
0.1	1.84	1.84	1.84	1.44	1.85
0.5	1.54	1.51	1.49	1.07	1.55
1.0	1.40	1.35	1.31	1.05	1.39
1.5	1.34	1.26	1.21	1.24	1.29
2.0	1.30	1.20	1.15	1.19	1.23

* K(z) is the "elliptical" K-correction (Sect. 4.1)

The desired ratio $R_{1\sigma}$ of the volumes is then

$$R_{1\sigma} = \frac{V(z(-\sigma)) - V(z_0)}{V(z_0) - V(z(+\sigma))} \quad (12)$$

In Table 1 the volume ratios are given for $z =$ 0.001, 0.1, 0.5, 1.0, 1.5, and 2.0, when $\sigma =$ 0.5 mag. The corresponding classical ratio for $\sigma =$ 0.5 mag is = 1.995, close to the calculated volume ratio for $z = 0.001$ in all Friedmann cases. We shall return to the last columns of Table 1 in Sects. 5 and 7 where the influence of the K-correction and non-zero cosmological constant are respectively discussed.

At increasing redshifts the ratios get progressively smaller which means that the Malmquist bias decreases (the volumes where the bright and faint Gaussian wings are sampled differ less). It is useful to inspect this table when one formally identifies redshift z with distance. Then the case $q_0 = 1$ has the same dependence of m on z as classically m has on distance r, and the strong dependence of the volume ratio on z hence reflects directly the "geometry" of such a z-space. For smaller q_0, both $m(z)$ dependence and geometry change, their effects working in opposite directions, and the net effect is that the volume ratios differ a little less from the classical one, in comparison with $q_0 = 1$. We illustrate the relevant trends in Fig. 3, where 3a shows the luminosity distance versus z, 3b shows the angular size distance versus z, and 3c shows the volume derivative dV/dz versus z, for a few Friedmann models. In all diagrams, we have also indicated the relation for the "classical" case, where $z \propto$ Euclidean distance.

4. Calculation of $< \log z >$ at $m \pm 1/2dm$

When one looks at the Hubble diagram as $\log z$ against m, and tries to compare the observed distribution of data points with some theoretical scheme, the basic task is calculation of the average value of $\log z$ at constant value of m. Already the classical Malmquist bias makes one anticipate that $\langle \log z \rangle$ at m is not the same as $\log z(m, M_0)$, where $z(m, M_0)$ has been solved from the Mattig equation (corresponding to an ideal standard candle with a zero-dispersion LF).

4.1. Basic formula

In this section we study the case of bolometric magnitude, ignoring any complications due to K-effect. It is simply assumed that in the observed sample, the galaxies at constant m, are found in

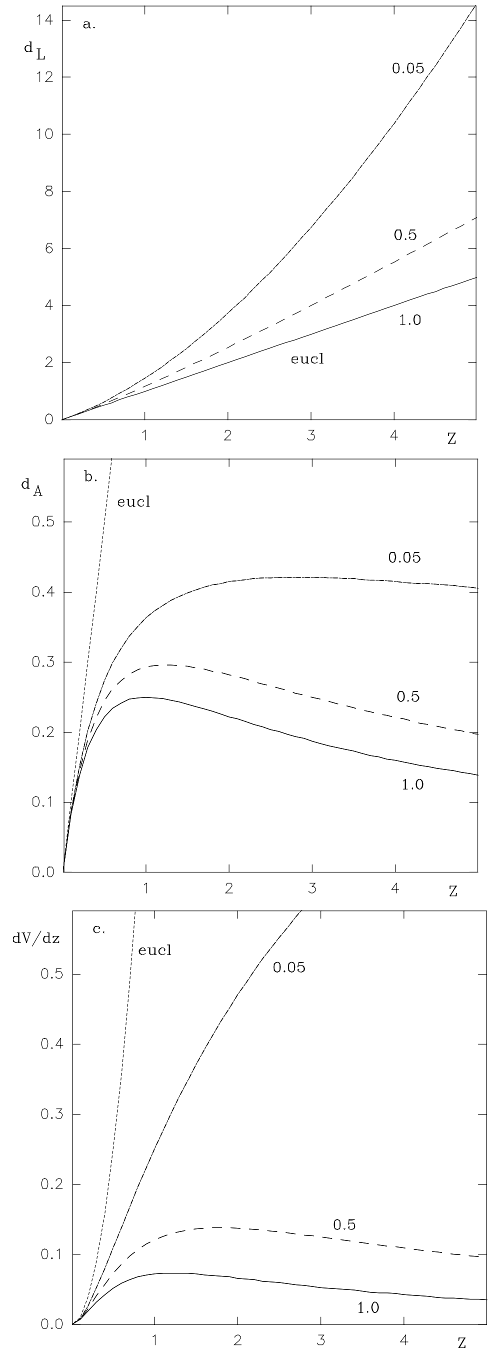

Fig. 3. These diagrams show how luminosity distance d_L, angular size distance d_A (both in unit of c/H_0), and derivative of comoving volume dV/dz (per unit solid angle) depend on redshift z, for Friedmann models with $q_0 = 0.05$, 0.5, and 1.0 ($\Lambda = 0$). "Eucl" means the case when z is proportional to distance in an Euclidean space – it forms an instructive comparison to what happens in a Friedmann universe. E.g., $q_0 = 1$ is closest to Euclidean as regards the luminosity distance, but farthest from Euclidean as regards the volume derivative.

space distributed according to z as expected on the basis of the co-moving volumes corresponding to dz and the space density of galaxies having absolute magnitude $M = m - f(z)$. It is useful to emphasize that a completeness in m is not required. It is, of course, not necessary that the sample contains all the existing standard candles in the sky having apparent magnitude $m \pm 1/2dm$. However, one must assume that the available sample galaxies at m faithfully follow the underlying z-distribution so that no selection effect distorts the distribution of z.

