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Letter to the Editor

An estimation of the mass of asteroid 20-Massalia derived from the Hipparcos minor planets data*

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Abstract. The ESA astrometry satellite Hipparcos has observed a set of 48 minor planets between 1989 and 1993. A close encounter between 20-Massalia and 44-Nysa, both planets observed by Hipparcos, enabled us to obtain a value for the mass of 20-Massalia. Due to another close approach with the large asteroid 4-Vesta, the value we obtained ($2.42 \pm 0.41\,10^{-12}$ $M_\odot$) is closely dependent on the value assigned to the mass of Vesta.

Key words: Hipparcos – asteroids – astrometry

1. Introduction

The Hipparcos mission was devoted to astrometric observations of stars. In addition, 48 minor planets belonging to the main belt of asteroids were also observed between 1989 and 1993. These minor planets have been chosen chiefly in order to allow a determination of the link between the Hipparcos reference frame and the dynamical one (Söderjhelm and Lindegren, 1982). Their magnitude ($<$ 12.5 in the case of the Hipparcos telescope) was not the only factor leading to this selection. The most important point was the number of observations per asteroid that the particular configuration of the Hipparcos satellite permitted (Bec-Borsenberger, 1992). However, since they provide a set of 2687 astrometric positions of very good accuracy (mean standard error of 15 milliarcsecond, 10 times lower than that obtained on Earth), the Hipparcos data of minor planets are also useful for improving the trajectories of these bodies. Moreover, close encounters occurring during the mission can provide valuable information about the masses of the minor planets involved.

Since fifteen years, several mass determinations of large minor planets have been made (e.g. Williams 1983, Scholl et al. 1987, Landgraf 1988, Goffin 1991, Viateau & Rapaport 1997). They usually use repeated, moderately close encounters of objects with the largest asteroids in order to accumulate their perturbing effects in the orbits of the test minor planets. For this purpose, the usual time span taken into account is typically about fifty years or even a century. Due to their accuracy, Hipparcos data of minor planets allowed us to examine the possibility of "instantaneous" mass determination. In this case, the time span corresponds to the mission duration, this is to say less than four years. This approach leads to significant results in the case of the 20 Massalia-44 Nysa close encounter, both asteroids observed by Hipparcos (this possibility was formerly suggested by Scholl and Schmadel). It proves that a short time span of observations of minor planets in the range of accuracy 10-100 milliarcsecond (mas) can provide opportunities for direct determinations of asteroid masses.

2. The O-C formulation

The published Hipparcos catalogue (ESA, 1997) gives two various reductions of the data as far as minor planets are concerned. This study is based on the NDAC data which are more numerous and seem to be more reliable than the FAST ones. Astrometric positions in the Hipparcos catalogue are given within the International Celestial Reference System (ICRS). The Hipparcos reference frame has been constructed in such a way that it coincides with the ICRS (Lindengren & Perryman, 1997). Thus, the rotation between the Hipparcos-ICRS reference frame and the dynamical one where ephemerides of minor planets are given is equivalent to a correction $\mathbf{\Theta}$. This correction has to be taken into account in the equations of condition.

In addition of $\mathbf{\Theta}$, the unknowns of the problem are the correction to the initial elements of each minor planet $\Delta \boldsymbol{u^0} = (\Delta l_0 + \Delta r, \Delta p, \Delta q, e\Delta r, \Delta a/a, \Delta e)$ and the correction $\Delta \boldsymbol{m}$ to the perturbing masses. The details of the whole calculation that leads to the system of equations of condition we use for Hipparcos minor planets can be found in (Bec-Borsenberger et al., 1995). The observed minus calculated position for each observation of a minor planet in case of Hipparcos data reduces to a projection over a Reference Great Circle (Hestroffer, 1997). It can be written as:

$$(O - C) = \mathbf{A}\Delta \boldsymbol{u^0} + \mathbf{B}\mathbf{\Theta} + \mathbf{C}\Delta \boldsymbol{m} \qquad (1)$$

