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ASTRONOMY AND ASTROPHYSICS

Scintillation in scalar-tensor theories of gravity

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Abstract. We study the scintillation produced by time-varying gravitational fields within scalar-tensor theories of gravity. The problem is treated in the geometrical optics approximation for a very distant light source emitting quasi plane monochromatic electromagnetic waves. We obtain a general formula giving the time dependence of the photon flux received by a freely falling observer. In the weak-field approximation, we show that the contribution to the scintillation effect due to the focusing of the light beam by a gravitational wave is of first order in the amplitude of the scalar perturbation. Thus scalar-tensor theories contrast with general relativity, which predicts that the only first-order effect is due to the spectral shift. Moreover, we find that the scintillation effects caused by the scalar field have a local character: they depend only on the value of the perturbation at the observer. This effect provides in principle a mean to detect the presence of a long range scalar field in the Universe, but its smallness constitutes a tremendous challenge for detection.

Key words: gravitation – relativity

1. Introduction

The plane optical wavefronts of a distant background light source become rippled when they cross a perturbation. For a distant perturbation, the focusing of light at the observer changes with the curvature of the ripples. It is the usual geometrical scintillation effect. It accounts, *e.g.*, for the twinkling of stars under the atmospheric turbulence.

The question whether gravitational waves can cause the light emitted by a distant source to scintillate is an old problem. In general relativity, it is well known from the early works by Zipoy (1966), Zipoy & Bertotti (1968) and Bertotti & Trevese (1972) that gravitational waves have no focusing property to the first order in their amplitude.

However, it has been recently pointed out by Faraoni (1996) that a first-order scintillation effect can be expected in scalar-tensor theories of gravity [1]. Furthermore, some actual improvements of the observational techniques renew the interest in the search of gravitational scintillation (Labeyrie 1993) and related effects (Fakir 1995).

The aim of the present work is to make a detailed analysis of the scintillation effect in monoscalar-tensor theories for a monochromatic electromagnetic wave propagating in a weak gravitational field. We adopt the point of view that the physical metric is the metrical tensor $g_{\mu\nu}$ to which matter is universally coupled. This basic assumption defines the usual "Jordan-Fierz" frame. We find a scintillation effect proportional to the value of the scalar field perturbation at the observer.

Our result contrasts with the zero effect found by Faraoni & Gunzig (1998) by using the "Einstein" conformal frame, in which the original physical metric $g_{\mu\nu}$ is replaced by a conformal one [2]. However, their negative conclusion is seemingly due to the fact that the authors do not take into account the changes in areas and other physical variables induced by the conformal transformation (Damour & Esposito-Farèse 1998).

The paper is organized as follows. In Sect. 2, we give the notations and we recall the fundamental definitions. In Sect. 3, we construct the theory of gravitational scintillation for a very distant light source emitting quasi plane electromagnetic waves. Our calculations are valid for any metric theory of gravity in the limit of the geometrical optics approximation. We obtain the variation with respect to time of the photon flux received by a freely falling observer as a sum of two contributions: a change in the scalar amplitude of the electromagnetic waves, that we call a geometrical scintillation, and a change in the spectral shift. We express each of these contributions in the form of an integral over the light ray arriving to the observer. In Sect. 4, we study the scintillation within the linearized weak-field approximation. We show that the geometrical scintillation is related to the Ricci tensor only. Thus we recover as a particular case the conclusions previously drawn by Zipoy and Zipoy & Bertotti for gravitational waves in general relativity. Moreover, we show that the contribution due to the change in the spectral shift is entirely determined by the curvature tensor. In Sect. 5, we apply the results of Sect. 4 to the scalar-tensor theories of gravity. We prove that these theories predict a scintillation effect of the first order, proportional to the amplitude of the scalar perturbation.

[1] On these theories, see, *e.g.*, Will (1993) and Damour & Esposito-Farèse (1992), and references therein.

[2] A clear distinction between the "Einstein" frame and the "Jordan" frame, may be found, *e.g.*, in Damour & Nordverdt (1993)

Furthermore, we find that this effect has a local character: it depends only on the value of the scalar field at the observer. Finally, we briefly examine the possibility of observational tests in Sect. 6.

2. Notations and definitions

The signature of the metric tensor $g_{\mu\nu}(x)$ is assumed to be $(+\ -\ -\ -)$. Indices are lowered with $g_{\mu\nu}$ and raised with $g^{\mu\nu}$.

Greek letters run from 0 to 3. Latin letters are used for spatial coordinates only: they run from 1 to 3. A comma (,) denotes an ordinary partial differentiation. A semi-colon (;) denotes a covariant partial differentiation with respect to the metric; so $g_{\mu\nu;\rho} = 0$. Note that for any function $F(x)$, $F_{;\alpha} = F_{,\alpha}$.

Any vector field w^ρ satisfies the following identity

$$w^\rho{}_{;\mu;\nu} - w^\rho{}_{;\nu;\mu} = -R^\rho_{.\sigma\mu\nu} w^\sigma \tag{1}$$

where $R^\rho_{.\sigma\mu\nu}$ is the Riemann curvature tensor (note that this identity may be regarded as defining the curvature tensor). The Ricci tensor is defined by

$$R_{\mu\nu} = R^\lambda_{.\mu\lambda\nu} \tag{2}$$

Given a quantity P, $\overline{P}$ denotes its complex conjugate.