Denoting $\log z$ by μ, the average value of $\log z$ for a fixed m can be calculated as:

$$\langle \log z \rangle_m = \frac{\int_{-\infty}^{\infty} \mu dV(\mu) G(m - f(\mu))}{\int_{-\infty}^{\infty} dV(\mu) G(m - f(\mu))} \tag{13}$$

In this expression, $dV(\mu) = (dV(\mu)/dz)(dz/d\mu)d\mu$, and the derivative $dV(\mu)/dz$ was given by Eq. (9).

4.2. Examples

In the displayed examples, we use two values of the dispersion σ for a standard candle: 0.3 mag and 0.5 mag. Of these, 0.3 mag may be regarded in practice as a very good one. It is assumed that $M_0 = -23$ and $H_0 = 50$ km s^{-1} Mpc^{-1}, and some results of calculation have been given for Friedmann models (of zero cosmological constant) with $q_0 = 0.25$, 0.5 and 0.75.

Figs. 4–5 show the difference between the actual $\langle \log z \rangle$ and the $\log z$ predicted by the Mattig $\log z$ vs. m relation which implicitly presupposes that the dispersion σ is zero. At zero redshift the difference is normalized to be zero by adding to the Mattig curve the classical Malmquist term $0.2 \times 1.382\sigma^2$: at small z the calculation correctly reproduces the classical Malmquist bias. Note that in practice, when one is not interested in the volume-limited average absolute magnitude of the standard candle, this adjustment has been left for Nature to do.

Conversely, one may read from Figs. 4–5 (by changing the sign) the *correction* which must be added to $\langle \log z \rangle$ at a given m if one wishes to compare a Hubble diagram for bolometric magnitudes with the predictions made by the Mattig curves. The correction depends on the correct Friedmann model itself (H_0, q_0, and more generally, Λ) and on the correct parameters of the standard candle (M_0, σ). In fact, because $m = M - 5 \log H_0 + 25 + F(q_0, \Lambda)$ and H_0 does not appear in the volume ratios, the correction at m is the same for any such choice of M_0 and H_0 giving $M_0 - 5 \log H_0 =$ fixed constant.

Let us consider only the case where standard candles may be observed in sufficient numbers already at small redshifts. There $m = M - 5 \log H_0 + 25 + 5 \log cz$, and the zero point $M_0 - 5 \log H_0 + 25$ is obtained without a separate determination of the distance scale (H_0) and the q_0 parameter. At small redshifts also the dispersion σ may be determined. Hence, in such a case an important feature of the standard classical Hubble diagram test is preserved: one does not need to know the value of H_0.

If one takes another value of $M_0 - 5 \log H_0$, e.g. another standard candle class (keeping H_0 and σ fixed) which is one magnitude brighter, then the corrections from Figs. 4–5 are valid for this class when one reads them at $m - 1$ instead of m (the intrinsically brighter class extends as deep into the redshift space at brighter apparent magnitude). Because the Malmquist correction to $\langle \log z \rangle$ depends on the Friedmann model, one needs a procedure where the corrections are calculated for a range of