For each date of observation, matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have to be computed: $\mathbf{A}$ is a matrix depending on the partial derivatives of

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* Based on data from the Hipparcos astrometry satellite.

the coordinates of the minor planet with respect to its initial elements, $\mathbf{B}$ is a matrix linking the Hipparcos reference frame and the dynamical one, and $\mathbf{C}$ is a matrix depending on the partial derivatives of the coordinates of the minor planet with respect to the perturbing mass m. Since we used the DE403 ephemeris of the Jet Propulsion Laboratory, which is constructed in order to coincide with the ICRS (Standish et al., 1995), in this study we actually set $\mathbf{\Theta} = 0$.

The following reduced model was established in order to study specific close encounters between two minor planets. For the test planet, we now write:

$$(O - C) = \mathbf{A}\boldsymbol{u}^{\mathbf{0}} + \mathbf{C}\Delta m_p \qquad (2)$$

where $\boldsymbol{u}^{\mathbf{0}}$ is the correction to the initial elements of the test minor planet and is m_p the mass of the perturbing planet. The perturbing effect of m_p is now introduced in the calculation of the position of the test minor planet. For doing so, an approximated value of m_p is computed by assuming a spherical shape with mean density of 2.5 g.cm^{-3}. IRAS values of diameters are used (Tedesco, 1989).

The epoch for all orbit corrections is t_0=JD 2446000.5, which is taken as the initial epoch for all integrations. This choice stems from the fact that the epoch should not be taken too close to the date of the encounter under study in order not to reduce its computed amplitude. The calculated positions of minor planets were obtained by numerical integration of the equations of motion. The perturbations from major planets Mercury to Pluto were taken into account when integrating these equations. In addition, the perturbations by minor planets 1-Ceres, 2-Pallas, 4-Vesta and 10-Hygiea were also systematically taken into account, since these four planets are the largest ones among minor planets and their masses have already been determined. We used the values recommended by the IAU commission 20 at the 1991 IAU General Assembly for the mass of Ceres, the values obtained by Standish & Hellings (1989) for the masses of Pallas and Vesta and by Scholl et al. (1987) for the mass of Hygiea (see Table 1). For the computation of perturbations by major planets, we used the DE403 ephemeris. Ephemerides of minor planets were computed at the Bureau des longitudes.

Table 1. Values of the masses of the four largest asteroids.

	Ceres	Pallas	Vesta	Hygiea
$(\times M_\odot)$	$5.0\ 10^{-10}$	$1.4\ 10^{-10}$	$1.5\ 10^{-10}$	$4.7\ 10^{-11}$

