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LETTER

Letter to the Editor

On the validity of the core-mass luminosity relation for TP-AGB stars with efficient dredge-up

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Abstract. We investigate the validity of the core mass – luminosity relation (CMLR), originally described by Paczyński (1970), for asymptotic giant branch stars under the presence of third dredge-up events. We find, that models with efficient third dredge-up with less massive cores than those associated with hot bottom burning (Blöcker and Schönberner, 1991) do not obey the linear CMLR.

Complete evolutionary calculations of thermal pulse stellar models which consider overshoot according to an exponential diffusive algorithm show systematically larger third dredge-up for lower core masses ($0.55\mathrm{M}_\odot \lesssim M_\mathrm{H} \lesssim 0.8\mathrm{M}_\odot$) than any other existing models. We present and discuss the luminosity evolution of these models.

Key words: stars: AGB and post-AGB – stars: evolution – stars: interiors

1. Introduction

A general relation between the core mass and the inter-pulse luminosity of asymptotic giant branch (AGB) stars, found by Paczyński (1970) as a result of numerical computations of stellar models, has been widely used for the interpretation of observational data. The general concept of a single linear core mass – luminosity relation (CMLR) for all AGB stars within a certain mass range has been confirmed by succeeding evolutionary calculations (Iben, 1977; Schönberner, 1979; Wood, 1981; Lattanzio, 1986; Boothroyd and Sackmann, 1988; Vassiliadis and Wood, 1993). Blöcker and Schönberner (1991), however, showed that massive AGB stars which suffer from envelope burning do not obey this relation. Models with a radiative zone above the H-burning shell, like those with lower core masses, do not show envelope burning and can be described by relations like (Blöcker, 1993):

$$L/\mathrm{L}_\odot = 62200(M_\mathrm{H}/\mathrm{M}_\odot - 0.487)\ , \qquad (1)$$

where $0.55\mathrm{M}_\odot \lesssim M_\mathrm{H} \lesssim 0.8\mathrm{M}_\odot$ is the mass of the hydrogen-exhausted core and L is the maximum pre-flash luminosity. Such linear relations have been derived from models which do not show any considerable third dredge-up. Note, that efficient dredge-up is required in order to explain the observational luminosity function of carbon stars (e.g. Marigo et al. (1996)).

It has been reported by Herwig et al. (1997) that the application of exponential diffusive overshoot, which is motivated by results of hydrodynamical simulations of convectively unstable surface layers (Freytag et al., 1996), leads to stronger third dredge-up events than previously found. We have further investigated the impact of this kind of overshoot on AGB stellar models. A detailed account of the new, more extended calculations will be given elsewhere. In this *Letter* the luminosity evolution of models with lower core masses and efficient dredge-up events is described. We show that strong dredge-up leads to the violation of the CMLR for models that have lower core masses than those associated with envelope burning. We compare our results with predictions obtained by the homology relations.

2. The models

The evolutionary code and the physical parameters [(Y, Z, α_MLT)=(0.28, 0.02, 1.7)] are the same as the one used by Herwig et al. (1997). Mixing has been treated in a time dependent manner by solving a diffusion equation for each isotope. The diffusion coefficient D depends on the assumed mixing model. For the regions which are immediately adjacent to convectively unstable zones we allow for overshooting by using a depth dependent diffusion coefficient. It has been derived by Freytag et al. (1996: Eq. 9) from their numerical simulations of two-dimensional radiation hydrodynamics of time-dependent compressible convection. In this overshoot prescription the coefficient f is a measure of the efficiency of the exponentially declining diffusive mixing beyond the boundary

