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Non instantaneous mixing: Deuterium burning in very low mass stars and brown dwarfs

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Abstract. We present results from detailed computations focused on chemical mixing in convective regions of very low mass stars (VLMs) and brown dwarfs (BDs) during deuterium burning. Computations are made both in the instantaneous and in the coupled–diffusive mixing schemes to check which mixing is required to treat these phases.

The convective timescale is much shorter than the nuclear time scales for the burning of the deuterium present in the forming cloud during the pre–MS phase, and instantaneous mixing is a satisfactory approximation. This is confirmed by explicit model computation. During the following MS phase, the two mixing schemes lead to different D–profiles along the structure; differences in terms of total stellar luminosity remain however negligible.

We conclude that, so far as we are dealing with VLMs and BDs evolutionary models, the treatment of chemical mixing does not influence either the pre-MS initial D-burning phase nor the hydrogen burning minimum mass and luminosity of the MS.

Key words: stars: interiors – stars: low-mass, brown dwarfs – stars: pre-main sequence

1. Introduction

The location of very low mass stars ($M \leq 0.15 M_{\odot}$, VLMs) and, mainly, brown dwarfs (BDs) in the HR diagram during the pre–Main Sequence phase of D-burning is one of the present–day important topics. This evolutionary phase is in fact both quite luminous and long lived, and search for these objects in the star formation regions has relatively good observational chances to be successfull (e.g. Luhman et al. 1998). In addition, a selfconsistent treatment of the role of deuterium during the main sequence proton-proton burning is needed to correctly determine the hydrogen burning minimum mass and luminosity. Chabrier & Baraffe (1997) found possible large effects when applying the instantaneous mixing approximation, but they used an approximate treatment of mixing which must be checked (Baraffe, private communication).

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In the vast majority of stellar evolutionary computations, mixing of chemicals inside convective regions is usually assumed to be an almost instantaneous process, much faster than chemical evolution due to nuclear reactions. Though this approximation turns out to be "reasonable" in most cases, there are also cases of interest in which it fails in providing physically correct results. As an example, treatment of lithium production in AGB requires a different approach (Sackman & Boothroyd 1995).

The code ATON2.0 (Ventura et al. 1998) implements both instantaneous mixing, and a diffusive algorithm which allows simultaneous and self–consistent treatment of nuclear burning and time dependent chemical mixing. We then have the possibility of performing the most straightforward test on the reliability of the instantaneous mixing scheme for the phases involving deuterium burning. In the present paper we compute full evolutionary sequences both in the instantaneous and in the diffusive mixing approximation. We compare the mixing timescale τ_{mix} and the timescale τ_{nuc} during the major phases involving deuterium inside VLMs and BDs. Although τ_{mix} is much longer than τ_{nuc} in the main sequence phases, we never found differences in terms of total stellar luminosity in the two cases. Qualitative differences are found in MS for the D–profiles inside the structures, but on quantitative grounds the results obtained turn out independent of the model chosen to describe turbulent mixing. The same conclusion are reached both in the Mixing Length Approximation (MLT, Cox & Giuli 1968), and in the Full Spectrum of Turbulence model (FST, Canuto & Mazzitelli 1991, 1992).

2. Chemical mixing

2.1. The convective time scale

In the standard (up today) treatment of chemical mixing inside convective regions of stars, flat profiles are generally assumed for all the elements explicitly accounted for. This approximation (instantaneous mixing) stems from the brevity of the convective time scale (defined as the ratio between the size of convective eddies and turbulent velocity) when compared to the nuclear burning time scale.

The mixing time above defined:

$$\tau_{mix} = \frac{l_d}{v_d}\,, \tag{1}$$

is not, however, uniquely defined, depending on the choice of the convective model. When computations are performed according to the MLT, in which one large eddy only is accounted for, τ_{mix} is the lifetime of the unique convective element. The situation changes in the FST framework, since the physical description of convection is more accurate, and the whole spectrum of eddy dimensions is accounted for to evaluate the convective flux. A detailed description of the physical assumptions of both convective models, and of the different values for turbulent velocity and diffusive scale length obtained, can be found in Ventura et al. (1998). No wonder, then, that the MLT and the FST τ_{mix} values differ from each other. Nevertheless, the main conclusions reached in the present paper for VLMs and BDs turn out to be independent of the convective model adopted.