The subscripts em and obs in formulae stand respectively for emitter and observer.

The constant c is the speed of light and $\hbar$ is the Planck constant divided by 2π.

3. General theory of the gravitational scintillation

In a region of spacetime free of electric charge, the propagation equations for the electromagnetic vector potential A_μ are (*e.g.*, Misner *et al.* 1973)

$$A^{\mu;\alpha}{}_{;\alpha} - R^\mu{}_\alpha A^\alpha = 0 \tag{3}$$

when A^μ is chosen to obey the Lorentz gauge condition

$$A^\mu{}_{;\mu} = 0 \tag{4}$$

It is convenient here to treat A_μ as a complex vector. Hence the electromagnetic field tensor $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \mathcal{R}e(A_{\nu;\mu} - A_{\mu;\nu}) \tag{5}$$

The corresponding electromagnetic energy-momentum tensor is defined by

$$T^{\mu\nu} = \frac{1}{4\pi}\left[-F^{\mu\rho}F^\nu_{.\rho} + \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}g^{\mu\nu}\right] \tag{6}$$

where $F^\nu_{.\rho} = g^{\nu\lambda}\,F_{\lambda\rho}$. The components of this tensor satisfy the conservation equations $T^{\mu\nu}{}_{;\nu} = 0$ as a consequence of Eqs. (3).

For an observer located at the spacetime point x and moving with the unit 4-velocity u^α, the density of electromagnetic energy flux is given by the Poynting vector

$$\mathcal{P}^\mu(x,u) = cT^{\mu\nu}(x)u_\nu(x) \tag{7}$$

and the density of electromagnetic energy as measured by the observer is

$$\mu_{el}(x,u) = T^{\mu\nu}(x)u_\mu u_\nu \tag{8}$$

In this paper, we use the geometrical optics approximation. So we assume that there exist wave solutions to Eqs. (3) which admit a development of the form

$$A^\mu(x,\xi) = [a^\mu(x) + O(\xi)]\exp(\frac{i}{\xi}\hat{S}(x)) \tag{9}$$

where $a^\mu(x)$ is a slowly varying complex vector amplitude, $\hat{S}(x)$ is a real function and ξ a dimensionless parameter which tends to zero as the typical wavelength of the wave becomes shorter and shorter. A solution like (9) represents a quasi plane, locally monochromatic wave of high frequency (Misner *et al.* 1973).

Let us define the phase S and the vector field k_α by the relations

$$S(x,\xi) = \frac{1}{\xi}\hat{S}(x) \tag{10}$$

and

$$k_\alpha = S_{,\alpha} \tag{11}$$

Inserting (9) into Eqs. (3) and (4), then retaining only the leading terms of order ξ^{-2} and ξ^{-1}, yield the fundamental equations of geometrical optics

$$k^\alpha k_\alpha = 0 \tag{12}$$

$$k^\alpha a^\mu_{;\alpha} = -\frac{1}{2}a^\mu k^\alpha_{;\alpha} \tag{13}$$

with the gauge condition

$$k_\alpha a^\alpha = 0 \tag{14}$$

Light rays are defined to be the curves whose tangent vector field is k^α. So the parametric equations $x^\alpha = x^\alpha(v)$ of the light rays are solutions to the differential equations

$$\frac{dx^\alpha}{dv} = k^\alpha(x^\lambda(v)) \tag{15}$$

where v is an affine parameter. Differentiating Eq. (12) and noting that

$$k_{\alpha;\beta} = k_{\beta;\alpha} \tag{16}$$

follows from (11), it is easily seen that k^α satisfies the propagation equations

$$k^\alpha k_{\beta;\alpha} = 0 \tag{17}$$

These equations, together with (12), show that the light rays are null geodesics.

Inserting (9) into (5) and (6) gives the approximate expression for $F_{\mu\nu}$

$$F_{\mu\nu} = \mathcal{R}e[i(k_\mu a_\nu - k_\nu a_\mu)e^{iS}] \tag{18}$$

and for $T^{\mu\nu}$ averaged over a period

$$T^{\mu\nu} = \frac{1}{8\pi} a^2 k^\mu k^\nu \tag{19}$$

where a is the scalar amplitude defined by [3]

$$a = (-a^\mu \overline{a}_\mu)^{1/2} \tag{20}$$

From (7) and (19), it is easily seen that the Poynting vector is proportional to the null tangent vector k^μ. This means that the energy of the wave is transported along each ray with the speed of light. Let us denote by $\mathcal{F}(x,u)$ the energy flux received by an observer located at x and moving with the 4-velocity u^α: by definition, $\mathcal{F}(x,u)$ is the amount of radiating energy flowing per unit proper time across a unit surface orthogonal to the direction of propagation. It follows from (8) and (19) that

$$\mathcal{F}(x,u) = c\mu_{el}(x,u) = \frac{c}{8\pi} a^2(x)(u^\mu k_\mu)^2_{obs} \tag{21}$$