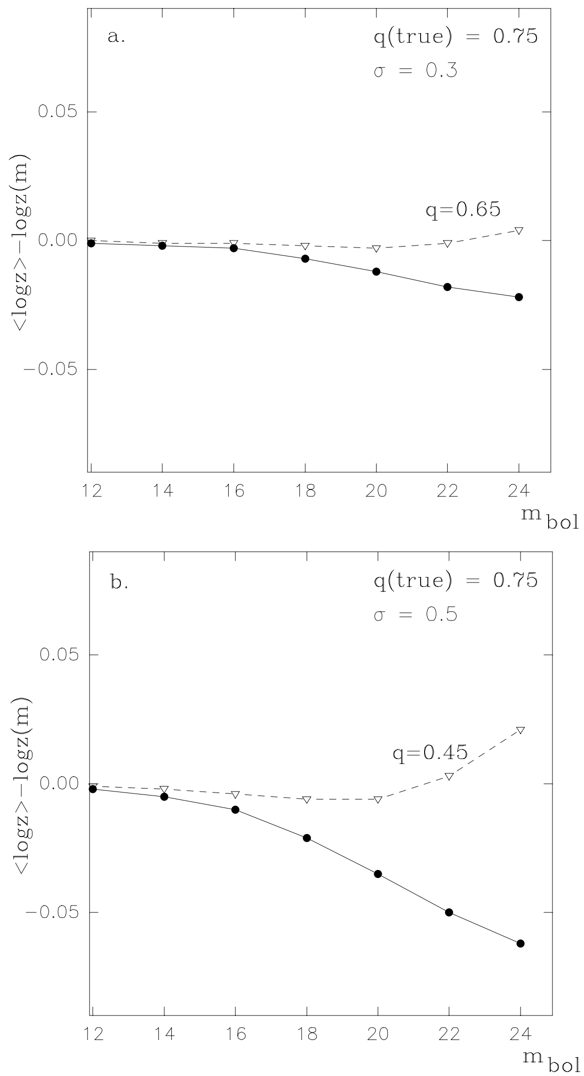

Fig. 4a and b. The difference $\langle \log z\rangle - \log z(m)$, given as dots, between the true $\langle \log z\rangle$ and the zero-dispersion (Mattig) prediction $\log z(m)$ for a standard candle class with $M_0 = -23$ and $\sigma = 0.3$ mag (**a**) and $\sigma = 0.5$ (**b**). It is assumed that the true parameters of the Friedmann universe are $q_0 = 0.75$ and $H_0 = 50$km s^{-1} Mpc^{-1}, and to the Mattig prediction has been added a constant term equal to the classical Malmquist bias. Note how the true $\langle \log z\rangle$ drops below the usual prediction at faint magnitudes. The open triangles show the corresponding difference for an erroneous value of q_0 which predicts the observed $\langle \log z\rangle$ at brighter magnitudes. For such standard candles $\langle \log z\rangle \approx 0.2$ around $m = 22$. As explained in the text, these corrections are valid for any standard candle and distance scale for which $M_0 - 5\log H_0 =$ constant $= -23 - 5\log 50$. If H_0 is kept constant, while M_0 is changed, one moves accordingly on the m-axis.

q_0 if one wishes to derive the value of q_0 from a high-z Hubble diagram.

It is illuminating to show in Figs. 4–5 also the curve for the apparent, erroneous value of q_0 which roughly "predicts" the run of $\langle \log z\rangle$ at brighter magnitudes, again on the basis of the zero-dispersion Mattig curve.

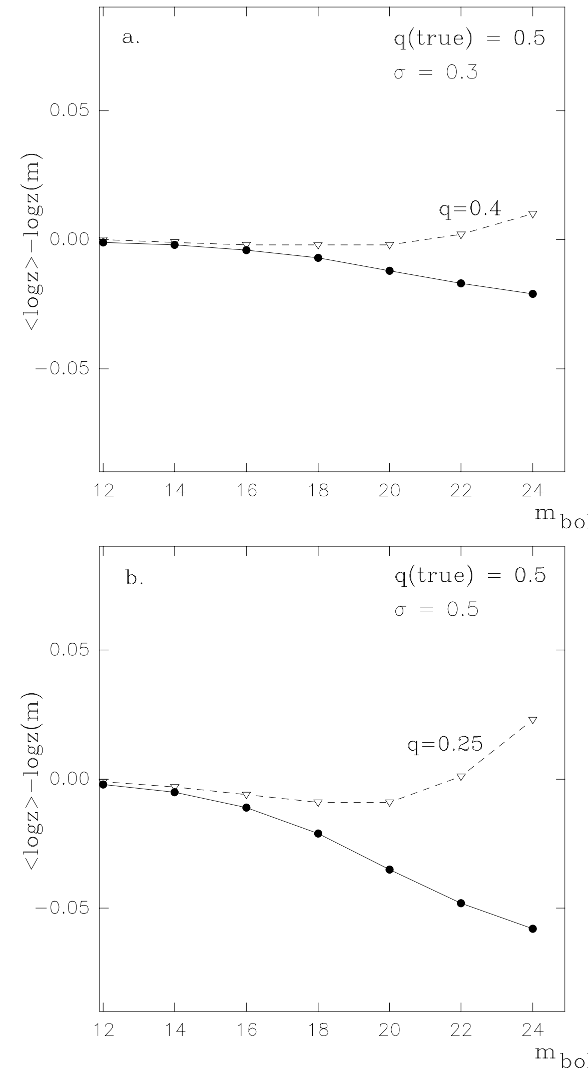

Fig. 5a and b. As Fig. 4, but now for the $q_0 = 1/2$ universe.

The results are such that the apparent value of q_0 goes down from the true one, and this effect is stronger for larger dispersion σ. With a very good standard candle with $\sigma = 0.3$, one would derive by the normal procedure from the Hubble diagram up to about $\langle z\rangle \approx 1.5$, that $q_0 = 0.65$ when it is actually 0.75. With $\sigma = 0.5$, the inferred value would drop down to 0.45 – 0.5, while $\sigma = 0.75$ would give $q_0 = 0.2$ – 0.3 (not displayed). If we are actually living in the Eistein-de Sitter universe ($q_0 = 1/2$), then $\sigma = 0.3$ would give that apparently $q_0 \approx 0.4$, while $\sigma = 0.5$ would perhaps make one believe that $q_0 \approx 0.25$. If $q_0 = 0.25$, then $\sigma = 0.3$ would produce an apparent $q_0 \approx 0.15$ (not displayed).

We emphasize that these results are concerned with observations below bolometric $m \approx 22$ (where $\langle \log z\rangle \approx 0.2$ or $z \approx 1.5$ with the adopted M_0 and H_0). At fainter magnitudes (for such a standard candle) the behaviour of $\langle \log z\rangle$ may not be described by a unique apparent q_0. In the next section it is shown that an improper treatment of the K-effect may substantially still enhance this source of error.

