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Main-sequence stars of 10 and 30 $M_\odot$: approaching the steady-state rotation

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Abstract. The evolution of the angular velocity profile in 10 and 30 $M_\odot$ main-sequence (MS) stars has been calculated from the initial uniform rotation to the asymptotic steady-state rotation. Following Zahn (1992) we assume that both transport of angular momentum and mixing of chemical elements are produced by rotationally induced meridional circulation and turbulent diffusion. It is shown that for a sufficiently large surface rotational velocity, whose value can be estimated *a priori*, the relaxation time for star's achieving the steady-state rotation is much shorter than the star's MS life-time. In this case the assumption that a star is in a state of stationary rotation from the very beginning of its MS evolution is quite reasonable. On the other hand, for a star rotating slowly one has to solve the nonstationary angular momentum transport equation simultaneously with the stellar evolution calculations. Despite the fact that the rate of mixing of chemical elements by meridional circulation is strongly reduced by horizontal erosion, diffusion-like abundance profiles of C and N are built up in the radiative envelope by the end of the star's MS life. The surface N abundance begins to increase after some delay time required for the diffusion wave to reach the atmosphere. If mixing penetrates the convective core the abundance of He is expected to behave like that of N. Internal gravity waves generated near the convective core border are shown to probably play an important role as another angular momentum transport mechanism, especially in the inner part of the envelope.

Key words: stars: early-type – stars: evolution – stars: interiors – stars: rotation

1. Introduction

Massive main-sequence (MS) stars, i.e. stars with $M \gtrsim 10\,M_\odot$ producing He at the expense of H in their convective cores, are observed as early B- or O-type stars. During the last two decades observational data have accumulated (see, for instance, a review paper of Lyubimkov (1996)) indicating that some additional mixing (in the sense that it is not included in the standard stellar structure and evolution calculations) is active in these stars. The most convincing arguments in favour of the existence of such mixing in massive MS stars are measurements of increased N and He abundances (sometimes accompanied by a deficit of C) in atmospheres of the OB-stars. Lyubimkov (1984, 1989) was the first to point out that atmospheres of a majority of (presumably) single early B-type MS stars were enriched in N and even in He and that those enrichments seemed to correlate with the stellar age and mass. A more recent re-analysis carried out by Gies & Lambert (1992) confirmed Lyubimkov's results on N.

Considerable He overabundances (up to three times the solar He abundance) were also found in luminous OB-stars (most of them are still on the MS) by the Munich group (Herrero et al. 1992). These stars are on average more massive than the early B-stars studied by Lyubimkov. The He abundance anomalies in the OB-stars were shown to be accompanied by mass discrepancies. It turned out that masses of stars derived from comparison of their position in the HR diagram with theoretical evolutionary tracks (the so-called "evolutionary masses") were systematically higher than "spectroscopic masses" estimated from stellar spectra and, independently, from the radiation-driven stellar wind theory.

Langer (1992), Weiss (1994) and Denissenkov (1994) proposed that both, helium and mass, discrepancies might be attributed to additional mixing (of as yet unknown nature) operating in the radiative envelopes of the OB-stars. Stellar material enriched in He becomes more transparent for radiation and this results in an increase of star's luminosity ($L \propto \mu^4$, where μ is the mean molecular weight (Kippenhahn & Weigert 1994)). As a consequence, the evolutionary mass can be overestimated. It should be noted that in his next publication Herrero (1994) reduced the previously announced mass discrepancies, however, without changing values of the He overabundances.

A massive MS star has a convective core surrounded by a radiative envelope. It is important to recall that almost no He is produced outside this convective core (see Fig. 3a below). Therefore, any mechanism of additional mixing must provide it with the ability to penetrate the core and, of course, to operate fast enough for material from deep layers to reach the atmosphere during the OB-stars MS life-times.

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The MS OB-stars are known to be very fast rotators, therefore, a natural idea is to connect additional mixing in their radiative envelopes with rotation. Several years ago Zahn (1992) elaborated an original scheme describing how the mixing might be initiated and sustained in a radiative zone of a single non-magnetic rotating star. His only assumption has been that turbulence induced by various instabilities associated with star's differential rotation is highly anisotropic. The resulting turbulent viscosity has a horizontal component strongly dominating over a vertical one. Among the instabilities induced by rotation Zahn has distinguished shear instability as possessing the shortest development time-scale, horizontal and vertical shear flows being naturally produced by the classical Eddington-Sweet meridional circulation (Eddington 1925; Vogt 1925; Sweet 1950) when it is redistributing angular momentum.