This formula enables us to determine the photon flux $\mathcal{N}(x,u)$ received by the observer located at x and moving with the 4-velocity u^α. Since the 4-momentum of a photon is $p^\mu = \hbar k^\mu$, the energy of the photon as measured by the observer is $cp^\mu u_\mu = c\hbar(u^\mu k_\mu)$. We have therefore

$$\mathcal{N}(x,u) = \frac{1}{8\pi\hbar} a^2(x)(u^\mu k_\mu)_{obs} \tag{22}$$

The spectral shift z of a light source (emitter) as measured by an observer is given by (*e.g.* G.F.R. Ellis, 1971)

$$1 + z = \frac{(u^\mu k_\mu)_{em}}{(u^\nu k_\nu)_{obs}} \tag{23}$$

Consequently, the photon flux $\mathcal{N}(x,u)$ may be written as

$$\mathcal{N}(x,u) = \frac{1}{8\pi\hbar} a^2(x) \frac{(u^\mu k_\mu)_{em}}{1+z} \tag{24}$$

The scalar amplitude a can be written in the form of an integral along the light ray γ joining the source to the observer located at x. Multiplying Eq. (13) by $\overline{a}_\mu$ yields the propagation equation for a

$$k^\alpha a_{;\alpha} \equiv \frac{da}{dv} = -\frac{1}{2} a k^\alpha_{;\alpha} \tag{25}$$

where d/dv denotes the total differentiation of a scalar function along γ. Then, integrating (25) gives

$$a_{|obs} = a_{|x_0} \exp\left(-\frac{1}{2}\int_{v_{x_0}}^{v_{obs}} k^\alpha_{;\alpha} \;\; dv\right) \tag{26}$$

where x_0 is an arbitrary point on the light ray γ.

In the following, we consider that the light source is at spatial infinity. We suppose the existence of coordinate systems x^α such that on any hypersurface $x^0 = const.$, $|g_{\mu\nu} - \eta_{\mu\nu}| = O(1/r)$ when $r = [\sum_{i=1}^3 (x^i)^2]^{1/2} \to \infty$, with $\eta_{\mu\nu} = diag(1,-1,-1,-1)$. We require that in such coordinate systems the quantities $k_{\alpha;\beta}$, $k_{\alpha;\beta;\gamma}$ and $a_{;\alpha}$ respectively fulfill the asymptotic conditions

$$\begin{cases} k_{\alpha;\beta}(x_0) &= O(1/|v_{x_0}|^{1+p}) \\ k_{\alpha;\beta;\gamma}(x_0) &= O(1/|v_{x_0}|^{2+p}) \\ a_{;\alpha}(x_0) &= O(1/|v_{x_0}|^{1+p}) \end{cases} \tag{27}$$

when $v_{x_0} \to -\infty$, with $p > 0$. Moreover, we assume that the scalar amplitude $a_{|x_0}$ in Eq. (26) remains bounded when $v_{x_0} \to -\infty$ and we put

$$\lim_{v_{x_0}\to-\infty} a_{x_0} = a_0 \tag{28}$$

It results from these assumptions that $a_{|obs}$ may be written as

$$a_{|obs} = a_0 \exp\left(-\frac{1}{2}\int_{-\infty}^{v_{obs}} k^\alpha_{;\alpha} \;\; dv\right) \tag{29}$$

Now, let us differentiate $k^\alpha_{;\alpha}$ with respect to v along γ. Applying (1) and (2), then taking (16) and (17) into account, we obtain the relation (Sachs 1961)

$$\frac{d}{dv}(k^\alpha_{;\alpha}) = -k^{\alpha;\beta}k_{\alpha;\beta} - R_{\alpha\beta}k^\alpha k^\beta \tag{30}$$

As a consequence, we can write

$$\int_{-\infty}^{v_{obs}} k^\alpha_{;\alpha}\, dv = -\int_{-\infty}^{v_{obs}} dv \int_{-\infty}^{v} [R_{\alpha\beta}(x^\lambda(v'))k^\alpha k^\beta + k^{\alpha;\beta}k_{\alpha;\beta}]dv' \tag{31}$$

The convergence of the integrals is ensured by conditions (27).

Eqs. (29) and (31) allow to determine the factor $a^2(x)$ in $\mathcal{N}(x,u)$ from the energy content of the regions crossed by the light rays and from the geometry of the rays themselves.

It is well known that $1/(1+z)$ (or $(1+z)$) can also be obtained in the form of an integral along the light ray γ (see *e.g.* Ellis 1971 or Schneider *et al.* 1992). However, the corresponding formula will not be useful for our discussion and we will not develop it here.

In fact, the scintillation phenomenon consists in a variation of $\mathcal{N}$ with respect to time. For this reason, it is more convenient to calculate the total derivative of $\mathcal{N}$ along the world-line $\mathcal{C}_{obs}$ of a given observer, moving at the point x with the 4-velocity u^α.

Given a scalar or tensorial quantity F, we denote by $\dot{F}$ the total covariant differentiation along $\mathcal{C}_{obs}$ defined by

$$\dot{F} \equiv u^\lambda F_{;\lambda} = \frac{\nabla F}{ds} \tag{32}$$

where $ds = (g_{\mu\nu}dx^\mu dx^\nu)^{1/2}$ is the line element between two events x^μ and $x^\mu + dx^\mu$ on $\mathcal{C}_{obs}$.