2.2. Diffusive mixing

According to first principles, when both nuclear reactions and turbulent mixing are present the local temporal variation of the i-th element can be described by a diffusion equation (Cloutman & Eoll 1976)

$$\frac{dX_i}{dt} = \left(\frac{\partial X_i}{\partial t}\right)_{nucl} + \frac{\partial}{\partial m}\left[(4\pi r^2\rho)^2 D \frac{\partial X_i}{\partial m}\right]\,, \tag{2}$$

where D is the diffusion coefficient, for which a local approximation is customarily used:

$$D = \frac{v_d \cdot l_d}{3}\,, \tag{3}$$

A detailed description of the physical and numerical problems with Eq. 2, and of the solution techniques, can be found in Ventura et al. (1998). Here, we simply recall that, in case of $\tau_{mix} << \tau_{nucl}$, we expect both instantaneous mixing and a diffusive approach to give, a priori, the same result. Use of Eqs. (1) and (2) would be then, in this particular case, unnecessary and time–wasting. On the contrary, were the mixing and nuclear time scales similar, the former approach would be incorrect, since mixing to the centre external matter richer in deuterium would lead to an overestimate of the central burning rates.

2.3. Deuterium burning in VLMs and BDs: the pre–MS phase

Let us first focus our attention on VLMs and BDs during their pre-MS phase. The only fusion reaction of interest from the point of view of the energy release is:

$$p + d \longrightarrow^3 He + \gamma\,, \tag{4}$$

Being the initial D-abundance in the forming cloud quite large ($X_D \sim 2 \cdot 10^{-5}$ in mass fraction) this phase lasts for longer than 10^6 yr in these objects (e.g. D'Antona & Mazzitelli, 1994), and thus it is important from an observational point of view. Reaction (4) is ignited at central temperatures $T_c > 8 \cdot 10^5 K$ (slightly increasing with total mass of the configuration). Particular care must be devoted to a physically correct treatment of D-burning, since its contribution can reach up to 80–90% of the total energy output, determining the stellar lifetimes in early pre–MS.

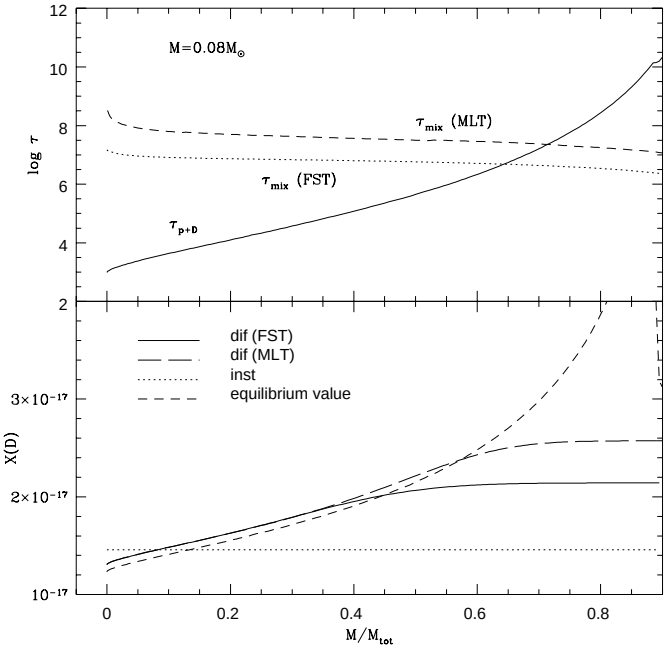

Fig. 1a and b. Top: Convective and nuclear time scales within the interior of a $M = 0.08 M_\odot$ star at an age of 10Gyr. Along a large fraction of the star the nuclear time scale for reaction (4) is much shorter than the mixing time scale, rendering unavoidable the solution of Eq. 2 to get the correct deuterium abundance. Bottom: deuterium abundances within the star obtained by adopting two schemes to deal with chemical mixing and two convective models; the equilibrium value, defined as in Eq. 6 is also shown.

When the only active process is D-burning, its equilibrium nuclear time scale is $\tau_{nucl} = 1/Y_p\lambda_{pd}$, where Y_p is the Hydrogen fractional abundance (in number) and λ_{pd} is the rate of reaction (4). As already stated, if $\tau_{mix} << \tau_{nucl}$ this phase can be addressed to within the instantaneous mixing scheme.

To correctly evaluate and compare the two above time scales, we used the most straightforward technique. We computed pre–MS evolutionary tracks of VLMs and BDs of solar abundance in both cases (instantaneous mixing and coupled diffusion). We made use of the **ATON 2.0** code: for all the details on the physical and numerical inputs of the code, see Ventura et al. (1998).