The basic assumption of horizontally dominating turbulence ensures that the star settles in a state of "shellular" rotation with the angular velocity Ω depending only on the distance from the center r. Zahn (1992) derived an expression for $\Omega(r)$ appropriate for shellular rotation. He also obtained an equation governing the transport of angular momentum by meridional circulation competing with turbulent diffusion, the latter being generated by shear instabilities. The stronger turbulence in the horizontal direction has also to be taken into account when one considers a redistribution of chemical elements by the meridional circulation. Chaboyer & Zahn (1992) have shown that it results in a reduction of the advective flow which carries a tracer with the meridional circulation, an effect called "horizontal erosion" by them.

Since 1992 several important modifications have been introduced to Zahn's original scheme: (*i*) transport by turbulent diffusion arising from the horizontal shear instability is no longer considered because shear flows on the level surfaces are thought to be effectively hindered by the strong turbulent viscosity in the horizontal direction (Zahn 1997); (*ii*) a new expression for the vertical component of the turbulent viscosity which takes into account the horizontal erosion and radiative leakage has been proposed by Talon & Zahn (1997); (*iii*) more recently, Maeder & Zahn (1998) developed a modified scheme which allows for the evolution of a star, i.e. for changing profiles of a star's structural parameters, and offers a solution of the problem of simultaneous treatment of meridional circulation and semiconvection.

Zahn's scheme gives a self-consistent solution of the problem of rotationally induced mixing in stellar radiative zones in the sense that the rotation profile $\Omega(r)$ is no longer chosen arbitrarily (for instance, $\Omega(r) =$ const, as was often assumed before) but instead is let to adjust itself with time as the angular momentum gets redistributed inside the star. A similar algorithm was used earlier by Endal & Sofia (1976), Pinsonneault et al. (1989) and, recently, by Fliegner et al. (1996), but they incorrectly described the angular momentum transport by meridional circulation as a purely diffusive process (Zahn 1992).

Recently, Zahn's scheme has been applied to study stellar evolution with rotation of 9, 20, 40 and 60 $M_\odot$ MS stars by Meynet & Maeder (1997) and to calculate the He and CNO abundance profiles built up by meridional circulation and turbulent diffusion in a 9 $M_\odot$ star by Talon et al. (1997). However, in those works approaching the steady-state rotation during which the function $\Omega(r)$ evolves from the assumed initial uniform rotation to an asymptotic distribution was not discussed. The authors had simply used the asymptotic solutions of Zahn's angular momentum transport equation as representative ones for $\Omega(r)$ and only allowed them to change slowly with time due to the evolution of the star's internal structure on the MS.

In this paper we discuss the massive MS stars' approach to the asymptotic steady-state rotation as well as some related problems. Because relaxation times required for the stars in question to reach the state of asymptotic rotation are found to be short compared to the MS life-times and, consequently, changes of the H (and He) abundance taking place during these times can be neglected, we use ZAMS models and do not follow the evolution of the stars. In order to find out how the results obtained depend on stellar mass and rotation rate we performed calculations for two values of M, 10 and 30 $M_\odot$, and for the 10 $M_\odot$ model two different values of the surface rotational velocity were considered.

2. Basic equations

We have modified the expression for the amplitude of the radial component of the meridional circulation velocity U derived by Zahn (1992, Eqs. (3.37–39)) to take into account effects of the radiation pressure which are important in massive MS stars. Following a procedure similar to that used by Zahn we find

$$U(r) = \frac{L}{Mg}\left(\frac{P}{c_P\rho T}\right)\frac{1}{\nabla_{\rm ad}-\nabla}\,(E_\Omega + E_\mu), \qquad (1)$$