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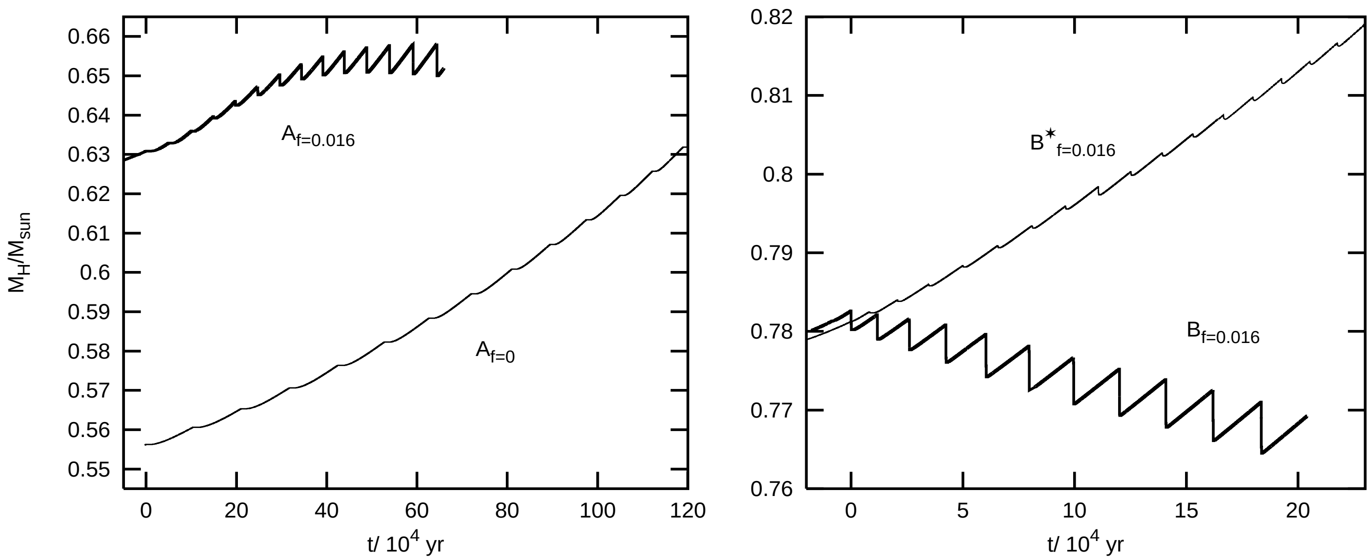

Fig. 1. Evolution of the core mass during the thermal pulse – AGB evolution of model sequences with initial masses of $3\mathrm{M}_\odot$ (*left*, label A) and $4\mathrm{M}_\odot$ (*right*, label B). The subscript indicates the treatment of overshoot: $\mathrm{f} = 0$ – no overshoot, $\mathrm{f} = 0.016$ overshoot with the given efficiency according to the method described in the text. The $\star$ denotes a sequence which has been computed with reduced numerical resolution. While overall TP-AGB evolution is not strongly affected by this, the third dredge-up is weaker by an order of magnitude (see text).

of convection. The findings presented in this paper are based on AGB models which have been computed with $f = 0.016$. If applied to core convection, this value of f reproduces the observed width of the main sequence (Schaller et al., 1992). A detailed discussion is given by Herwig (1998). Standard models, which have been computed for comparison, are referred to as $f = 0$ models.

Before we comment on the core mass evolution of our model sequences (Fig. 1), recall, that two main processes are altering the core mass: (a) hydrogen shell burning during the interpulse phase increases the mass while (b) the third dredge-up occurring shortly after the thermal pulse (TP) reduces the mass.

We have computed two AGB model sequences with $f = 0.016$: at the first TP sequence $\mathrm{A}_{\mathrm{f}=0.016}$ exhibits a core mass $M_{\mathrm{H1}} = 0.631\mathrm{M}_\odot$ and sequence $\mathrm{B}_{\mathrm{f}=0.016}$ has $M_{\mathrm{H1}} = 0.783\mathrm{M}_\odot$. Sequence $\mathrm{A}_{\mathrm{f}=0.016}$ experiences dredge-up from the third TP on with steadily increasing efficiency (Fig. 1). At the eleventh TP the dredge-up parameter λ[1] has reached unity. Already at the first pulse of sequence $\mathrm{B}_{\mathrm{f}=0.016}$ the dredge-up parameter exceeds unity. Starting from $\lambda = 1.6$ it gradually decreases to 1.3 at the last TP computed so far. This enhanced efficiency of dredge-up, compared to existing calculations, is caused by the application of the exponential diffusive overshoot algorithm and will be further described in a forthcoming paper. The $f = 0$ sequence $\mathrm{A}_{\mathrm{f}=0}$ with $M_{\mathrm{H1}} = 0.556\mathrm{M}_\odot$ shows no third dredge-up (Fig. 1). The sequence $\mathrm{B}^\star_{\mathrm{f}=0.016}$ has been computed with identical assumptions as sequence $\mathrm{B}_{\mathrm{f}=0.016}$ with the only difference that our improved adaptive step and grid size algorithm (Herwig, 1998) has not been applied. As a result, the dredge-up remains weak ($\lambda \simeq 0.12$). Sequence $\mathrm{B}^\star_{\mathrm{f}=0.016}$ serves only for comparison with sequence $\mathrm{B}_{\mathrm{f}=0.016}$ as an example of a TP-AGB evolution with only weak dredge-up but otherwise identical properties.