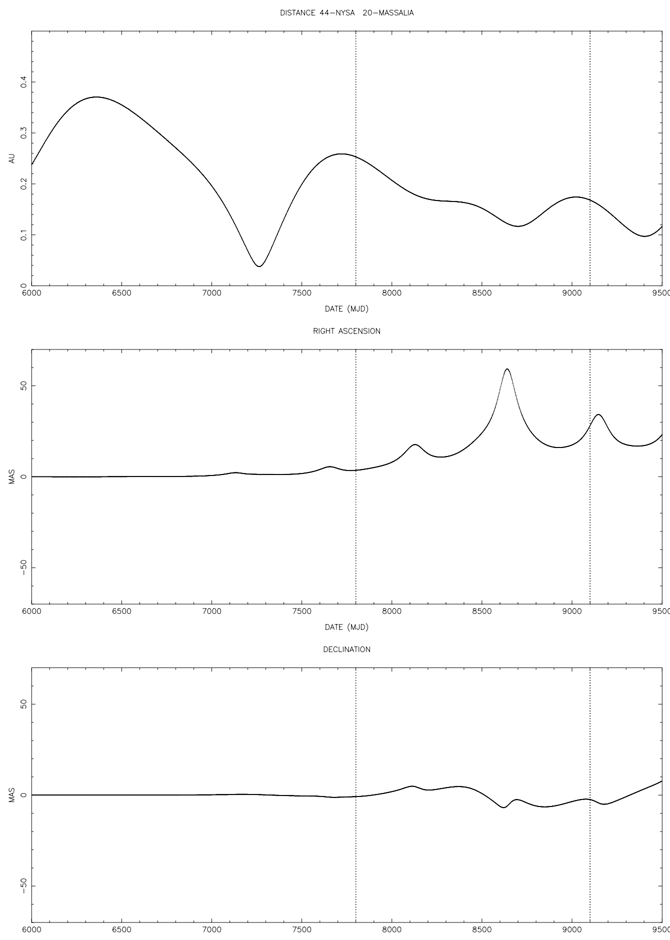

Fig. 1. Relative distance 20-Massalia 44-Nysa with respect to time (up, in AU) and differences in right ascension and declination between the perturbed orbit of 44-Nysa and the non perturbed one, assuming a value of $2.26\ 10^{-12}$ M$_\odot$ for the mass of Massalia (values in mas). The vertical lines give the begining and the end of the Hipparcos mission.

3. The close encounter 20-Massalia 44-Nysa

A selection procedure was used in order to determine the close encounters with expected effects on the minor planet trajectories larger than 5 mas (Bange and Bec-Borsenberger, 1997). For this purpose, we used a criterion based on the maximum of deflection as pointed out by Hoffmann (Hoffmann, 1989) in order to evaluate the efficiency of each close encounter. This criterion takes into account, not only the minimal distance, but also the square of the encounter velocity and the estimated mass of the perturbing body. The higher the criterion, the larger the expected effect of the test orbit perturbation. The whole set of the 48 Hipparcos minor planets was considered, and the minimal distance and encounter velocity for each pair of minor planets was computed by numerical integration of the equations of motion. For this computation, the integration step was set to one day.

This method predicted measurable effects for the pair 20-Massalia 44-Nysa. Actually, for this pair, a succession of three close encounters occurred just before or during the Hipparcos mission, with their minimal distances equal to 0.037 AU (JD 2447262.5); 0.116 AU (JD 2448700.5) and 0.096 AU (JD 2449402.5) (see Table 2). The encounter velocity was particularly low: 1.65 km s^{-1} (according Hoffmann (1989), the average encounter velocity between asteroids is about 5 km s^{-1}). The diameters of both planets, as given by Tedesco (1989), are 151 km (Massalia) and 73.3 km (Nysa). The test asteroid is consequently about 9 times less massive than the perturbing one. Thus, this case can be regarded as the most favourable one for

Table 2. Close encounters of minor planets with 44-Nysa before or during the Hipparcos mission. v_r is the encounter velocity, $\Delta\alpha_{max}$ and $\Delta\delta_{max}$ are the maximal expected perturbations on right ascension and declination during the mission.

Perturbing planet	Diameter km	Estimated mass 10^{-12} M$_\odot$	Date of close encounter JD	Minimal distance AU	v_r km s^{-1}	$\Delta\alpha_{max}$ mas	$\Delta\delta_{max}$ mas
4-Vesta	501.0	140.00	2446104.5	0.176	3.80	49.10	11.21
13-Egeria	215.0	6.54	2447568.5	0.124	5.61	2.19	0.25
20-Massalia	151.0	2.26	2447262.5	0.037	1.65	59.28	6.91
20-Massalia	151.0	2.26	2448700.5	0.116	1.45	″	″
20-Massalia	151.0	2.26	2449402.5	0.096	1.08	″	″
29-Amphitrite	219.0	6.91	2448200.5	0.124	3.38	1.22	0.47
129-Antigone	125.0	1.28	2446756.5	0.196	7.32	0.08	0.02
471-Papagena	139.0	1.76	2448482.5	0.149	5.79	0.11	0.06

Table 3. Orbital elements of 20-Massalia and 44-Nysa (rad).