where

$$\begin{aligned} E_\Omega &= \frac{8}{3}\,\frac{\Omega^2 r^3}{GM}\left(1-\frac{\Omega^2}{2\pi G\rho}-\frac{\varepsilon}{\varepsilon_{\rm m}}\right) \\ &- \frac{\rho_{\rm m}}{\rho}\left\{\frac{r}{3}\frac{d}{dr}\left[H_T\frac{d(\frac{\Theta}{\delta})}{dr}-(\chi_T+1-\delta)\frac{\Theta}{\delta}\right]\right. \\ &- \left. 2\frac{H_T}{r}\frac{\Theta}{\delta}+\frac{2}{3}\Theta\right\} \\ &- \frac{\varepsilon}{\varepsilon_{\rm m}}\left[H_T\frac{d(\frac{\Theta}{\delta})}{dr}+(\varepsilon_T-\chi_T-1+\delta)\frac{\Theta}{\delta}\right] \\ &- \frac{\Omega^2}{2\pi G\rho}\Theta, \end{aligned} \qquad (2)$$

and

$$\begin{aligned} E_\mu &= \frac{\rho_{\rm m}}{\rho}\left\{\frac{r}{3}\frac{d}{dr}\left[H_T\frac{d(\frac{\Lambda}{\delta})}{dr}-(\chi_T+1)\frac{\Lambda}{\delta}-\chi_\mu\Lambda\right]\right. \\ &- \left. 2\frac{H_T}{r}\frac{\Lambda}{\delta}\right\} \\ &+ \frac{\varepsilon}{\varepsilon_{\rm m}}\left[H_T\frac{d(\frac{\Lambda}{\delta})}{dr}+(\varepsilon_T-\chi_T-1)\frac{\Lambda}{\delta}\right. \\ &+ \left.(\varepsilon_\mu-\chi_\mu)\Lambda\right]. \end{aligned} \qquad (3)$$

In formula (1) L is the luminosity at radius r, M the mass enclosed within a sphere of radius r (the Lagrangian mass coordinate), $g = GM/r^2$ the local gravity, $P = (\mathcal{R}/\mu)\rho T + \frac{1}{3}aT^4$ the total pressure, c_P the specific heat, $\nabla_{\rm ad} = (d\ln T/d\ln P)_{\rm ad}$ the adiabatic temperature gradient and ∇ is the actual gradient, which in a stellar radiative zone equals the radiative one $\nabla_{\rm rad} = 3/(16\pi Gac)(\kappa P/T^4)(L/M)$, where κ is the opacity. Other quantities have their usual meaning. In expressions (2) and (3) we have used the same notations for the measure of the horizontal fluctuations of density $\Theta = \tilde{\rho}/\rho = (r^2 d\Omega^2)/(3gdr)$ and of the mean molecular weight $\Lambda = \tilde{\mu}/\mu$ as in Zahn's paper. H_T is the temperature scale height and $\varepsilon_{\rm m} = L/M$ and $\rho_{\rm m} = M/(\frac{4\pi}{3}r^3)$ give the mean energy production rate and the mean density, respectively. ε is the local nuclear energy generation rate, $\chi = (4acT^3)/(3\kappa\rho)$ the radiative conductivity, symbols ε_T, χ_T and ε_μ, χ_μ being used to denote their logarithmic derivatives with respect to T and μ. The quantity $\delta = -(\partial\ln\rho/\partial\ln T)_{\mu,P} = (4-3\beta)/\beta$, with $\beta = (\frac{\mathcal{R}}{\mu}\rho T)/P$, approaches the limit $\delta = 1$ as soon as a contribution of the ideal gas pressure becomes dominating over that of the radiation one. In this case our Eqs. (2) and (3) turn into Zahn's Eqs. (3.37) and (3.38) with necessary small corrections applied to (3.37) (Zahn's private communication; see also Maeder & Zahn (1998)).

The evolution of the angular velocity profile can be calculated by solving the angular momentum transport equation

$$\frac{\partial}{\partial t}(\rho r^2\Omega) = \frac{1}{5r^2}\frac{\partial}{\partial r}(\rho r^4\Omega U) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4\nu_{\rm v}\frac{\partial\Omega}{\partial r}\right), \qquad (4)$$

where $\nu_{\rm v}$ is the vertical component of the turbulent viscosity (Zahn 1992, 1997). This is a nonlinear fourth order *PDE*. For $\nu_{\rm v}$, which is assumed to be of the same order of magnitude as the turbulent diffusivity in the vertical direction $D_{\rm v}$, we take the expression used by Talon et al. (1997)

$$\nu_{\rm v} \approx D_{\rm v} \approx \frac{8Ri_{\rm c}}{5}\frac{\left(r\frac{d\Omega}{dr}\right)^2}{N_T^2/(K+D_{\rm h}) + N_\mu^2/D_{\rm h}}. \qquad (5)$$