None of the models presented here shows envelope burning (hot bottom burning, HBB) because the core masses are too low.

3. The core mass – luminosity relation

3.1. Results from the models

In Fig. 2 we show the core masses and luminosities for each computed TP. The values refer to the end of the interpulse period when the luminosity is predominantly generated in the hydrogen burning shell and has reached its maximum.

Due to dredge-up the core-mass growth ΔM_{H} per TP is smaller compared to sequences without dredge-up or even negative. The luminosities and core masses of the model sequences with efficient dredge-up (lines with filled symbols in Fig. 2) are not reproduced by the linear CMLR (solid line, Eq. 1). They violate the classical relation.

During the first few pulses of sequence $\mathrm{A}_{\mathrm{f}=0.016}$ dredge-up is negligible. The relation bends asymptotically towards the linear relation (solid line, Eq. 1). As the dredge-up gains strength from pulse to pulse this trend turns into an upwards inflection. At the last pulses, shown in Fig. 2, the dredge-up parameter slightly exceeds unity (M_{H} does not grow anymore) and the luminosity is still increasing. For sequence $\mathrm{B}_{\mathrm{f}=0.016}$ the dredge-up parameter always clearly exceeds unity. Accordingly the core mass is *reduced* after each TP. However, the corresponding luminosities are still *increasing* (Fig. 2). This CMLR runs almost perpendicular to the linear relation.

The relation of sequence $\mathrm{A}_{\mathrm{f}=0}$ (no third dredge-up) runs parallel to the relation given by Eq. 1. The small offset is due to the different opacities used. The relation labeled S97 in Fig. 2 has

[1] λ is defined as the ratio of core mass decrease by dredge-up to core mass growth by hydrogen burning during the TP cycle.

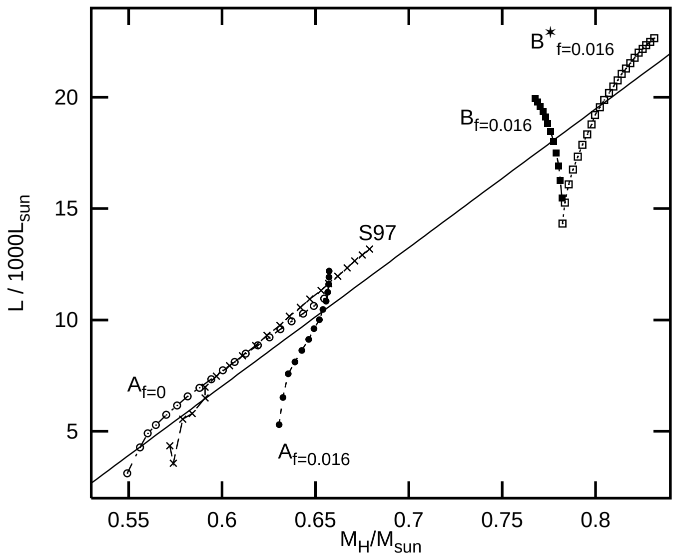

Fig. 2. Interpulse luminosity vs. core mass for the two A and B sequences introduced in Sect. 2. Each symbol represents one TP. S97 (crosses) refers to a $3\mathrm{M}_\odot$ AGB model sequence of Straniero et al. (1997). The solid line represents Eq. 1.