In Eq. (24), the quantity $c\hbar(u^\mu k_\mu)_{em}$ is the energy of a photon emitted by an atom of the light source as measured by an

[3] We introduce a minus sign in (20) because Eqs. (12) and (14) imply that a^μ is a space-like vector when the electromagnetic field is not a pure gauge field.

observer comoving with this atom. So $(u^\mu k_\mu)_{em}$ is a constant which depends only on the nature of the atom (this constant characterizes the emitted spectral line). Consequently, the change in the photon flux with respect to time is simply due to the change in the scalar amplitude a and to the change in the spectral shift z. From (24), we obtain at each point x of $\mathcal{C}_{obs}$

$$\frac{\dot{\mathcal{N}}}{\mathcal{N}} = 2\frac{\dot{a}}{a} + (1+z)\frac{d}{ds}\left(\frac{1}{1+z}\right) \quad (33)$$

Henceforth, we shall call the contribution $2\dot{a}/a$ in Eq. (33) the geometrical scintillation because the variations in a are related to the focusing properties of light rays by gravitational fields (see G.F.R.Ellis 1971 and references therein; see also Misner *et al.* 1973).

Let us now try to find expressions for $\dot{a}/a$ and $\frac{d}{ds}(1+z)^{-1}$ in the form of integrals along γ. In what follows, we assume that the ray γ hits at each of its points $x(v)$ a vector field v^μ which satisfies the boundary condition

$$v^\mu(x_{obs}) = u^\mu_{obs} \quad (34)$$

Let us emphasize that v^μ can be chosen arbitrarily at any point x which does not belong to the world line $\mathcal{C}_{obs}$ (for example, $v^\mu(x)$ could be the unit 4-velocity of an observer at x, an assumption which is currently made in cosmology; however we shall make a more convenient choice for v^μ in what follows).

It results from the boundary conditions (27) and (34) that $\dot{a}/a$ may be written as

$$\left.\frac{\dot{a}}{a}\right|_{obs} = \int_{-\infty}^{v_{obs}} \frac{d}{dv}[v^\mu(\ln a)_{;\mu}]dv \quad (35)$$

Thus we have to transform the expression

$$\frac{d}{dv}[v^\mu(\ln a)_{;\mu}] = k^\alpha(v^\mu(\ln a)_{;\mu})_{;\alpha} \quad (36)$$

taken along γ. Of course, we must take into account the propagation equation (25) which could be rewritten as

$$k^\alpha(\ln a)_{;\alpha} = -\frac{1}{2}k^\alpha_{;\alpha} \quad (37)$$

Noting that

$$k^\alpha(v^\mu(\ln a)_{;\mu})_{;\alpha} = k^\alpha v^\mu(\ln a)_{;\mu;\alpha} + k^\alpha v^\mu_{;\alpha}(\ln a)_{;\mu} \quad (38)$$

then using the relation

$$F_{;\alpha;\beta} = F_{;\beta;\alpha} \quad (39)$$

which holds for any scalar F, we find

$$\frac{d}{dv}[v^\mu(\ln a)_{;\mu}] = v^\mu[k^\alpha(\ln a)_{;\alpha}]_{;\mu} + [k,v]^\mu(\ln a)_{;\mu} \quad (40)$$

where the bracket $[k,v]$ of k^α and v^β is the vector defined by

$$[k,v]^\mu \equiv k^\alpha v^\mu_{;\alpha} - v^\alpha k^\mu_{;\alpha} \quad (41)$$

Taking (37) into account, it is easily seen that

$$\frac{d}{dv}[v^\mu(\ln a)_{;\mu}] = -\frac{1}{2}v^\mu k^\alpha_{;\alpha;\mu} + [k,v]^\mu(\ln a)_{;\mu} \quad (42)$$

Now, using the identity (1) and the definition (2) yields

$$\frac{d}{dv}[v^\mu(\ln a)_{;\mu}] = \frac{1}{2}R_{\mu\nu}k^\mu v^\nu - \frac{1}{2}v^\mu k^\alpha_{;\mu;\alpha} + [k,v]^\mu(\ln a)_{;\mu} \quad (43)$$

Let us try to write the term $-\frac{1}{2}v^\mu k^\alpha_{;\mu;\alpha}$ in the form of an integral along γ. In agreement with (27), we have at any point $x(v)$ of γ:

$$v^\mu k^\alpha_{;\mu;\alpha} = \int_{-\infty}^{v} \frac{d}{dv}(v^\mu k^\alpha_{;\mu;\alpha})dv = \int_{-\infty}^{v} k^\lambda(v^\mu k^\alpha_{;\mu;\alpha})_{;\lambda}dv \quad (44)$$

A tedious but straightforward calculation using (1), (2) and (17) leads to the following result

$$-\frac{d}{dv}(v^\mu k^\alpha_{;\mu;\alpha}) = (R_{\rho\sigma;\mu} - R_{\mu\rho;\sigma})v^\mu k^\rho k^\sigma + R_{\rho\sigma}k^\rho v^\mu k^\sigma_{;\mu} + v^\mu(k^{\alpha;\beta}k_{\alpha;\beta})_{;\mu} - [k,v]^\mu k^\alpha_{;\mu;\alpha} \quad (45)$$

In the above formulae v^μ is an arbitrary vector. So we can choose v^μ so that the transport equations [4]

$$[k,v]^\mu = 0 \quad (46)$$

are satisfied along the ray γ. Since (46) is a system of first order partial differential equations in v^μ, there exists one and only one solution satisfying the boundary conditions (34). With this choice, $2\dot{a}/a$ is given by the integral formula:

$$2\left.\frac{\dot{a}}{a}\right|_{obs} = \int_{-\infty}^{v_{obs}} R_{\mu\nu}k^\mu v^\nu dv + \int_{-\infty}^{v_{obs}} dv \int_{-\infty}^{v} [(R_{\rho\sigma;\mu} - R_{\mu\rho;\sigma})v^\mu k^\rho k^\sigma + R_{\rho\sigma}k^\rho v^\mu k^\sigma_{;\mu} + v^\mu(k^{\alpha;\beta}k_{\alpha;\beta})_{;\mu}]dv' \quad (47)$$