5. Including K-effect and other corrections

A realistic situation involves magnitudes defined by a fixed wavelength band, which in the presence of redshift requires one to take into account the K-effect. We do not see in the sky the bolometric magnitude $m_{\rm bol}$, but

$$m_{\rm obs}(z) = m_{\rm bol}(z) + K(z) \quad (14)$$

In the calculation using bolometric magnitude, the distribution of $\log z$ at m was derived from the knowledge of the luminosity function $G(M)$ where $M = m - f(z)$. With the K-correction, one instead has to use $M = m - K(z) - f(z)$ in order to get the absolute magnitude that defines the luminosity function.

Generally, if $K(z)$ increases with z so that the galaxies get faint more rapidly than just due to the bolometric factor, then the backside volume effectively decreases in Friedmann models, and the trend noted with bolometric magnitudes is still strengthened. At a fixed observed $m_{\rm obs}$ the absolute magnitude $M_0 - \sigma$ is reached at a smaller z than if the K-correction would be zero ($m_{\rm obs} = M_0 - \sigma + f(z) + K(z)$).

5.1. *"Elliptical" K_V-corrections*

We illustrate this effect using the K-correction for the V-magnitudes of elliptical galaxies. Coleman et al. (1980) give such corrections up to $z = 2.0$ in their Table 7, corresponding to measurements of the bulge of M31. Without evolution, the K-effect would make an elliptical galaxy fainter by 3.46 and 5.07 mag at $z = 1.0$ and 2.0, respectively, as compared with the pure effect of the bolometric factor. As we make calculation only up to $\langle z \rangle \approx 1.5$, it is sufficient to extrapolate the K-corrections up to $z = 2.5$.

Table 1 shows the ratios of foreground and background volumes in this case for $q_0 = 0.5$. Note how the ratios change in comparison to the bolometric case. The change of the trend around $z \approx 1.5$ is caused by the fact that there the change of bolometric magnitude with z becomes more rapid than the change in the K-correction.

It is a practice to include the K-correction in the theoretical zero-dispersion Mattig curve and compare this with $(z, m_{\rm obs})$ data in the Hubble diagram (e.g. Yoshii & Takahara 1988). This means that the Mattig equation becomes

$$m_{\rm obs} - K(z) - M_0 = f(z) \quad (15)$$

from which the $\log z(m)$ relation is solved. Now the relevant question is: How much do the actual $\langle \log z \rangle$ vs. $m_{\rm obs}$ relation, derived as discussed above, and this K-shifted zero-dispersion curve differ from each other?

Fig. 6 shows similarly as Figs. 4 and 5 the difference between the actual $\langle \log z \rangle$ and the $\log z$ predicted by the K-shifted Mattig $\log z$ vs. m relation. In comparison with the bolometric magnitudes, the "apparent" q_0 inferred using the K-shifted curves becomes still smaller, and the effect may be quite significant even for good standard candles. For example, if q_0 is actually 1.0 and $\sigma = 0.3$ mag, the usual procedure would make one infer that $q_0 \approx 0.5$. If q_0 is 0.5, then one might derive that $q_0 \approx 0.15$.

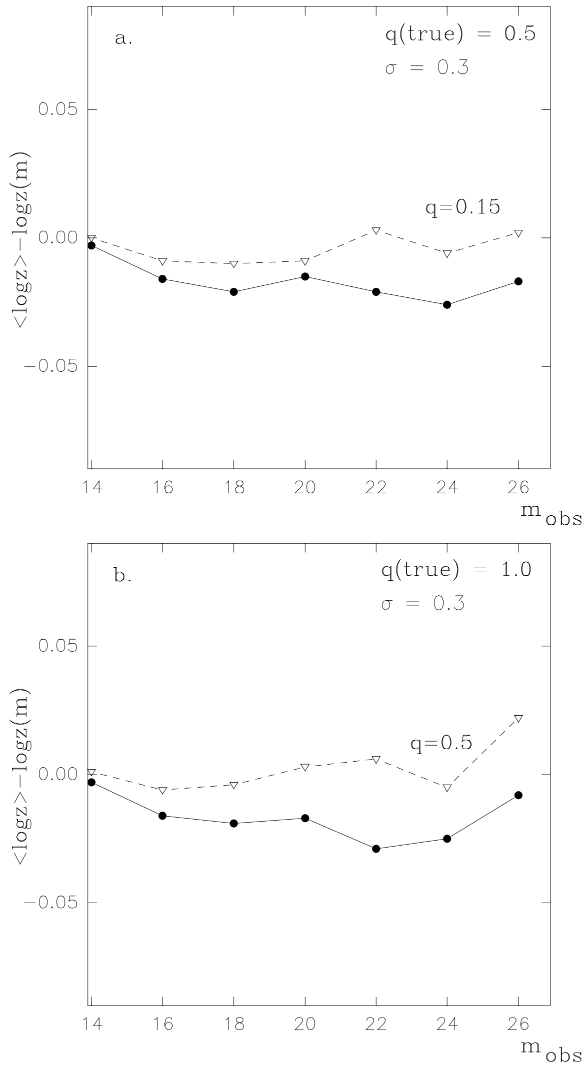

Fig. 6a and b. The difference between $\langle \log z \rangle$ and the zero-dispersion, K-corrected (Mattig) prediction for a standard candle class with $M_0 = -23$ and $\sigma = 0.3$ mag and "elliptical" K-correction for V-magnitudes (dots). The Friedmann universe has $H_0 = 50$ km s^{-1} Mpc^{-1}and $q_0 = 0.5$ (upper panel) or = 1.0 (lower panel). The open triangles show the corresponding difference for an apparent value of q_0 which roughly predicts the observed $\langle \log z \rangle$ at brighter magnitudes. Around $m_{\rm obs} = 26$, $\langle \log z \rangle \approx 0.12$ or $\langle z \rangle \approx 1.3$.