The results we obtain for a $M = 0.1 M_\odot$ star, when coupled diffusive mixing (the most correct procedure in any case) is applied, are the following: during the phase of D-burning we find a large difference between τ_{mix} and τ_{nucl}, the former being six orders of magnitude lower: ($\tau_{mix} \sim 10^7 s$, $\tau_{nucl} \sim 10^{13} s$) we therefore deduce that an instantaneous mixing scheme would have been sufficient to deal with D-burning in these stars and these phases. In fact, instantaneous mixing models gave the same results. Analogously when τ_{mix} is computed within the

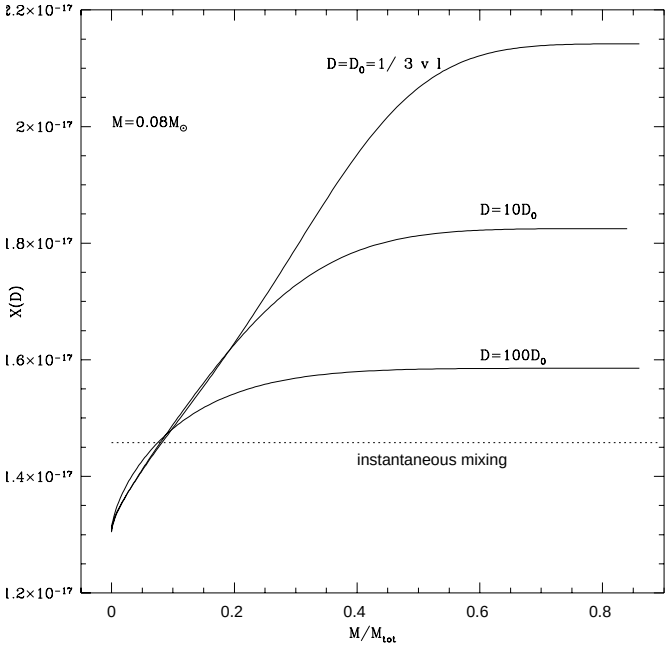

Fig. 2. Deuterium profiles obtained for different values of the diffusive coefficient D. We note that larger values of D, simulating a faster mixing, lead to lower abundances in the external layers and a flatter profile all over the star.

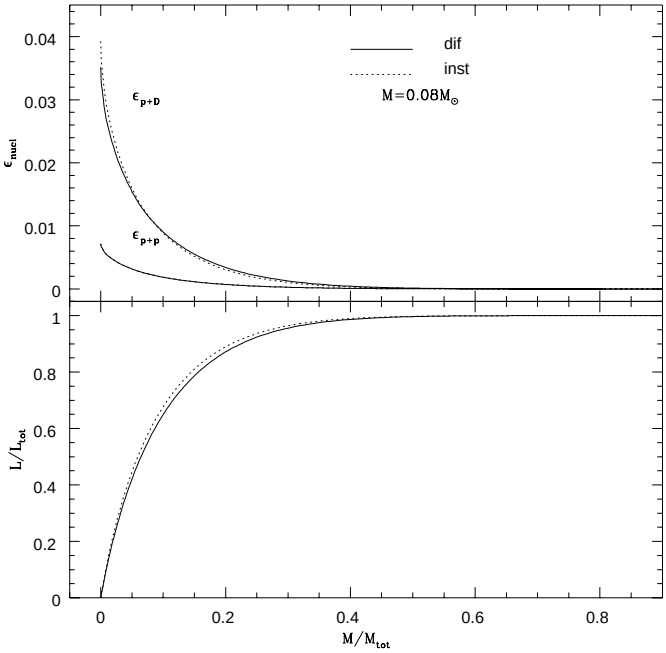

Fig. 3a and b. Top: Energy generation rates for reactions (4) and (5) within a $M = 0.08M_{\odot}$ star at an age of 10Gyr; we note that the rate of reaction (4) is larger in the innermost regions of the star in the instantaneous mixing case (dotted line), due to an overestimate of the central deuterium abundance. Bottom: Fraction of total luminosity as a function of mass in the interior of the same star: we see that more than $\sim 40\%$ (in mass) of the star contributes to the total luminosity.

MLT for turbulent convection, although τ_{mix} in MLT is much longer ($\sim 10^8 s$) due to the different convective scale length and turbulent velocity.

The same conclusions hold for lower masses: detailed computations performed for a $M = 0.05M_{\odot}$ structure gave analogous results. The computations performed with the two different algorithms however show small differences (always $\leq 2\%$) in the time-luminosity evolution. These differences are due to the different numerical treatment only.

2.4. D-burning in VLMs and BDs: the MS phase

As soon as the initial deuterium is all consumed, the VLMs and BDs resume contractions towards the MS. At the start of proton-proton burning, deuterium is now produced by:

$$p + p \longrightarrow d + e^{+} + \nu_e \,, \tag{5}$$

and is consumed by reaction (4) (the following destruction of ^3He is important, in an Hubble time, only for stars of mass $M \geq 0.15M_{\odot}$). Now the time scale for D–equilibrium results smaller than the mixing time scale at temperatures $T \geq 1.8 \cdot 10^6 K$, thus rendering compulsory, at least if we wish to obey first principles, the solution of the diffusive algorithm to get the correct deuterium profile along the central regions of the star. The problem of finding out the correct deuterium abundances in the central regions, and its implications on the determination of the stellar luminosity has been recently pointed out by Chabrier & Baraffe (1997). It goes without saying that what stated above holds only if deuterium is explicitly accounted for in the nuclear network of the code; if not, it is implicitly assumed that it is at its equilibrium value everywhere, this assumption being well justified (see Fig. 1b) in the innermost regions of the star, where most of the energetic release comes from.