Orbital element (JD 2446000.5)	20-Massalia	44-Nysa
Mean anomaly	0.65300608	0.56726578
Argument of perihelion	4.44789510	5.96774086
Longitude of asc. node	3.61597468	2.29918696
Inclination	0.01219953	0.06467134
Eccentricity	0.14569056	0.15182786
Semimajor axis (AU)	2.40793155	2.42214574

a mass determination, since reciprocal perturbation of the orbit of Massalia by Nysa can be neglected. Moreover, Nysa can be considered as a quasi stellar object, so that the phase effect is negligible (Hestroffer & Mignard, 1997). The NDAC reduction provides 53 observations of Nysa, from date JD 2448021.5 to JD 2448808.5 almost with a good repartition. Orbital elements of both minor planets at initial epoch JD 2446000.5 are given in Table 3.

The expected effects of the mass of Massalia on the trajectory of Nysa were first analysed. For doing so, we have computed two distinct orbits of Nysa. The first one did not take into account any perturbation by 20-Massalia, whereas the second one did. A value of $2.26\ 10^{-12}$ M$_\odot$ for the mass of Massalia was used in this computation (spherical approximation). Under this approximation, the maximal expected perturbations reached 59.28 mas in right ascension and 6.91 mas in declination during the Hipparcos mission. They are visualized on Fig. 1, along with the relative distance between both asteroids. The same procedure was used for the other close encounters involving Nysa during the Hipparcos mission, in order to evaluate their efficiency (see Table 2 for the results).

4. Resolution and discussion

In a first step, only corrections to the orbital elements of Nysa ($\Delta\boldsymbol{u}^{\mathbf{0}}$) are computed and Δm_p in Eq. (2) is set to 0. After examination of the post-fit residuals, one observation was rejected, showing significant deviation. The solution of system (2) then

Table 4. Rms of the residuals (mas) for the orbit of Nysa for various values of the mass of 20-Massalia (given in 10^{-12} M$_\odot$).

mass	0.00	0.77	1.55	2.33	3.10	3.88	4.65	5.43
rms	32.59	30.47	29.08	28.53	28.87	30.06	32.02	34.60

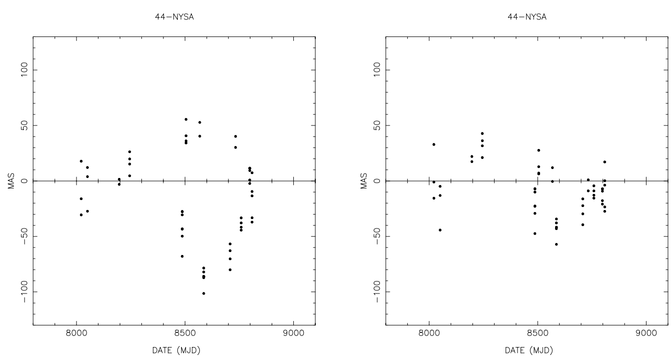

Fig. 2. Residuals of the observations of Nysa without any perturbation by Massalia (*left*) and assuming a value of $2.42\ 10^{-12}$ M$_\odot$ for its mass (*right*).

leads to the following corrections for the orbit of Nysa (mas):

$$\begin{cases} \Delta l_0 + \Delta r = -144.4 \pm 525.3 & e\Delta r = -24.42 \pm 24.18 \\ \Delta p = -13.60 \pm 5.38 & \Delta a/a = 9.02 \pm 31.38 \\ \Delta q = -4.16 \pm 3.98 & \Delta e = 1.21 \pm 21.03 \end{cases}$$

The only correction to be well determined is Δp. We use this value in order to improve the orbit of Nysa. The computation of the perturbing mass of Massalia is then performed on this new orbit. For doing so, different orbits of 44-Nysa are calculated, using different values for the mass of Massalia and the same set of 52 observations. The root mean square of the residuals (rms) for the various 44-Nysa orbit solutions is computed for several values of the perturbing mass (Table 4). The mass of Massalia and its mean error are then determined by the method of Herget (1972). A value of $2.42 \pm 0.41\ 10^{-12}$ M$_\odot$ is obtained. Fig. 2 shows the values of the residuals after the first iteration with or without perturbation by Massalia. The effects of the perturbation are more perceptible in the second part of the mission.