This was derived for the first time by Talon & Zahn (1997). In formula (5) $Ri_{\rm c} \approx 1/4$ is the critical Richardson number, $K = \chi/(c_P\rho)$ the radiative diffusivity, $N_T^2 = (g/H_P)\delta(\nabla_{\rm ad} - \nabla)$ and $N_\mu^2 = (g/H_P)\varphi\nabla_\mu$ are the T- and μ- components of the squared Brunt-Väisälä (buoyancy) frequency, H_P being the pressure scale height, and $\varphi = (\partial\ln\rho/\partial\ln\mu)_{P,T} = 1$ for a mixture of ideal gas and radiation. Unfortunately, there is no expression for the horizontal component of the turbulent diffusivity $D_{\rm h}$ deduced from first principles. Zahn (1992) has proposed to use the estimate

$$D_{\rm h} \approx \frac{rU}{C_{\rm h}}\left[\frac{1}{3}\frac{d\ln(\rho r^2 U)}{d\ln r} - \frac{1}{2}\frac{d\ln(r^2\Omega)}{d\ln r}\right] \qquad (6)$$

with a free parameter $C_{\rm h} \approx 1$. Making use of formula (6) we explicitly assume that it is the meridional circulation which is responsible for producing and sustaining a state of differential rotation on the level surfaces and, therefore, $D_{\rm h}$ is likely to be proportional to U.

The right hand side of Eq. (4) includes two terms, the first one describing advection by meridional circulation and the second one modelling vertical turbulent diffusion of the angular momentum. Due to effects of horizontal erosion the contribution of meridional circulation to the *rhs* of the equation of nuclear kinetics takes a diffusion form (Chaboyer & Zahn 1992)

$$\frac{\partial y_i}{\partial t} = \left(\frac{\partial y_i}{\partial t}\right)_{\rm nucl} + \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left[\rho r^2(D_{\rm v} + D_{\rm eff})\frac{\partial y_i}{\partial r}\right], \qquad (7)$$

where $(\partial y_i/\partial t)_{\rm nucl}$ is a source/sink term from nuclear reactions, y_i the relative abundance of the i-th nuclide (by particle number), and

$$D_{\rm eff} = \frac{|rU|^2}{30D_{\rm h}}. \qquad (8)$$

Eqs. (1–8) supplemented with appropriate initial and boundary conditions (see the next section) make a completely self-consistent set.

3. Further assumptions and simplifications

As said in the introduction, we have used ZAMS model stars and have not followed their evolution because our objective has been to calculate stars' approaching the steady-state rotation which was found to happen rather quickly compared to the MS life-times. Two ZAMS models of $M = 10$ and $30\,M_\odot$ for the initial hydrogen and heavy element relative mass abundances $X = 0.70$, $Z = 0.02$ have been prepared. The stellar evolution code we used for this is an updated version of the same one already used by Denissenkov & Weiss (1996). The modifications, which include the employment of the latest OPAL opacity (Rogers & Iglesias 1992; Iglesias & Rogers 1996) and equation of state (Rogers et al. 1992) tables, have been described in Schlattl et al. (1997). With these models all necessary stellar structure parameters entering Eqs. (1–8) and independent of the rotation state were calculated once and considered to be fixed furtheron.

We did not modify the equation of hydrostatic equilibrium to take into account effects of rotation on the stars' structure because according to Meynet & Maeder (1997) these are very small.

Assuming that convection penetrates up to a level $\sim 0.1\,H_P$ into the radiative envelope we have applied the inner boundary conditions for Eq. (4) at a radius $r_{\rm i} = r_{\rm c} + 0.1H_P$, a little above the convective core border $r_{\rm c}$, which allowed us to avoid a singularity when calculating U with formula (1). We have used the following inner boundary conditions:

$$\begin{cases} \partial\Omega/\partial r = 0, \text{ and} \\ \Omega = \Omega_{\rm c}, \end{cases} \qquad (9)$$

where

$$\frac{d\Omega_{\rm c}}{dt} = \frac{(\rho r^4\Omega U)_{r=r_{\rm c}}}{5\int_0^{r_{\rm c}}\rho r^4\,dr}. \qquad (10)$$

Relation (10) results from the following assumptions: (*i*) the convective core rotates as a rigid body due to a large turbulent viscosity of convective origin; (*ii*) the star conserves its angular momentum, i.e. mass loss is ignored (we discuss this point in more detail in the conclusion).