With a larger σ, the apparent values of q_0 would become still lower (e.g. with $\sigma = 0.5$ mag, $q_0 = 1.0$ would drop down to ≈ 0.15).

5.2. *Why not make K-corrections to observed magnitudes and then compare with "bolometric" predictions?*

At this point it is useful to consider the question: What if one first constructs the $\log z$ vs. m_c diagram where $m_c = m_{\rm obs} - K(z)$ and then compares the resulting run of data points with the more fundamental bolometric predictions of Sect. 4? Using this, perhaps "more natural" procedure, does one generally get the same result on q_0, as with the $\log z$ vs. $m_{\rm obs}$ diagram? This question

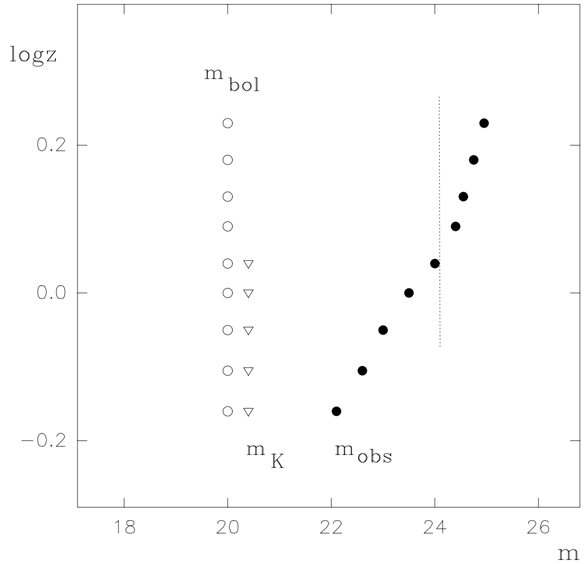

Fig. 7. Part of a schematic Hubble diagram showing how making negative K-corrections to observed magnitudes (dots) affected by a magnitude limit (here = 24 mag) does not reproduce the distribution of $\log z$ for bolometric magnitudes (open circles), but a truncated distribution results (triangles). In this case the selection bias influences the cosmological Malmquist bias. This implies that one should use in the analysis of the Hubble diagram $\log z$ versus $m_{\rm obs}$ (and the results of Sect. 5.2), and not $\log z$ versus K-corrected m_K (with the bolometric results of Sect. 4), especially close to the magnitude limit.

is relevant, because if the answer is positive, it is sufficient to work with the results discussed for bolometric magnitudes.

However, the answer is negative, which may be seen as follows: The procedure with bolometric magnitudes requires that the distribution of $\log z$ at constant m does not depend on anything else than space geometry, bolometric factor $f(z)$, and luminosity function. A constant m_c is made from galaxies with different $m_{\rm obs}$ and the distribution of $\log z$ at m_c now comes to depend also on the relative completeness of the sample at different $m_{\rm obs}$. For example, if the K-correction increases towards high z, the high-z wing of the $\log z$-distribution at a constant m_c will be underpopulated relative to the small-z wing. This is schematically illustrated in Fig. 7, where are shown 1) a distribution of $\log z$ at a constant bolometric magnitude (open circles), 2) positions of these galaxies in the Hubble diagram influenced by the K effect (dots), together with a vertical line indicating the magnitude limit, and 3) the galaxies after K-correction made for the *observed* galaxies (triangles). Clearly, making K-corrections does not reproduce the bolometric distribution, because many high-z galaxies have been lost beyond the magnitude limit. By the way, here we have a situation where the two types of biases together influence interpretation of the Hubble diagram even in the sense of $\log z$ vs. m.

It should be added that a similar problem appears also if the K-corrections have not a positive, but negative sign (like typically radio loud quasars in V-band; e.g. Fig. A1 in Teerikorpi 1981). In this case, a lower part of the $\log z$ distribution at faint apparent magnitudes corrected for the K-effect is missing, because of the magnitude limit.

In the approach of Sect. 5.1. the magnitude incompleteness does not appear for the same reason as in the formalism using bolometric magnitude (in fact, instead of $m = M - f(z)$, one has $m = M - f(z) + K(z) = m - f'(z)$, and galaxies with apparent magnitude m are gathered unaware of whether the faintness is due to the cosmological bolometric factor f or the K-factor K).

It may be concluded that the $\log z$ vs. $m_{\rm obs}$ approach is really the recommended one, and the theoretical expectations must be calculated in the manner described in Sect. 5.1. Of course, when one works sufficiently far away from the magnitude limit, then the K-corrected magnitudes m_c, together with the bolometric predictions (Sect. 4), should give the same estimate of q_0.