We focused our attention on a $M = 0.08M_{\odot}$ star which should be extremely sensitive to differences in deuterium abundances in the central regions. In Fig. 1a it is shown a comparison between the time-scales involved in the computations along the whole star, at an age of 10Gyr. The time-scale of D-burning is much shorter than the convective time scale in a large fraction of the star, up to $\sim 60\%$ in mass, while the velocity of the mixing process is larger in the outer layers, with temperatures $T \leq 1.8 \cdot 10^6 K$. Note again the difference in terms of τ_{mix} obtained by adopting either the FST or the MLT to describe turbulent convection. The non instantaneous mixing scheme will produce a deuterium abundance profile within the star, as shown in Fig. 1b (continuous line). In the same figure we also show what we obtain with the instantaneous mixing scheme (dotted line) and the equilibrium abundance (long-dashed line), defined as

$$X_D = X_p \frac{\lambda_{pp}}{\lambda_{pD}} \tag{6}$$

Notice that the D abundance we obtain with the instantaneous mixing scheme is not the average value of the equilibrium abundances in the whole star, but it does account only for the most central layers, as in the outer layers ($M/M_{tot} > 0.4$) p-p burning is not efficient, and the equilibrium deuterium abun-

dance is not reached. In fact only in the centre burning is so fast that D can indeed reach equilibrium concentration, that is where $\tau_{p+D} < \tau_{mix}$. With the diffusive approach we obtain slightly larger D abundances in the external regions in the MLT case, consistently with the longer mixing times (see Fig. 1a).

To check the validity of our diffusive scheme we artificially rised our diffusive coefficient D all over the star. The results of these attempts are shown in Fig. 2: we see that for larger values of D the deuterium profile we obtain tends to the instantaneous approximation one, this confirming that the instantaneous mixing approximation stands on the assumption that mixing is everywhere much faster than any nuclear reaction, which is equivalent to assume an infinite value of D in Eq. 2.

The contributions to the total energy of reactions (4) and (5) obtained by adopting the two schemes for chemical mixing can be seen in Fig. 3a. The ϵ_{p+p} is the same in both cases, while the differences in terms of deuterium abundance outlined in Fig. 1a lead to a larger ϵ_{p+D} in the instantaneous mixing case in the central region ($\sim 10\%$ in mass); the situation is reversed in the most external layers, due to the larger D abundances provided by the diffusive approach.

Fig. 3b shows that the whole inner region (up to $\sim 40\%$ in mass) contributes to the total luminosity, so that, though the instantaneous mixing scheme gives an energy generation rate which is more peaked in the innermost layers $\sim 10\%$ (in mass) of the star, the net effect is that the total luminosity is the same in both cases, i.e. $\log(L/L_{\odot}) = -3.879$

3. Conclusions

In this paper we have presented new calculations to describe the pre-MS and MS phases of nuclear burning inside VLMs and BDs.

We have shown that during the pre-MS phase the D-burning time-scale is always at least 6 orders of magnitude larger then the mixing time-scale during the major phase of D-burning, thus implying that the whole convective region (i.e. the whole star) is well homogenized, and rendering useless the solution of the diffusive algorithm. Time-luminosity relationships for these stars during their approach to the MS confirm what stated above.

Regarding the following MS phase, Chabrier & Baraffe (1997) predicted possible large differences in term of total stellar luminosity obtained with the two schemes for mixing at the stellar/substellar transition, which is extremely sensitive to any parameter affecting thermal equilibrium. These differnces were due to a slight overestimation of the D abundance in their instantaneous mixing scheme (Baraffe, private communication).

On the contrary, during the MS phase we find that the instantaneous mixing approximation leads to a slightly incorrect deuterium abundance profile within the star since, during this phase, D-destruction can be much faster than the convective time-scale, expecially in the central regions of the star. And yet, this does not lead to any meaningful difference in terms of total luminosity, since D-burning is also active in more external regions off centre, where the diffusive approach provides larger deuterium abundances (and then larger energy generation rates).

The results we obtained are independent of the convective model adopted, since detailed computations performed both with the FST and MLT convective model for these kind of stars led to similar results.

We conclude that there is no important role of the treatment of chemical mixing in determining either the pre-MS initial D-burning phase nor the hydrogen burning minimum mass and luminosity of the MS.

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