Now we look for an integral form for the total derivative $\frac{d}{ds}(1+z)^{-1}$ along $\mathcal{C}_{obs}$. Henceforth, we suppose for the sake of simplicity that the observer is freely falling, *i.e.* that $\mathcal{C}_{obs}$ is a timelike geodesic. So we have

$$\dot{u}^\alpha = u^\lambda u^\alpha_{;\lambda} = 0 \quad (48)$$

Since $(u^\mu k_\mu)_{em}$ is a constant characterizing the observed spectral line (see above), it follows from (23) and (48) that

$$\frac{d}{ds}\left(\frac{1}{1+z}\right)_{obs} = \frac{1}{(u^\alpha k_\alpha)_{em}}(u^\mu u^\nu k_{\mu;\nu})_{obs} \quad (49)$$

Given an arbitrary vector field v^μ fulfilling the boundary condition (34), Eq. (49) may be written as

$$\frac{d}{ds}\left(\frac{1}{1+z}\right)_{obs} = \frac{1}{(u^\alpha k_\alpha)_{em}}\int_{-\infty}^{v_{obs}} k^\lambda(v^\mu v^\nu k_{\mu;\nu})_{;\lambda}dv \quad (50)$$

[4] These equations mean that $v^\mu = \alpha\eta^\mu$, where $\alpha = const.$ and η^μ is a connection vector of the system of light rays associated with the phase function S (see, *e.g.*, Schneider *et al.* 1992).

Using (1), (17) and (41), a straightforward calculation gives the general formula

$$\frac{d}{ds}\left(\frac{1}{1+z}\right)_{obs} = \frac{1}{(u^\alpha k_\alpha)_{em}} \int_{-\infty}^{v_{obs}} \{-R_{\mu\rho\nu\sigma} v^\mu v^\nu k^\rho k^\sigma$$

$$+ (k^\lambda v^\mu_{;\lambda})(v^\nu k_{\mu;\nu}) + v^\mu [k,v]^\nu k_{\mu;\nu}\} dv \quad (51)$$

which holds for any freely falling observer.

Now let us choose for v^μ the vector field defined by (46) and (34).

We obtain

$$(1+z)\frac{d}{ds}\left(\frac{1}{1+z}\right)_{obs} =$$

$$\frac{1}{(u^\lambda k_\lambda)_{obs}} \int_{-\infty}^{v_{obs}} [-R_{\mu\rho\nu\sigma} v^\mu v^\nu \; k^\rho k^\sigma \; + v^\mu v^\nu k^\alpha_{;\mu} k_{\alpha;\nu}] dv \quad (52)$$

4. Weak-field approximation

Now we assume the gravitational field to be very weak. So we put

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (53)$$

where $h_{\mu\nu}$ is a small perturbation of the flat spacetime metric $\eta_{\mu\nu}$, and we systematically discard the terms of order $h^2, h^3, ...$ in the following calculations. Thereafter, we suppose that any quantity T (scalar or tensor) may be written as

$$T = T^{(0)} + T^{(1)} + O(h^2) \quad (54)$$

where $T^{(0)}$ is the unperturbed quantity in flat spacetime and $T^{(1)}$ denotes the perturbation of first-order with respect to $h_{\mu\nu}$. Henceforth, indices will be lowered with $\eta_{\mu\nu}$ and raised with $\eta^{\mu\nu} = \eta_{\mu\nu}$.

We shall put for the sake of simplicity

$$K_\mu = k^{(0)}_\mu = S^{(0)}_{,\mu} \quad (55)$$

Neglecting the first order terms in h, Eq. (12) gives $K^\alpha K_\alpha = 0$, whereas Eq. (17) reduces to the equation of a null geodesic in flat spacetime related to Cartesian coordinates

$$K^\alpha K_{\beta,\alpha} = 0 \quad (56)$$

In agreement with the assumptions made in Sect. 3 to obtain Eqs. (29) and (31), we consider that at the zeroth order in $h_{\mu\nu}$, the light emitted by the source is described by a plane monochromatic wave in a flat spacetime. So we suppose that the quantities K_μ, $a^{(0)\mu}$ and consequently $a^{(0)}$ are constants throughout the domain of propagation.

Moreover, we regard as negligible all the perturbations of gravitational origin in the vicinity of the emitter (this hypothesis is natural for a source at spatial infinity) and the quantity a_0 in Eqs. (28) and (29) is given consequently by

$$a_0 = a^{(0)} = const. \quad (57)$$

Furthermore, it results from $K_\mu = const.$ that $k_{\alpha;\beta} = O(h)$. Therefore, terms like $k^\alpha_{;\mu} k_{\alpha;\nu}$ or $R_{\rho\sigma} k^\rho v^\mu k^\sigma_{;\mu}$ are of second order and can be systematically disregarded.