5.3. Luminosity evolution, galactic and intergalactic extinction

The present paper deals with the fundamental modifications that the classical Malmquist bias experiences, when standard candles are put into a non-classical, here Friedmann, space from small to high redshifts. In this first step, a few basic assumptions are still taken over from the classical case: 1) the standard candle class itself and its space number density do not change with the look-back time, 2) the intergalactic space is clean from extinction, and 3) there is also no variable foreground galactic extinction screen. Here we make just a few comments on these topics.

As far as the observer is concerned, a change of the intrinsic luminosity with redshift has similar consequencies for the $\langle \log z \rangle$ vs. $m_{\rm obs}$ diagram as a K-effect, and must be included in the analysis formally as was done in Sect. 5.1. The influence of intergalactic extinction is like that of K-correction and z-dependent evolution. Though it is not presently known how important this extinction is, it would in any case work in the same direction as an increasing K-term, i.e. pulling down the apparent value of q_0.

How should one make galactic extinction corrections? Consider two regions of the sky, one (A) without extinction and another (B) with a constant extinction δm. At A, the galaxies falling on the apparent magnitude m have the same distribution of $\log z$, as the galaxies at B, found at the apparent magnitude $m + \delta m$. Hence, one may freely combine in the $\log z$ vs. m diagram the galaxies at A and at B, just by using for the latter the corrected $m_c = m - \delta m$. In this manner, the $\langle \log z \rangle$ vs. m relation is not affected by galactic extinction, if each m is accurately corrected (so, the procedure differs from K-, evolution, and intergalactic extinction corrections).

6. Bias in angular size distances

Qualitatively speaking, luminosity distance in a Friedmann universe (when $q_0 < 1$) increases more rapidly than "redshift distance", while angular size distance increases more slowly (Fig. 3). How does this influence the volume ratios and the bias?

Let us assume that we have a standard rod (which we look perpendicularly, something like the apparent major axis of a disc

Table 2. 1σ volume ratios for angular size*

z/q_0	0.05	0.5	1.0	0.5 & σ = 0.05	Ω_l = Ω_m = 0.5
0.001	2.00	2.00	2.00	1.41	2.00
0.1	2.21	2.22	2.24	1.49	2.21
0.2	2.48	2.57	2.67	1.59	2.50
0.3	2.83	3.16	3.63	1.75	2.92
0.4	3.30	4.58	—	1.99	3.67
0.5	4.00	—	—	2.45	5.64
0.6	5.12	—	—	4.01	—
0.7	7.43	—	—	—	—

$*$ Empty entries mean that the upper redshift $z(+\sigma)$ exceeds the value $z_{\rm min}$ where angular size has its minimum.

galaxy) which has the linear size $\langle \log D \rangle = D_0$ and gaussian dispersion $\sigma_{\log D}$. Classically, the 1σ volume ratio is:

$$R_{1\sigma} = \frac{10^{3\sigma} - 1}{1 - 10^{-3\sigma}} \quad (16)$$

This ratio has the same value as for magnitudes with $\sigma_M = 5 \times \sigma_{\log D}$ (Eq. 8). In order to compare the volume ratios with those of Table 1, we hence take $\sigma_{\log D} = 0.1$. Such a standard rod corresponds in accuracy to a standard candle with $\sigma_M = 0.5$ mag. Because of the minimum angular size, we consider only redshifts where $z(+\sigma) < z_{\rm min}$. Table 2 shows that even at such relatively low redshifts the volume ratio (which now *increases* with z) deviates strongly from the classical one. In the fifth column the calculation for $q_0 = 0.5$ has been repeated for a better standard rod with $\sigma = 0.05$, while the last column refers to a case with non-zero cosmological constant (Sect. 7).

We can understand why the deviation from the classical value is larger when q_0 is larger, by looking at Fig. 1: both the angular size distance and the volume derivative increasingly deviate from the "euclidean" curve when q_0 is increased. The generally large deviation makes one to expect a large influence of the Malmquist bias on the $\langle \log z \rangle$ vs. $\log \Theta$ ($\Theta =$ angular size) curves already at small z. Because the volume ratio increases with z, the effect on the Malmquist bias is different than in the case of standard candles: the bias increases with decreasing $\log \Theta$, and ignoring the bias will lead to a too large value of q_0.

When we are looking at the data as $\log z$ vs. $\log \Theta$, the expected angular size minimum (or angular size distance maximum; Fig. 3) at $z_{\rm min}$ complicates the situation. Hence, it is reasonable to restrict oneself here to relatively small $\langle \log z \rangle$ and to show that even well before the influence of the maximum in d_A starts to be felt, the Malmquist bias is important. When one calculates $\langle \log z \rangle$ at a fixed $\log \Theta$, using the formula analogous to Eq. (13), a possible choice for the upper integration limit is the $z_{\rm min}$ corresponding to q_0. Then at large $\log \Theta$ the rods originating from the "backside" or decreasing part of the d_A-curve, do not interfere. Because of the strong deviation from the zero-dispersion Mattig curve already at rather small distances, even with this simplification, which slightly underestimates the effect, the curves are quite revealing.

