Letter to the Editor

Gamma-ray bursts from X-ray binaries

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Abstract. A weakly magnetized ($\sim 10^7$G) neutron star, slowly spun up by accretion in an X-ray binary, crosses the instability boundary for r-mode instability at P=1–2 msec. The amplitude of the oscillation, which initially increases only at the spinup time scale, is secularly unstable due to the negative temperature dependence of the viscosity in neutron star matter, and diverges after a few hundred years. Angular momentum loss by the gravitational wave causes strong differential rotation, in which the magnetic field is wound up to 10^{17}G on a time scale of a few months. When this field becomes unstable to buoyancy instability, a surface field strength of a few 10^{16}G is produced on a time scale of seconds, which then powers a GRB with energies of 10^{51}–10^{52} and duration of 1–100 sec.

Key words: gamma-ray bursts – X-ray binaries – gravitational waves – magnetic fields – instabilities

1. Introduction

A significant difficulty with several proposed mechanisms for gamma-ray bursts (GRB, Fishman & Meegan 1995) is that only a small amount of baryons ($\lesssim 10^{-5}\,M_\odot$) can be involved if the observed relativistic expansion speeds are to be produced (Shemi & Piran 1990, Paczyński 1990). Scenarios involving the coalescence of two neutron stars (Narayan, Paczyński & Piran 1992, Ruffert & Janka, 1998), or special types of supernovae (Paczyński 1998, MacFadyen & Woosley 1998) have been proposed, but both involve potentially large amounts of 'contaminating' baryons. A baryon-poor mechanism is the rapid spindown of a pulsar with a rotation period on the order of a millisecond and a magnetic field strength of 10^{16-17}G (Usov 1992). The problem with this idea is finding a convincing scenario for magnetizing a millisecond pulsar sufficiently rapidly to such a field strength (Kluźniak & Ruderman 1998, Dai & Lu 1998).

A neutron star with a mass of $1.4\,M_\odot$, a radius of $10\,\mathrm{km}$ and a rotation period of a millisecond contains a rotational energy of 2×10^{52} erg, the right amount to power a gamma-ray burst. Known neutron stars with such periods are the milliscond pulsars, believed to be descendants of X-ray binaries such as Sco X-1 or SAX J1808.4-3658 (P=2.5 msec, Wijnands & van

der Klis 1998). Accretion of angular momentum from the mass transfering secondary has slowly spun up the neutron star in such a system over a period of 10^8–10^9 yr. When the secondary has been evaporated (Ruderman et al. 1989, Tavani & London 1993), a single millisecond pulsar remains. Its rotation energy is slowly dissipated by pulsar radiation. If the magnetic dipole moment of the star is μ, the electromagnetic torque braking these 'cosmic flywheels' is (Shapiro & Teukolsky 1983).

$$T_{\mathrm{EM}} = \frac{2}{3}\frac{\mu^2\Omega^3}{c^3}, \tag{1}$$

where Ω is the star's rotation rate. If $B_{\mathrm{s}} = 10^8 B_8$G is the star's surface field strength, and $P = 10^{-3}P_{-3}$ s its rotation period, the spin-down time scale by this torque is $4 \times 10^9\,\mathrm{yr}P_{-3}^2/B_8^2$. Extraction of the rotation energy on the short time scale of a GRB requires a much higher field strength, of the order 10^{16}G. If the star rotates differentially by an amount $\Delta\Omega/\Omega \sim 1$, field strengths up to 10^{17}G are produced by winding-up of an initially weak field. Such fields are also strong enough to overcome the stable density stratification in the star, and float to the stellar surface (Kluźniak & Ruderman 1998) to produce dipole fields of the order 10^{16}G. The problem of making a GRB from an X-ray binary is thus reduced to finding a plausible way to make the star rotate differentially.