According to our general assumption in this section, the unit 4-velocity of the observer may be expanded as

$$u^\alpha_{obs} = U^\alpha + u^{(1)\alpha}_{obs} + O(h^2) \quad (58)$$

at any point of $\mathcal{C}_{obs}$, with the definition

$$U^\alpha = u^{(0)\alpha}_{obs} \quad (59)$$

It follows from (48) and from $g_{\alpha\beta} u^\alpha u^\beta = 1$ that

$$U^\alpha = const. \quad (60)$$

and

$$\eta_{\alpha\beta} U^\alpha U^\beta = 1 \quad (61)$$

From these last equations, we recover the fact that the unperturbed motion of a freely falling observer is a time-like straight line in Minkowski space-time.

Now we have to know the quantities v^μ occurring in Eqs. (47) and (52) at the lowest order. An elementary calculation shows that, in Eqs. (46), the covariant differentiation may be replaced by the ordinary differentiation. So we have to solve the system

$$\frac{dv^\alpha}{dv} = v^\mu k^\alpha_{,\mu} \quad (62)$$

together with the boundary conditions (34).

Assuming the expansion

$$v^\mu = v^{(0)\mu} + v^{(1)\mu} + O(h^2) \quad (63)$$

it is easily seen that the unique solution of (62) and (34) is such that at any point of the light ray γ, the components $v^{(0)\mu}$ are constants given by

$$v^{(0)\mu} = U^\mu \quad (64)$$

Neglecting all the second order terms in (47) and (52), we finally obtain

$$2\frac{\dot{a}}{a}\bigg|_{obs} = \int_{-\infty}^{v_{obs}} R^{(1)}_{\mu\nu} K^\mu U^\nu dv$$

$$+ \int_{-\infty}^{v_{obs}} dv \int_{-\infty}^{v} (R^{(1)}_{\rho\sigma,\mu} - R^{(1)}_{\mu\rho,\sigma}) U^\mu K^\rho K^\sigma dv' \quad (65)$$

and

$$(1+z)\frac{d}{ds}\left(\frac{1}{1+z}\right)_{obs} =$$

$$-\frac{1}{U^\lambda K_\lambda} \int_{-\infty}^{v_{obs}} R^{(1)}_{\mu\rho\nu\sigma} U^\mu U^\nu K^\rho K^\sigma dv \quad (66)$$

all the integrations being performed along the unperturbed path of light.

In Eq. (66) $R^{(1)}_{\mu\rho\nu\sigma}$ denotes the linearized curvature tensor of the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, *i.e.*

$$R^{(1)}_{\mu\rho\nu\sigma} = -\frac{1}{2}(h_{\mu\nu,\rho\sigma} + h_{\rho\sigma,\mu\nu} - h_{\mu\sigma,\nu\rho} - h_{\nu\rho,\mu\sigma}) \tag{67}$$

and $R^{(1)}_{\mu\nu}$ is the corresponding linearized Ricci tensor

$$R^{(1)}_{\mu\nu} = \eta^{\alpha\beta} R^{(1)}_{\alpha\mu\beta\nu} \tag{68}$$

It is worthy to note that the components $R^{(1)}_{\mu\rho\nu\sigma}$ and $R^{(1)}_{\mu\nu}$ are gauge-invariant quantities. Indeed, under an arbitrary infinitesimal coordinate transformation $x^\alpha \to x'^\alpha = x^\alpha + \xi^\alpha(x)$, $h_{\mu\nu}(x)$ transforms into $h'_{\mu\nu}(x) = h_{\mu\nu}(x) - \xi_{\mu,\nu} - \xi_{\nu,\mu}$, and it is easily checked from (67) and (68) that

$$R^{(1)}_{\mu\rho\nu\sigma}(h'_{\alpha\beta}) = R^{(1)}_{\mu\rho\nu\sigma}(h_{\alpha\beta}) \tag{69}$$

$$R^{(1)}_{\mu\nu}(h'_{\alpha\beta}) = R^{(1)}_{\mu\nu}(h_{\alpha\beta}) \tag{70}$$

This feature ensures that the right-hand sides of Eqs. (65) and (66) are gauge-invariant quantities.

Eq. (65) reveals that the first order geometrical scintillation effect depends upon the gravitational field through the Ricci tensor only. On the other side, it follows from (66) that the part of the scintillation due to the spectral shift depends upon the curvature tensor.

These properties have remarkable consequences in general relativity. Suppose that the light ray γ travels in regions entirely free of matter. Since the linearized Einstein equations are in a vacuum

$$R^{(1)}_{\mu\nu} = 0 \tag{71}$$

it follows from Eq. (65) that

$$2\frac{\dot{a}}{a} = 0 + O(h^2) \tag{72}$$

As a consequence, $\dot{\mathcal{N}}/\mathcal{N}$ reduces to the contribution of the change in the spectral shift

$$\frac{\dot{\mathcal{N}}}{\mathcal{N}} = -\frac{1}{U^\lambda K_\lambda} \int_{-\infty}^{v_{obs}} R^{(1)}_{\mu\rho\nu\sigma} U^\mu U^\nu K^\rho K^\sigma dv \tag{73}$$

From (72), we recover the conclusion previously drawn by Zipoy (1966) and Zipoy & Bertotti (1968): within general relativity, gravitational waves produce no first order geometrical scintillation.

5. Application to the scalar-tensor theories

The general theory developed in the above sections is valid for any metric theory of gravity. Let us now examine the implications of Eqs. (65) and (66) within the scalar-tensor theories of gravity.