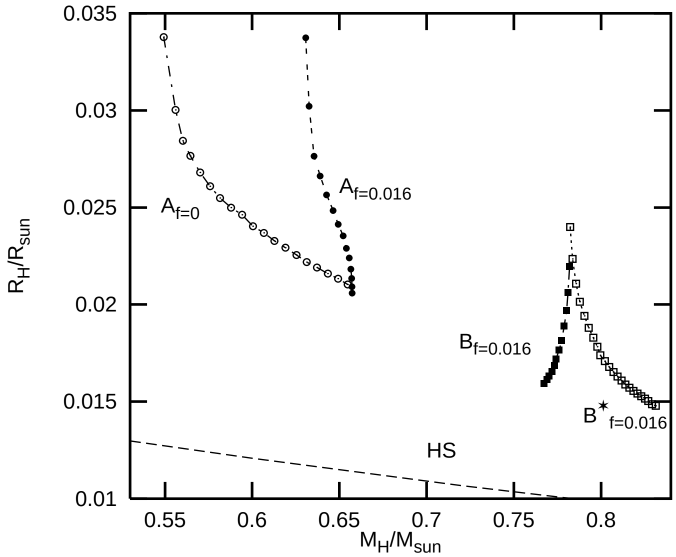

Fig. 3. Core radius (R_H) vs. core mass for the sequences described in Sect. 2. The radius is largest at the first TP. The line HS gives the mass – radius relation of Hamada and Salpeter (1961) for white dwarfs.

been taken from Straniero et al. cite*straniero:97. Their models exhibit dredge-up with $\lambda < 0.46$, leading only to a slight upturn. We conclude that *with the occurrence of sufficiently efficient dredge-up the AGB models do not follow the classical core mass – luminosity relation anymore.* As will be explained below, this deviation from the linear CMLR has a different physical origin than the one due to HBB for massive AGB stars.

3.2. Explanation from homology relations

Kippenhahn (1981), extending previous works by Refsdal & Weigert (1970), has demonstrated how homology relations can provide an expression for the luminosity of shell burning stellar models. E.g. the homology relation for the luminosity reads

$$l(\frac{r}{R_\mathrm{H}}) \approx M_\mathrm{H}^{\sigma_1} R_\mathrm{H}^{\sigma_2} \ . \tag{2}$$

Here l and r are the luminosity and radius at homologous points of two models. R_H and M_H are the core radius and the core mass. Thus, in the case of an interpulse AGB model shortly before the next TP, l at the upper boundary of the shell is practically the surface luminosity. Kippenhahn (1981) has derived for the exponents of Eq. 2 the expressions

$$\sigma_1 = \frac{4n+\nu}{N} \ , \sigma_2 = \frac{3-\nu-3n}{N}\beta \tag{3}$$

with

$$N = (4-3\beta)(1+n) + (1-\beta)(\nu-4) \ , \tag{4}$$

where $\beta := \frac{P_\mathrm{gas}}{P}$ is the gas pressure fraction and a, b, n, and ν are the exponents of the power-law approximations for the opacity and the energy production rate:

$$\kappa = \kappa_0 P^a T^b \ , \epsilon = \epsilon_0 \rho^{n-1} T^\nu \ . \tag{5}$$

Suitable estimates for the exponents in the approximations for κ and ϵ are $a = b = 0$ for electron scattering and $\nu = 14, n = 2$ for CNO cycle. For $\beta \to 0$ (radiation pressure is dominant, large core masses) the dependence of the luminosity on the core radius decreases ($\sigma_2 = 0$) and the homology method predicts a linear CMLR ($\sigma_1 = 1$). However, it has been shown by Blöcker and Schönberner (1991) that models with core masses larger than $M_\mathrm{H} \approx 0.8\mathrm{M}_\odot$ violate the linear CMLR anyway due to a different reason: the convective envelope region can penetrate into the hydrogen burning shell which leads to enhanced H-burning.

In the range of core masses considered here ($0.55 \lesssim M_\mathrm{H}/\mathrm{M}_\odot \lesssim 0.8$) radiation pressure is smaller (β is larger) and the dependence of L on R_H remains important (while HBB does not occur). For $\beta = 0.6$ (which roughly corresponds to $M_\mathrm{H} \approx 0.8\mathrm{M}_\odot$) the homology method predicts the core mass – core radius – luminosity relation

$$L \propto M_\mathrm{H}^2 R_\mathrm{H}^{-1} \ . \tag{6}$$

Thus, for models of lower core mass the homology method cannot predict any CMLR without an additional relation between core mass and core radius. I. e. *the luminosity is not a function of the core mass alone.*

Therefore, we display the core radii vs. core masses for our model sequences in Fig. 3. Sequence $\mathrm{A_{f=0}}$ shows a strong core radius decrease over the first few pulses when the core mass growth is rather small. Afterwards, the core mass – core radius relation (CMCRR) is almost linear. The models of sequences $\mathrm{A_{f=0.016}}$ and $\mathrm{B_{f=0.016}}$ also show a steady radius decrease, although their core masses evolve quite differently. For example, when the dredge-up parameter λ has reached unity ($\Delta M_\mathrm{H}/\mathrm{M}_\odot = 0$, sequence $\mathrm{A_{f=0.016}}$) the core radius continues to decrease steadily from pulse to pulse leading to a vertical CMCRR. This leads, according to Eq. 6, to continuously increasing