The outer boundary conditions applied near the stellar surface were

$$\begin{cases} \partial\Omega/\partial r = 0, \text{ and} \\ U = 0. \end{cases} \quad (11)$$

These are direct consequences from the above assumption (*ii*).

As the initial condition the uniform rotation law $\Omega(r) = \Omega_0$ was used, where a value of the quantity Ω_0 consistent with other assumptions was calculated as explained below.

First, for a specified value of the surface angular velocity $\Omega_{\rm s}$ an asymptotic solution of Eq. (4) was found by solving the stationary equation

$$-\frac{1}{5}\Omega U = \nu_{\rm v}\frac{d\Omega}{dr} \quad (12)$$

which is deduced from (4) by assuming that the partial derivative with respect to time equals zero (cf. Zahn 1992; Urpin et al. 1996; Talon et al. 1997). The boundary problem for the third order *ODE* (12) (the boundary conditions being now reduced to the first row from (9) and (11) supplemented with the requirement that $\Omega = \Omega_{\rm s}$ at the surface) was solved by a shooting method. After that the appropriate value of Ω_0 was chosen to satisfy the angular momentum conservation law.

To simplify calculations we have assumed that the radiative diffusivity K is much greater than the turbulent diffusion coefficient $D_{\rm h}$. Another simplification was ignoring energy and μ-gradient producing nuclear reactions in the radiative envelope. Let us emphasize again that we did not follow stellar evolution and therefore our "radiative envelope" was actually a zone stretching from a position of the convective core border at its maximum extent up to the stellar surface. The standard stellar evolution calculations show that during star's MS life the convective core shrinks and leaves behind itself a zone of variable chemical composition which becomes later a semiconvective zone in massive MS stars. We do not discuss here the rather difficult problem of how the additional mixing succeeds to overcome the μ-gradient barrier (cf. Maeder & Zahn 1998) but instead address the problem of whether the mixing is fast enough to bring products of nuclear reactions from the radius $r_{\rm i}$ to the surface while the star is on the MS. In this respect our second simplification is permissible.

The above two simplifications allow us to put $E_\mu = 0$ and $N_\mu^2 = 0$ and instead of (5) to make use of the approximate formula

$$\nu_{\rm v} \approx D_{\rm v} \approx \frac{8Ri_{\rm c}}{5}\frac{K}{N_T^2}\left(r\frac{d\Omega}{dr}\right)^2. \quad (13)$$

Solutions of the full nonstationary problem (Eqs. (1–4), (6), (8–11) and (13)) were obtained by a Newton relaxation method.

Before coming to a discussion of results of the calculations we are going to write down consistency criteria ensuring that Zahn's scheme does work. These must be checked *a posteriori*.

$$D_{\rm v} \gtrsim \frac{1}{3}\nu Re_{\rm c}, \quad (14)$$

where ν is the ordinary (i.e. molecular plus radiative) viscosity and $Re_{\rm c} \approx 10$ the critical Reynolds number. If condition (14) is not fulfilled then turbulent motions responsible for the turbulent diffusion transport will dissipate through viscous friction.

The second inequality

$$D_{\rm v} \ll K \quad (15)$$

must be fulfilled to justify that effects of heat transfer by turbulence are ignored.

$$K \gg D_{\rm h} \quad (16)$$

is our simplifying assumption which together with the assumption of a zero μ-gradient transforms (5) into (13).

$$D_{\rm v} \ll D_{\rm h}. \quad (17)$$

This is Zahn's basic assumption which makes it possible to formulate the whole problem as one-dimensional.