The class of theories that we consider here is described by the action [5]

$$\mathcal{J} = -\frac{1}{16\pi c}\int d^4x\sqrt{|g|}\left[\Phi R - \frac{\omega(\Phi)}{\Phi}\Phi^{,\alpha}\Phi_{,\alpha}\right] + \mathcal{J}_m(g_{\mu\nu}, \psi_m) \tag{74}$$

where R is the Ricci scalar curvature ($R = g^{\mu\nu}R_{\mu\nu}$), Φ is the scalar gravitational field, g is the determinant of the metric components $g_{\mu\nu}$, $\omega(\Phi)$ is an arbitrary function of the scalar field Φ, and $\mathcal{J}_m$ is the matter action. We assume that $\mathcal{J}_m$ is a functional of the metric and of the matter fields ψ_m only. This means that $\mathcal{J}_m$ does not depend explicitly upon the scalar field Φ (it is the assumption of universal coupling between matter and metric).

We consider here the weak-field approximation only. So we assume that the scalar field Φ is of the form

$$\Phi = \Phi_0 + \phi \tag{75}$$

where ϕ is a first order perturbation of an averaged constant value Φ_0. Consequently, the field equations deduced from (74) reduce to the following system:

$$R^{(1)}_{\mu\nu} = 8\pi\Phi_0^{-1}(T^{(0)}_{\mu\nu} - \frac{1}{2}T^{(0)}\eta_{\mu\nu}) + \Phi_0^{-1}(\phi_{,\mu\nu} + \frac{1}{2}\Box\phi\,\eta_{\mu\nu}) \tag{76}$$

$$\Box\phi = \frac{8\pi}{2\omega(\Phi_0)+3}T^{(0)} \tag{77}$$

where $T^{(0)}_{\mu\nu}$ is the energy-momentum tensor of the matter fields ψ_m at the lowest order, $T^{(0)} = \eta^{\alpha\beta}T^{(0)}_{\alpha\beta}$ and $\Box$ denotes the d'Alembertian operator on Minkowski spacetime: $\Box\phi = \eta^{\alpha\beta}\phi_{,\alpha\beta}$.

It is easily seen that any solution $h_{\mu\nu}$ to the field equations (76) is given by [6]

$$h_{\mu\nu} = h^E_{\mu\nu} - \frac{\phi}{\Phi_0}\eta_{\mu\nu} \tag{78}$$

where $h^E_{\mu\nu}$ is a solution to the equations

$$R^{(1)}_{\mu\nu}(h^E_{\alpha\beta}) = 8\pi\Phi_0^{-1}(T^{(0)}_{\mu\nu} - \frac{1}{2}T^{(0)}\eta_{\mu\nu}) \tag{79}$$

which are simply the linearized Einstein equations with an effective gravitational constant $G_{eff} = c^4\Phi_0^{-1}$. Indeed, inserting (78) in (67) yields the following expression for the curvature tensor

$$R^{(1)}_{\mu\rho\nu\sigma}(h_{\alpha\beta}) = R^{(1)}_{\mu\rho\nu\sigma}(h^E_{\alpha\beta}) + \frac{1}{2}\Phi_0^{-1}(\eta_{\mu\nu}\phi_{,\rho\sigma} + \eta_{\rho\sigma}\phi_{,\mu\nu} - \eta_{\mu\sigma}\phi_{,\nu\rho} - \eta_{\nu\rho}\phi_{,\mu\sigma}) \tag{80}$$

[5] For details see Will (1993) and references therein. The factor $-(16\pi c)^{-1}$ in the gravitational action is due to the fact that we use the definition of the energy-momentum tensor given in Landau & Lifshitz (1975).

[6] This transformation can be suggested by the conformal transformation of the metric which passes from the Jordan-Fierz frame to the Einstein frame.

from which one deduces the Ricci tensor

$$R^{(1)}_{\mu\nu}(h_{\alpha\beta}) = R^{(1)}_{\mu\nu}(h^E_{\alpha\beta}) + \Phi_0^{-1}(\phi_{,\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\Box\phi) \tag{81}$$

Then substituting for $R^{(1)}_{\mu\nu}(h_{\alpha\beta})$ from its expression (81) into the field equations (76) gives Eqs. (79), thus proving the proposition.

The decomposition (78) of the gravitational perturbation $h_{\mu\nu}$ implies that each term contributing to $\dot{\mathcal{N}}/\mathcal{N}$ can be split into a part built from the Einsteinian perturbation $h^E_{\mu\nu}$ only and into an other part built from the scalar field ϕ alone. In what follows, we use the superscript ST for a functional of a solution $(h_{\mu\nu}, \phi)$ to the field equations (76)-(77) and the superscript E for the same kind of functional evaluated only with the corresponding solution $h^E_{\mu\nu}$.