2. Runaway gravitational wave instability

Rotating neutron stars are naturally unstable to excitation of their oscillation modes by the emission of gravitational waves (Shapiro & Teukolsky 1983). A mode is excited like a squeaking brake. The deformation of the oscillation star provides surface roughness and the gravitational wave the torque coupling the star to the vacuum outside, analogous to the frictional coupling of a brake lining. Rotational modes (Rossby waves) turn out to be much more effectively excited than the pulsation modes of the star (Andersson 1997, Andersson et al. 1998). Modes with angular quantum numbers $l = m = 2$ grow fastest. The unstable displacements in these modes are mainly along horizontal surfaces. Their frequency in an inertial frame is $2/3\,\Omega$, and in the corotating frame $-4/3\,\Omega$ (retrograde). In the absence

of viscous damping, the growth rate of the most unstable mode is (Andersson et al. 1998)

$$\sigma_{\mathrm{GW}} = 5 \times 10^{-2} P_{-3}^{-6} \quad \mathrm{s}^{-1}. \tag{2}$$

Including viscous damping, the growth rate is

$$\sigma = \sigma_{\mathrm{GW}} - 1/\tau_{\mathrm{v}}, \tag{3}$$

where τ_{v} is the viscous damping time $\tau_{\mathrm{v}} = (\Delta R)^2/\nu$, and $\Delta R \approx 0.5R$ is the width of the unstable eigenfunction. For the kinematic viscosity ν the standard value quoted (van Riper 1991) for a neutron superfluid is $\nu \approx 2 \times 10^7 T_7^{-2} \ \mathrm{cm}^2\mathrm{s}^{-1}$ at an average density of $3 \times 10^{14} \ \mathrm{g} \ \mathrm{cm}^{-3}$, where $T = 10^7 T_7$ is the temperature. This expression holds at low temperatures; at temperatures above $\sim 3 \times 10^9$K the viscosity starts increasing strongly with T. During the spinup by accretion, the temperature of the star is determined by the accretion rate, $T_7 \approx \dot{M}_{-9}^{1/4}$, where $\dot{M} = 10^{-9} \dot{M}_{-9} \ \mathrm{M}_{\odot}/\mathrm{yr}$ is the mass accretion rate. With the typical value $\dot{M}_{-9} = 1$, the critical rotation period for instability to set in is 3 msec.

SAX J1808.4-3658 has a period shorter than this, and millisecond radio pulsars are known with periods as short as 1.5 msec. This suggests that the instability actually sets in at somewhat shorter periods. This is possible if the standard value underestimates the viscosity somewhat. Assuming that this is the case, I will, in the following, allow the viscosity to be increased, by a constant factor f_{v}. For example, with $f_{\mathrm{v}} = 30$ the critical period is 1.5 msec at an accretion rate $\dot{M}_{-9} = 1$.

A star slowly spun up by accretion (on a time scale of 10^9 yr) would become weakly unstable to the gravitational wave mechanism as its period decreases below the critical value. The unstable mode dissipates energy by viscous friction. Together with the negative temperature dependence of the viscosity this makes the mode amplitude secularly unstable, leading to a runaway of the mode amplitude in a finite time (this has been noted independently by Levin, 1998). The mode energy E is governed by

$$\mathrm{d}E/\mathrm{d}t = \sigma E, \tag{4}$$

The rotation rate of the star, assumed uniform at this stage, is determined by the accretion torque T_{a} and the gravitational wave angular momentum loss T_{GW}:

$$I\mathrm{d}\Omega/\mathrm{d}t = T_{\mathrm{a}} - T_{\mathrm{GW}} = (GMR)^{1/2}\dot{M} - \frac{3}{2}\frac{\sigma_{\mathrm{GW}}}{\Omega}E. \tag{5}$$

The heating is determined by viscous dissipation of the mode:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \frac{\nu}{(\Delta R)^2}E - L_{\nu}, \tag{6}$$