4. Results of the calculations

In Fig. 1 solutions of the nonstationary problem are presented for the $30\,M_\odot$ model. The surface rotational velocity $V_{\rm s} = 470\,{\rm km\,s^{-1}}$ has been chosen as large as those observed in the most rapidly rotating OB-stars. It corresponds to $\Omega_{\rm s} = 10^{-4}\,{\rm s^{-1}}$ and to the ratio of the centrifugal acceleration to the gravity on the stellar equator $\eta_{\rm s} = \Omega_{\rm s}^2 R^3/GM \approx 0.26$. The solid line in Fig. 1a gives the steady-state asymptotic solution (Eq. (12)). One can see that in this particular case the nonstationary solutions approach the stationary one very quickly (numbers along the lines show their respective ages in years). The MS life-time for the $30\,M_\odot$ model is $\tau_{\rm MS} = 5.9\cdot 10^6$ years, so, for $\Omega_{\rm s} = 10^{-4}\,{\rm s^{-1}}$ it takes only about 1% of the MS life-time for the $30\,M_\odot$ star to settle in the steady-state rotation. This "relaxation" time can be roughly estimated *a priori* as the ratio $\tau_{\rm rel} \approx R/U(\Omega_{\rm s})$ (see below) which is approximately proportional to $\Omega_{\rm s}^{-2}$ (Eqs. (1–2)) for a fixed stellar mass. Obviously, the more rapidly rotating stars reach the state of asymptotic rotation quicker than stars rotating slower.

In panels b and c the evolution of the meridional circulation velocity and of the total diffusion coefficient are shown. For the initial uniform rotation law the quantity U changes sign near the stellar surface. This is a well-known result being due to the negative term $-\Omega^2/2\pi G\rho$ in brackets in the first row of expression (2) (Gratton 1945; Öpik 1951). In the classical description with the uniform rotation law applied, the meridional circulation currents consist of two zones, the mass of the outer zone being much smaller than that of the inner zone (Pavlov & Yakovlev 1978). Slow mixing across the "quiet" layer separating these zones can occur only as a purely microscopic diffusion process (Charbonnel & Vauclair 1992). In the past there were even some speculations that the quiet layer could prevent the radiative envelope of a single rotating MS star from being fully mixed (Vauclair 1988; Leushin et al. 1989). However, in Zahn's scheme, as our calculations show (see also Urpin et al. 1996; Talon et al. 1997), the position of the quiet layer shifts towards