As in Sect. 4, we add to the $\log z$ from the Mattig curve the classical (small-z) Malmquist bias which for diameters is $(1/5) \times 1.382 \times (5\sigma_{\log D})^2 = 6.908\sigma^2_{\log D}$. Fig. 8 shows the diameter Hubble diagrams for $q_0 = 0.05$ and 0.3, assuming that the dispersion $\sigma_{\log D} = 0.06$ (corresponding in classical accuracy to $\sigma_M = 0.3$ mag). In addition to the $\langle \log z \rangle$ prediction, the Mattig curves for the true and "apparent" q_0's are shown. The curves indicate that well above the angular size minimum, the run of $\langle \log z \rangle$ for such a good standard rod is given by apparent $q_0 \approx 0.5$ when true $q_0 = 0.05$, and by $q_0 \approx 1.0$ when true $q_0 = 0.3$.

In practice, one has more often presented the diameter-redshift diagram as $\log \Theta$ versus z (though not always; Pen 1997) This approach is natural when the data extend beyond the expected $z_{\rm min}$ and when the problem with a limiting angular size is insignificant. Even then, one should get a consistent result using the $\log z$ vs. $\log \Theta$ approach above the region of the minimum angular size. However, as was shown above, ignoring the Malmquist bias could invalidate such an attempt, bringing about confusing and inconsistent results.

7. Notes on non-zero cosmological constant

It is interesting to know in which way a non-zero cosmological constant influences the Malmquist bias: the dependence of both distance modulus and comoving volume derivate on redshift rather strongly depend on how large is the contribution of the cosmological constant term Ω_l to the total $\Omega_{\rm tot} = \Omega_m + \Omega_l$.

It is sufficient here to state the expected trends, on the basis of how the cosmological constant changes the luminosity distance vs. redshift, angular size distance vs. redshift, and comoving volume derivative vs. redshift relations.

See e.g. Figs. 5 (angular size distance) and 6 (volume derivative) in the review by Carroll et al. (1992), where the curve A corresponds to the case $\Omega_{\rm tot} = 1$ and $\Omega_l = 0$, while the curve E refers to $\Omega_{\rm tot} = 1$ and $\Omega_l = 0.99$. There is a significant difference between these two curves already below $z = 1$. If one considers open universes ($\Omega_{\rm tot} \leq 1$) with non-zero Ω_l, both luminosity distance and volume derivate increase steeper (at small and moderate redshifts) than in zero-Ω_l spaces. Steeper luminosity distance means deviation away from, while steeper volume derivative means approaching the "classical" case, hence it is not immediately clear which is the net result.

Calculating the 1σ volume ratios for the flat case $\Omega_m + \Omega_l = 1$ shows that the steeper volume derivate is actually slightly more important (volume ratios shift a little towards the classical one). Angular size distance vs. redshift relation also is steepened ($d_A = d_L/(1+z)^2$) by the non-zero cosmological constant, but now this means a change towards the classical case (cf. Fig. 1b). Because also the volume derivative goes towards the classical case, this is also the net result. These conclusions are verified in the last columns of Tables 1 and 2, which show the 1σ volume ratios for the case where $\Omega_m = \Omega_l = 0.5$.

In order to calculate the volume ratios and the theoretical $\langle \log z \rangle$ expectations when Ω_l is not zero, one needs the general

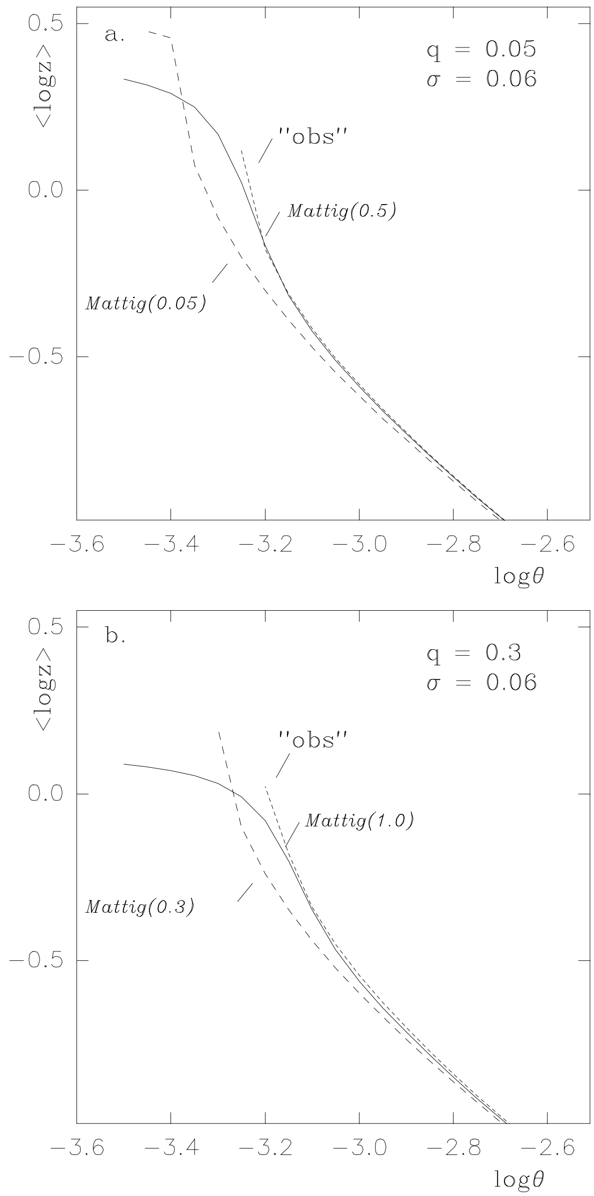

Fig. 8a and b. $\log z$ vs. log(angular size) relations for a standard rod with dispersion $\sigma_{\log D} = 0.06$, at large angular sizes where $\langle z \rangle < 1$. In **a** true $q_0 = 0.05$ and in **b** $q_0 = 0.3$. The integration has been extended up to $z_{\min}$, hence the "obs" curves slightly underestimate the bias in the small-$\log \Theta$ end. Corresponding zero-dispersion Mattig curves are also shown, as well as the Mattig curves which rather well fit the predicted run of $\langle \log z \rangle$: "apparent" $q_0 \approx 0.5$ in a. and 1.0 in b.