where $U \approx 3 \times 10^{43} T_7^2$ is the internal energy integrated over the star (van Riper 1991). $L_{\nu} \approx 10^{40} T_9^8$ represents the loss by neutrino cooling (van Riper 1991), and becomes important only at high temperatures. The energy losses by radiation at the surface can be neglected on the time scale of interest. The solution of these equations is shown in Fig. 1. A few hundred years after the rotation rate has crossed the instability threshold, the mode amplitude diverges. During the final runaway phase the temperature is so high that viscous damping is weak and the mode grows on a time scale of the order of minutes.
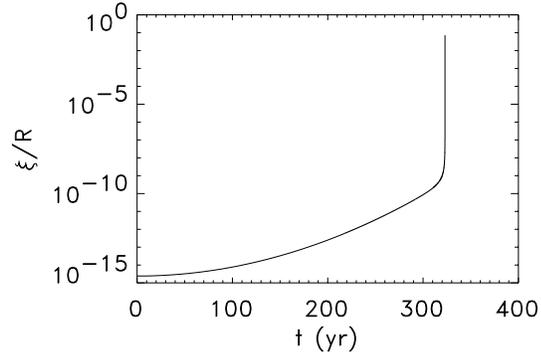
Fig. 1. Growth of the r-mode oscillation amplitude of a neutron star due to gravitational wave emission. Time in years since onset of instability, amplitude is the fractional displacement amplitude. The final runaway is due to the low viscosity of the interior, as it is heated by viscous dissipation of the mode
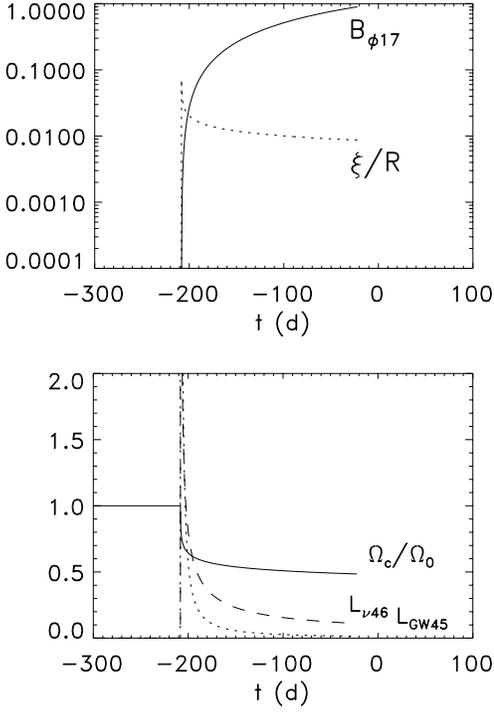
3. Differential rotation and winding-up of the magnetic field

The angular momentum loss by gravitational wave emission varies through the star, depending on the shape of the unstable eigenfunction. This loss does not match the angular momentum distribution of a uniformly rotating star, so differential rotation is induced. This winds the initial magnetic field into a toroidal (azimuthal) field. Because the rotation period is so short, the nonaxisymmetric components of the initial field are quickly eliminated by magnetic diffusion (Rädler 1981) in a process similar to 'convective expulsion' of magnetic fields (Weiss 1966); the remaining axisymmetric and nearly azimuthal field is as sketched in Kluźniak & Ruderman (1998). To represent these processes in a simple model, I divide the star into two zones with boundary at $r \approx 0.5R$, such that the outer shell has approximately the same moment of inertia as the core, and rotating at rates Ω_{s} and Ω_{c} respectively. The angular momentum loss by gravitational waves is taken to be restricted to the core for simplicity. If the initial radial field component is B_r, the azimuthal field component increases as

$$\mathrm{d}B_{\phi}/\mathrm{d}t = (\Omega_{\mathrm{s}} - \Omega_{\mathrm{c}})B_r. \tag{7}$$

Between core and shell the magnetic torque $T_{\mathrm{m}} = \frac{2}{3}r^3 B_r B_{\phi}$ acts, opposing the differential rotation.