In order to perform the calculation of the integrals (65) and (66), we note that $K^\mu F_{,\mu}$ is the usual total derivative of the quantity F along the unperturbed ray path, which implies that

$$\int_{-\infty}^{v_{obs}} K^\mu F_{,\mu} dv = F_{obs} - F_{(-\infty)} \tag{82}$$

The 4-vector K^μ (supposed here to be future oriented, *i.e* such that $K^0 > 0$) gives the direction of propagation of the light coming from the observed source. For a given observer moving with the 4-velocity U^μ, let us put

$$N^\mu = (\eta^{\mu\nu} - U^\mu U^\nu)\frac{K_\nu}{(U^\lambda K_\lambda)} = \frac{K^\mu}{(U^\lambda K_\lambda)} - U^\mu \tag{83}$$

We have $\eta^{\mu\nu}N_\mu N_\nu = -1$. Since N^μ is orthogonal to U^μ by construction, N^μ can be identified to the unit 3-vector $\boldsymbol{N}$ giving the direction of propagation of the light ray in the usual 3-space of the observer.

Using (76), (77), (80) and the assumption $\phi_{(-\infty)} = 0$, it is easily seen that (65) and (66) can be respectively written as

$$2\frac{\dot{a}}{a}\bigg|^{ST}_{obs} = 2\frac{\dot{a}}{a}\bigg|^{E}_{obs} + \frac{\dot{\phi}}{\Phi_0}\bigg|_{obs} \tag{84}$$

and

$$(1+z)\frac{d}{ds}\left(\frac{1}{1+z}\right)^{ST}_{obs} = (1+z)\frac{d}{ds}\left(\frac{1}{1+z}\right)^{E}_{obs} + \frac{1}{2\Phi_0}(\dot{\phi} - \boldsymbol{N}.\boldsymbol{\nabla}\phi)_{obs} \tag{85}$$

where

$$\dot{\phi} = U^\mu \phi_{,\mu} \tag{86}$$

and

$$\boldsymbol{N}.\boldsymbol{\nabla}\phi = N^\mu \phi_{,\mu} \tag{87}$$

As a consequence, the rate of variation in the photon flux as received by the observer is given by the general formula

$$\frac{\dot{\mathcal{N}}}{\mathcal{N}}\bigg|^{ST}_{obs} = \frac{\dot{\mathcal{N}}}{\mathcal{N}}\bigg|^{E}_{obs} + \frac{1}{2\Phi_0}(3\dot{\phi} - \boldsymbol{N}.\boldsymbol{\nabla}\phi)_{obs} \tag{88}$$

In a vacuum ($T^{(0)}_{\mu\nu} = 0$), the metric $h^E_{\mu\nu}$ satisfies the linearized Einstein field equations (71) and Eq. (84) reduces to

$$2\ \frac{\dot{a}}{a}\bigg|^{ST}_{obs} = \frac{\dot{\phi}}{\Phi_0}\bigg|_{obs} \tag{89}$$

In Eq. (88), $(\dot{\mathcal{N}}/\mathcal{N})^E_{obs}$ is reduced to the term given by Eq. (73), where $R^{(1)}_{\mu\rho\nu\sigma}$ is constructed with $h^E_{\mu\nu}$.

It follows from (89) that contrary to general relativity, the scalar-tensor theories (defined by (74)) predict the existence of a first-order geometrical scintillation effect produced by gravitational waves. This effect is proportional to the amplitude of the scalar perturbation. It should be noted that an effect of the same order of magnitude is also due to the change in the spectral shift.

To finish, let us briefly examine the case where the scalar wave ϕ is locally plane (it is a reasonable assumption if the source of gravitational wave is far from the observer). Thus we can put in the vicinity of the observer located at the point x_{obs}

$$\phi = \phi(u) \tag{90}$$

where u is a phase function which admits the expansion

$$u(x) = u(x_{obs}) + L_\mu(x^\mu - x^\mu_{obs}) + O(|x^\mu - x^\mu_{obs}|^2) \tag{91}$$

with

$$L_\mu = const. \tag{92}$$

It follows from Eq. (77) with $T^{(0)} = 0$ that L_μ is a null vector of Minkowski spacetime.

Replacing K_μ by L_μ in (83) defines the spacelike vector P^μ, which can be identified with the unit 3-vector $\boldsymbol{P}$ giving the direction of propagation of the scalar wave in the 3-space of the observer. Then introducing the angle α between $\boldsymbol{N}$ and $\boldsymbol{P}$, a simple calculation yields

$$\frac{\dot{\mathcal{N}}}{\mathcal{N}}\bigg|^{ST}_{obs} = \frac{\dot{\mathcal{N}}}{\mathcal{N}}\bigg|^{E}_{obs} + (1+\cos^2\frac{\theta}{2})\ \frac{\dot{\phi}}{\Phi_0}\bigg|_{obs} \tag{93}$$

This formula shows that the contribution of the scalar wave to the scintillation cannot be zero, whatever be the direction of observation of the distant light source.

6. Are observational tests possible?

It follows from our formulae that the scintillation effects specifically predicted by scalar-tensor theories are proportional to the amplitude of the scalar field perturbation at the observer. This *local* character casts a serious doubt on the detectability of these effects, since the scalar field perturbation is very small for a localized source of gravitational waves.

Indeed, one can put in most cases $\phi/\Phi_0 \sim \alpha^2 h$, where α^2 is a dimensionless constant coupling the scalar field with the metric gravitational field (see, *e.g.*, Damour & Esposito-Farèse 1992). Experiments in the solar system and observations of binary pulsars like PSR 1913+16 indicate that $\alpha^2 < 10^{-3}$. Consequently, setting $h \sim 10^{-22}$ for gravitational waves emitted by localized sources gives $\phi/\Phi_0 < 10^{-25}$ and the effect is much too weak to be detected.

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