Assuming Massalia to be an asteroid with a 151 ± 11 km diameter, the value of its density deduced from our result is 2.67

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Table 5. Influence of the value of the mass of Vesta on the determination of the mass of Massalia (values in solar mass).

Mass of Vesta (assumed)	Mass of Massalia (computed)	Resulting density g.cm^{-3}
$1.30\ 10^{-10}$	$2.70\ 10^{-12}$	2.98
$1.35\ 10^{-10}$	$2.63\ 10^{-12}$	2.90
$1.40\ 10^{-10}$	$2.56\ 10^{-12}$	2.82
$1.45\ 10^{-10}$	$2.49\ 10^{-12}$	2.75
$1.50\ 10^{-10}$	$2.42\ 10^{-12}$	2.67
$1.55\ 10^{-10}$	$2.35\ 10^{-12}$	2.59

$\pm$ 1.06 g.cm^{-3}. It belongs to the S class (Tholen and Barucci, 1989). Unfortunately, direct determinations of class S asteroid masses are rare. Their densities are usually supposed to be higher than those of class C asteroids (Standish & Hellings, 1989), which are typically 2.5 (present value of Ceres). Nevertheless, keeping into mind that asteroid densities are subject to large uncertainties due to both the uncertainties on mass values and the irregular shapes of these bodies, our result seems to be realistic.

5. Perturbations of 44-Nysa by other minor planets

Vesta is one of the three largest asteroids, with a diameter of 501 km. A close encounter between Vesta and Nysa also took place before the Hipparcos mission (JD 2446104.5) at a minimal distance of 0.176 AU and relative velocity of 3.80 km s^{-1} (quite slow). In order to estimate the influence of the mass of Vesta on our solution, various solutions for the mass of Massalia were computed with different values for the mass of Vesta through the same method than described above. Results are given in Table 5, showing a compensation between the masses of Vesta and Massalia with respect to their effects upon the orbit of Nysa: the lower the value retained for the mass of Vesta, the higher the mass of Massalia. Vesta is known to be an atypical asteroid in terms of mass and spectra. According to successive determinations, its mass varies from $1.2\ 10^{-10}$ M$_\odot$ (Hertz, 1968) to $1.5\ 10^{-10}$ M$_\odot$ (Standish & Hellings, 1989). Our result suggests that this last value might be more reliable.

Our preliminary study also revealed a close approach between 13-Egeria and Nysa just before the Hipparcos mission, with minimal distance equal to 0.1237 AU, and a close approach between 29-Amphitrite and Nysa during the mission, with minimal distance equal to 0.1239 AU (see Table 2). The orbit of Nysa was recalculated including perturbations by these two minor planets but no significant difference was found for the value of the mass of Massalia. As shown in Table 2, the theoretical effects of such close encounters were expected to reach a maximum of 2,19 mas for the first one and 1.22 mas for the second one ($\Delta\alpha_{max}$ for both). Consequently, we did not examine the close encounters with minor planets 129-Antigone and 471-Papagena, two less massive asteroids, which occurred before or during the Hipparcos mission, since their effects, as shown in Table 2, are negligible.

6. Conclusion

The study of the close encounters occurring between the minor planets observed by Hipparcos can be of interest in determining the mass of a minor planet. The close encounter between 20-Massalia and 44-Nysa led us to give the first direct determination of the mass of Massalia, a 151 km large asteroid. The value we found ($2.42 \pm 0.41\ 10^{-12}$ M$_\odot$) seems to be in agreement with present mass determinations of asteroids in terms of density and mean error. However, the small number of observations available is responsible for the high value of the uncertainty of our result.

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