The growth of the unstable mode is limited at large amplitude by nonlinear mode couplings. Since these have not been computed yet, I parametrize them by adding a term $-\sigma_{\mathrm{GW}}E^2/E_{\mathrm{s}}$ to the right hand side of (4), where E_{s} is the level at which the mode saturates. The displacement amplitude of the mode at this level is $\xi_{\mathrm{s}} \approx (E_{\mathrm{s}}/M)^{1/2}/\Omega$. The evolution of the azimuthal field strength, core rotation rate, gravitational wave luminosity and neutrino luminosity during the runaway phase is shown in Fig. 2 for a case with an assumed saturation level $\xi/R = 0.03$. The luminosities peak when the mode first reaches this level. After this, the mode amplitude decreases as the core spins down by the gravitational wave torque.
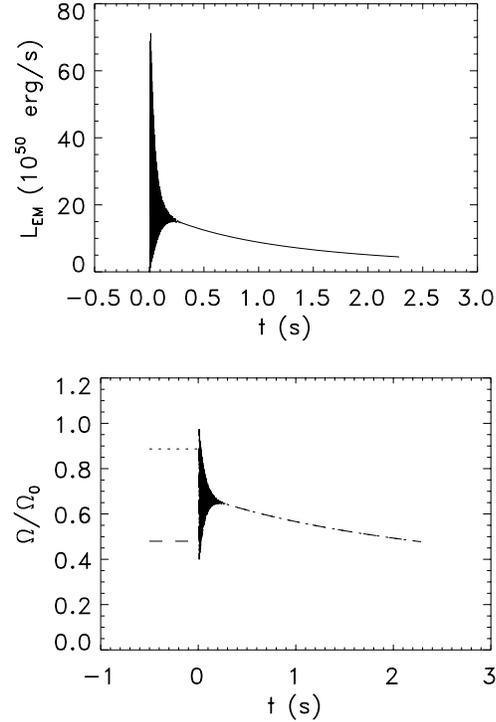
Fig. 2. *top*: Winding up of the magnetic field of the star (solid, in units of 10^{17}G) by differential rotation in the saturated phase of the gravitational wave instability. This phase of the process ends (at $t = 0$) when the magnetic field strength exceeds the critical value for buoyant instability of the field. *bottom*: Core rotation rate Ω_c (relative to its initial value Ω_0, solid), gravitational wave (dotted) and neutrino (dashed) luminosities (in units of 10^{44} and 10^{45} erg/s, respectively)

Fig. 3. *top*: gamma-ray burst luminosity (in units of 10^{50} erg/s). The ringing is an Alfvénic oscillation due to the coupling between core and shell by the strong radial field component. An ad hoc damping term has been added to represent magnetic dissipation inside the star, making the ringing decay during the burst. *bottom*: evolution of the rotation rates of shell (dotted) and core (dashed) during the burst. The strong radial field quickly couples the two

4. Buoyant instability of the toroidal field

The increasing azimuthal magnetic field inside the star does not manifest itself at the surface until it becomes unstable to buoyancy instability. For this to happen the field has to become strong enough (Kluźniak & Ruderman 1998), of the order $B_c = 10^{17}$G, to overcome the stable stratification associated with the increasing neutron/proton ratio with depth. The growth time τ_B of the instability is of the order of the Alfvén travel time, $\tau_B \sim R/(V_A - V_{Ac})$, where V_{Ac} is the Alfvén speed at the critical field strength B_c. Buoyancy instability is generally nonaxisymmetric, forming loops of field lines which erupt through the surface (e.g. Matsumoto & Shibata, 1992). The action of the radial velocities in the eruption process on the azimuthal field produces a strong radial field component, which couples to the differential rotation. The whole process is likely to yield a complex time dependent field configuration, beyond the scope of this investigation, but important aspects can be illustrated with a simple model. Let B_r be a representative value for the amplitude of the radial field component developing in the star. Its growth is due to the buoyant instability, but limited to a value of the order of the azimuthal field strength:

$$\mathrm{d}B_r/\mathrm{d}t = (v_A - v_{Ac})(B_\phi^2 - B_r^2)^{1/2}/R, \qquad (v_A > v_{Ac})$$
$$= 0 \qquad (v_A < v_{Ac}). \qquad (8)$$

The radial field component appearing at the surface will have a complicated structure, of which only the lower multipole components contribute to the field at the light cylinder, where the electromagnetic energy emission starts. To take this into account, I assume an effective dipole field strength at the surface, $B_s = \epsilon B_r$, with $\epsilon < 1$. Including the electromagnetic torque (1) due to this surface field, the rotation rates of the core and shell are given by

$$I_s \mathrm{d}\Omega_s/\mathrm{d}t = -T_m - T_{EM}, \qquad (9)$$

$$I_c \mathrm{d}\Omega_c/\mathrm{d}t = T_m - T_{GW}. \qquad (10)$$

The solution of Eqs. (8–10) starting at the time when the critical field strength B_c is reached, is shown in Fig. 3 for a case with $\epsilon = 0.3$. The final rise of the burst is on an Alfvén time scale $\tau_A = R/v_A \approx 0.5$ msec. The ringing seen in the light curve is an Alfvénic oscillation, in which the shell and the core exchange angular momentum by the magnetic torque T_m. In this simple model, it is a regular oscillation; in reality, the angular momentum exchange between different parts of the star will lead to a much more complicated variation, but on comparable time scales. Observations show clear evidence for such sub-msec variability in the central engine of GRB (e.g. Bhat et al. 1992, Walker et al. 1998).