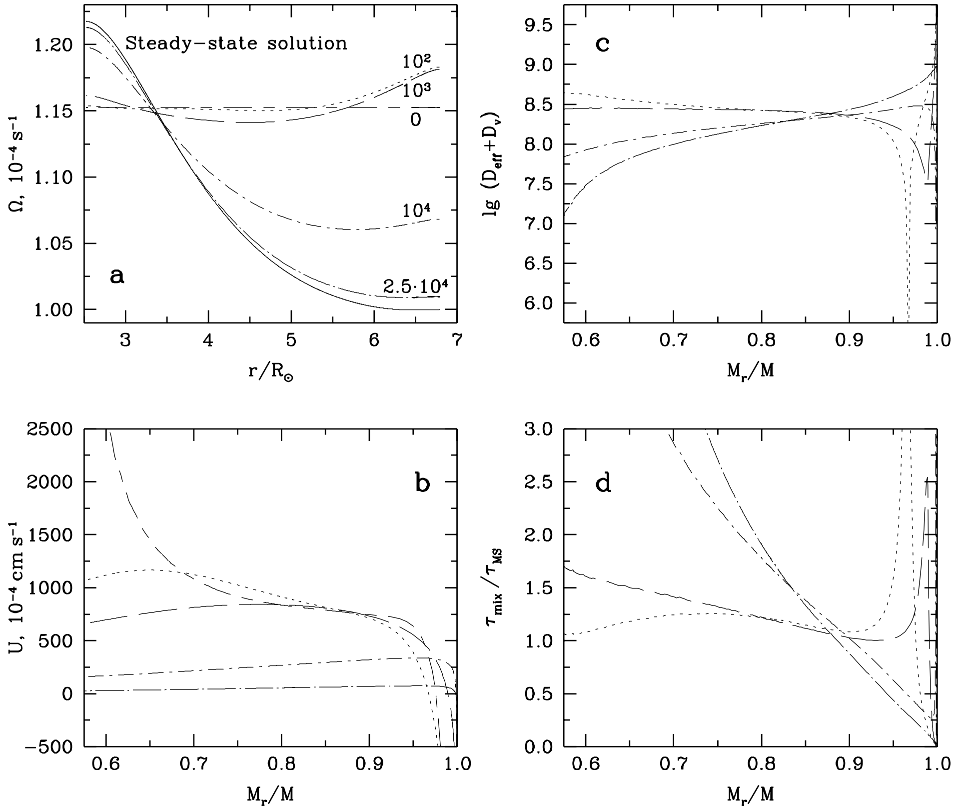

Fig. 1a–d. Approaching the steady-state rotation by the $30\,M_\odot$ star rotating with surface velocity $V_S = 470\,\mathrm{km\,s^{-1}}$ (corresponds to $\Omega_S = 10^{-4}\,\mathrm{s^{-1}}$). The asymptotic stationary solution is plotted with the solid line in panel **a**. Numbers along the lines in panel **a** give ages in years of the respective angular velocity profiles. Panels **b**, **c** and **d** show the evolution of the meridional circulation velocity, of the total diffusion coefficient and of the ratio of the time-scale for mixing chemical elements, defined as $\tau_{\rm mix} = (R-r)^2/(D_{\rm eff}+D_{\rm V})$, to the star's MS life-time, respectively. In all the panels lines of the same type have the same age. In this and in the next figures where the coordinate $r/R_\odot$ or M_r/M is used as abscissa it always starts on the left approximately from the convective core border or, more precisely, from the point where $r = r_{\rm C} + 0.1\,H_p$ (see text)

the stellar surface as the rotation profile evolves. As a result, the outer zone gradually (and, for a large enough value of Ω_S, rather quickly) becomes smaller in mass and then disappears completely. Simultaneously with these changes a characteristic value of U (say, in the middle of the envelope) decreases by a factor of about 50. As long as the rotation profile stays close to the initial flat one, $D_{\rm eff}$ (i.e. meridional circulation) remains the main contributor to the total diffusion coefficient. On the other hand, for $\Omega(r)$ approaching the asymptotic distribution, $D_{\rm V}$ (turbulent diffusion) begins to play a dominant role. Finally, the sum $D_{\rm eff}+D_{\rm V}$ is found to decrease near the convective core border by ~ 1.5 dex compared to the case of uniform rotation (Fig. 1c).

In Fig. 1d we have plotted the ratio $\tau_{\rm mix}/\tau_{\rm MS}$ against the relative mass coordinate, the mixing time being here defined as $\tau_{\rm mix} = (R-r)^2/(D_{\rm eff}+D_{\rm V})$. We see that even for the rotation profiles lying very close to the flat one, $\tau_{\rm mix}$ exceeds $\tau_{\rm MS}$ in the greater part of the inner envelope. This result strongly contrasts with the classical (Eddington-Sweet) estimate of the mixing time

$$\tau_{\rm ES} \approx \frac{\tau_{\rm KH}}{\eta_S},$$

where $\tau_{\rm KH} = GM^2/RL$ is the Kelvin-Helmholtz time. For our $30\,M_\odot$ ZAMS model $\tau_{\rm KH} \approx 3.4\cdot10^4$ years, so, for $\eta_S = 0.26$ and $\tau_{\rm MS} = 5.9\cdot10^6$ years we get $\tau_{\rm ES}/\tau_{\rm MS} \approx 0.02$! A main reason for such the large difference in the mixing time-scales (Zahn's scheme versus the classical description) is the horizontal erosion responsible for the factor $|rU|/30D_{\rm h}$ appearing in (8) which considerably reduces the product $|rU|$, the classical estimate of the mixing rate. It is this factor that causes a substantial diminution of $D_{\rm eff}$ even for the nearly uniform rotation.

It should be noted that in real evolving stellar models the advection and diffusion will never reach a stage where they exactly balance each other so as to make the time derivative in (4) equal to zero. Therefore, it would be more correctly to consider $\tau_{\rm rel}$ not as a time to achieve a stationary regime (which is never met) but instead as a characteristic time for the change of the inner rotation distribution.

In Fig. 2 the same parameter distributions as those shown in Fig. 1 are presented for the $10\,M_\odot$ model rotating with surface velocity $V_S = 230\,\mathrm{km\,s^{-1}}$ ($\Omega_S = 9\cdot10^{-5}\,\mathrm{s^{-1}}$). We have also performed calculations for $V_S = 460\,\mathrm{km\,s^{-1}}$ ($\Omega_S = 1.8\cdot10^{-4}\,\mathrm{s^{-1}}$).