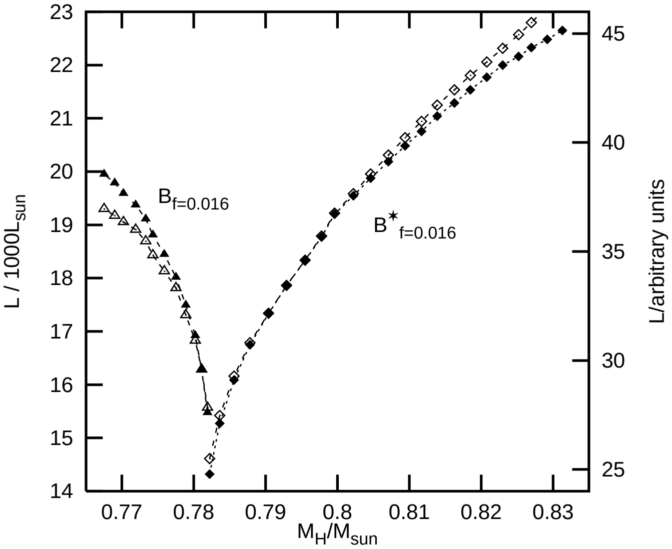

Fig. 4. Interpulse luminosities of the full numerical model sequences $\mathrm{B}_{\mathrm{f}=0.016}$ and $\mathrm{B}^{\star}_{\mathrm{f}=0.016}$ in comparison with the "semi-analytical" luminosities. The filled symbols repeat the respective lines from Fig. 2 and refer to the left ordinate. The open symbols show the values according to Eq. 6 if the core radius and core mass of the numerical models (as shown in Fig. 3) are inserted.

luminosities at constant core mass which in turn leads to the deviation from the linear CMLR. Also for sequence $\mathrm{B}_{\mathrm{f}=0.016}$ and $\mathrm{B}^{\star}_{\mathrm{f}=0.016}$, Fig. 3 shows a steady core radius decrease per pulse despite the large difference in both λ and core mass evolution. The CMCRR of a model sequence with efficient dredge-up is most importantly determined by the dependence of the core mass evolution on the dredge-up parameter.

From Eq. 6 the importance of the CMCRR for the understanding of the deviations from the linear CMLR found in the models with efficient dredge-up is evident. It is illustrative to insert the core masses and core radii of the sequences $\mathrm{B}_{\mathrm{f}=0.016}$ and $\mathrm{B}^{\star}_{\mathrm{f}=0.016}$ (see Fig. 3) into Eq. 6. In Fig. 4 one can see that the resulting "semi-analytical" luminosities do reproduce the functional dependence of luminosity and core mass for both cases. The fact that the homology relations reproduce the deviations from the CMLR illustrates also the different physical origin of this deviation compared to the previously mentioned one due to HBB (Blöcker and Schönberner, 1991).

4. Conclusions

We have shown that models with efficient dredge-up and core masses smaller than associated with HBB, do not obey the Paczyński core mass – luminosity relation because their core evolution with respect to its mass and radius is so different.

Our finding does not depend on the precise physical circumstances under which the third dredge-up occurs. The deviations from the CMLR reported here should be a general feature applying to all stars which are believed to suffer efficient dredge-up (and no HBB). Thus, it should be critically reviewed that synthetic models assume both considerably efficient dredge-up *and* a linear core mass – luminosity relation to be valid (Marigo et al., 1996; Groenewegen et al., 1995). Another immediate consequence is that core masses based on observed luminosities and the CMLR are probably too large. Finally, the deviation from the CMLR described here should be preserved during the post-AGB evolution, in contrast to that due to HBB. A physically related situation of post-AGB stars with identical core mass and different initial masses has been studied by Blöcker and Schönberner (1990) and Blöcker (1995). The different history leads to remnants with different core radii and consequently different luminosities.

Since the pioneering calculations of Paczyński (1970) the core-mass luminosity relation has been widely used and became a basic ingredient for many applications. However, it seems to turn out that the existence of such a general relation should be doubted for the large fraction of AGB stars which suffer either hot bottom burning or efficient dredge-up.

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