Some 70% of the initial rotation energy of the star is released during the burst. The remaining 30% is released in the form of gravitational waves and neutrinos, mostly during the peak of the gravitational wave instability (Fig. 2) a few months before the electromagnetic burst.

5. Discussion

A number of parameters are involved in the caculations presented, in particular the initial field strength B_0 of the star, the assumed saturation amplitude ξ_s of the r-mode oscillation, the critical field strength for buoyant instability B_c and the viscosity factor f_v. Variation of B_0 has little effect on the duration and amplitude of the burst; it mainly affects the duration of the winding-up phase (Fig. 2), which decreases with increasing B_0. The saturation amplitude also has little effect, and also influences mainly the duration of the winding-up phase (decreasing with ξ_s). The value of B_c is more critical: if it is taken higher than about 3×10^{17}G, there is not enough energy in the differential rotation to reach buoyant instability. On the other hand, lowering B_c has little effect on the burst duration and amplitude; as buoyant instability sets in, the field is quickly amplified to a few times 10^{17}G by the differential rotation, and the rest of the burst is the same as before.

The most important parameter is the initial rotation rate Ω_0, i.e. the rate at the time when r-mode instability sets in. This rate is determined by the viscosity at that time, which depends on the assumed viscosity factor f_v and, through the temperature dependence of the viscosity, on the accretion rate (actually, the average of $\dot{M}$ over a thermal time scale preceding the instability). The total energy of the burst increases as Ω_0^2, the duration of the burst decreases as Ω_0^{-2}. The peak burst luminosity, which is critical for the detection of GRB, therefore increases as Ω_0^4. In the above, I have tuned f_v to get a critical rotation period of 1.5 msec, a value suggested by the known periods of msec pulsars. In the present scheme, these would be the remnants of X-ray binaries that managed to escape becoming GRB. Theoretical work to determine if such a high viscosity factor is realistic would be desirable, however.

At the distance of the secondary star ($\sim 10^{11}$cm), the radiation energy density during the burst is of the same order as the binding energy density of the secondary. It is therefore likely that a significant fraction of the secondary's mass is stripped in the process, or that it is destroyed altogether.

Since the life time of an X-ray binary ($\sim 10^8 - 10^9$ yr) is modest on a cosmological scale, the GRB rate predicted by the model would roughly scale as the cosmological star formation rate. X-ray binaries are observed in the galactic disk, so that the model puts GRB in galactic disks as well, as observed. They would not be closely related to individual star forming regions (cf. Paczyński, 1998), however, since the life time of these regions is less than the age of X-ray binaries. The neutron star binary merging scenario is more likely to produce GRBs somewhat outside galaxies, since the typical kick of 300 km/s received by the last-formed neutron star would have caused such binaries to travel ~ 100 kpc before merging. (X-ray binaries are a special population that must have received small kicks, since typical ones would have unbound these binaries).

After the burst there remains a slowly rotating hot neutron star with a (dipole) field strength of a few 10^{16}G. Due to its violent origin, this field configuration is likely to be unstable and rearrange itself on various time scales, including ones much longer than that of the burst. This predicts that most bursts would show low-level activity continuing after the main burst, probably decaying as a power law of time. The rate of decay would depend on the decay rate of the field strength. If this decay last sufficiently long, the remnant would behave like a soft gamma repeater, as in the model of Thompson & Duncan (1995). Perhaps some of the current soft gamma repeaters and anomalous X-ray pulsars (van Paradijs et al. 1995, Mereghetti et al. 1998, Kouveliotou et al. 1998), with present field strengths of $\sim 3 \times 10^{14}$G, may actually be the remnants of GRB that took place in our galaxy 10^5–10^6 years ago.

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