formula for the volume element dV, giving the derivative dV/dz for the comoving volume (e.g. Carroll et al. 1992):

$$dV = \left(d_M^2 / \left[1 + k (H_0/c)^2 d_M^2 \right]^{1/2} \right) d(d_M) d\Omega \qquad (17)$$

Then calculation of $\langle \log z \rangle$ proceeds exactly as in Sects. 4 and 6, using in addition formulae (2) - (5). Here we do not present such calculations, but merely point out some qualitative trends which are useful to know.

Take again the hypothetical situation where it is considered as a fact that $k = 0$ ($\Omega_m + \Omega_m = 1$), but one does not know the relative contributions to $\Omega_{\rm tot}$ coming from mass and cosmological constant. Assume that one wishes to determine the fraction Ω_l using standard candles and rods in the classical sense ($\langle \log z \rangle$ vs. m, $\langle \log z \rangle$ vs. $\log \Theta$), but one is not aware of the cosmological Malmquist bias. If in reality $\Omega_l = 0$, then straightforward use of zero-dispersion curves makes one conclude from the standard candles (assume bolometric magnitudes) that $\Omega_l > 0$. On the contrary, standard rods (c.f. Sect. 6) would seem to require $\Omega_l < 0$! This contradiction could only be resolved by taking into account the cosmological Malmquist biases for magnitudes and sizes.

8. Discussion and conclusions

As the present paper introduces the cosmological Malmquist bias as a theoretical phenomenon, we do not make a detailed study of its consequencies on the various previous investigations of the Hubble diagram - it is important that in future works its influence is recognized whenever one is compelled to use the $\langle \log z \rangle$ vs. m approach. One fact which complicates matters is that different authors use different sets of K- and evolution corrections, not always explicitly published, and the analysis of any Hubble diagram must combine such corrections with the Malmquist bias model here introduced.

However, for the purpose of illustration, one may look at the well known Hubble diagram in V-magnitude for giant ellipticals and radio galaxies, published by Yoshii & Takahara (1988) together with various zero-dispersion theoretical curves (their Fig. 3). On the basis of the above discussion and assuming that $\sigma = 0.3$, one may conclude that their non-evolving, but K-corrected V-curve of $q_0 = 1.0$ should be shifted down close to the curve of $q_0 = 0.5$, while the latter is further shifted towards smaller q_0.

The theoretical evolution corrections for elliptical galaxies strongly counteract the K_V-effect, though they do not totally cancel it. Hence, the curves with the adopted evolution corrections should be shifted in the same sense as the non-evolving curves, somewhat increasing the pressure towards higher q_0 and/or more recent galaxy birth epoch z_f.

It should be noted that though with a sufficiently small σ the present effect goes to zero, the larger effects accompanying the larger σ's are quite relevant in practice: one would certainly be happy to find a true cosmological standard candle with, say, a moderate dispersion $\sigma = 0.5$. E.g., around $\langle z \rangle = 2$ the difference between the usual bolometric Mattig curves for $q_0 = 0.2$ and 0.5 is about 0.5 mag, and one would expect that a sufficiently large sample of such candles would easily make a difference between these cases. However, forgetting the Malmquist bias in the $\log z$ vs. m diagram would necessarily lead to an erroneous conclusion, and the more so if one puts "elliptical" K-corrections directly to the Mattig curves, as is often done.

The main conclusions of the present study are:

- The cosmological Malmquist bias is an essential part of the theory of gaussian standard candles (or rods) in the Friedmann and other cosmological models.

- There is no question that the usual zero-dispersion Mattig (1958) curves should be corrected by a non-constant Malmquist bias term in the m–$\log z$ Hubble diagram. Making no correction is equivalent to the silent assumption that the bias is constant as is the case classically when the space distribution is uniform.
- The deviation of the cosmological bias from the constant classical bias depends strongly on the dispersion σ of the gaussian luminosity function. If σ is larger than or equal to 0.3 mag, neglecting this bias in the analysis of the Hubble diagram can lead to significant errors in the derived value of the deceleration parameter.
- K-effect and luminosity evolution require special attention: they cannot be just put into the Mattig curve, not even to the deformed Mattig curve which includes the effect of the bolometric magnitude Malmquist bias. Nor should one make direct corrections to the observed magnitudes and then make a straightforward comparison with the (deformed) Mattig curve. Instead, one must explicitly calculate the predicted $\langle \log z \rangle$ at fixed observed apparent magnitude m.
- It is worthwhile to consider always two complementary aspects of the Hubble diagram as a cosmological test. These are the $\log z$ vs. m and m vs. $\log z$ approaches, which are influenced, respectively, by the Malmquist bias of the first kind (here discussed) and the Malmquist bias of the second kind, also called the selection bias.

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