A comparison of the numerical results for the two chosen M values and for the two different rotational velocities for the $10\,M_\odot$ model led us to the following rough estimate of the relaxation time:

$$\tau_{\rm rel} \approx R/U(\Omega_S) \approx \tau_{\rm ES} \propto \tau_{\rm KH}\Omega_S^{-2}\left(\frac{M^2}{R^3}\right). \qquad (18)$$

This time is much shorter than the mixing time $\tau_{\rm mix}$ defined above because the angular momentum transport by meridional circulation is not affected by the horizontal erosion (Eq. (4)). It is the difference in the rates of chemical mixing and of angular momentum transport that is considered as the main advantage of Zahn's new scheme, particularily, for the interpretation of a rather quick spin-down of the Sun and solar-like stars accompa-

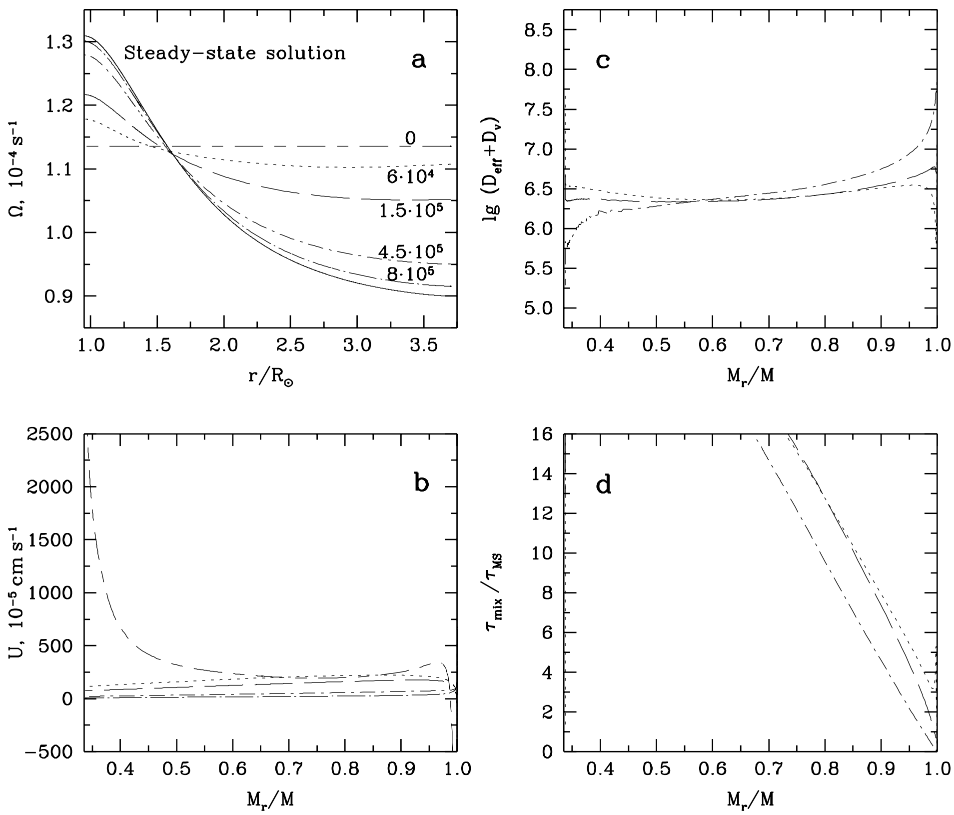

Fig. 2a–d. The same as in Fig. 1 but for the $10\,M_\odot$ star with $V_{\rm S} = 230\,{\rm km\,s^{-1}}$ ($\Omega_{\rm S} = 9 \cdot 10^{-5}\,{\rm s^{-1}}$)

nied by a much slower depletion of their surface Li abundance (Zahn 1997).

If we take the mass-radius relation of Beech & Mitalas (1994) $R \propto M^{0.52}$ valid for MS stars in the mass range $15 \le M/M_\odot \le 60$, then $M^2/R^3 \propto M^{0.44}$ and, consequently, in the first approximation $\tau_{\rm rel} \propto \tau_{\rm KH}\Omega_{\rm S}^{-2}$. This relation holds (within an uncertainty factor of $3 \div 4$) in our numerical calculations. For example, the $10\,M_\odot$ model has $\tau_{\rm KH} \approx 1.5 \cdot 10^5$ years (the Kelvin-Helmholtz time increases with decreasing M on the MS), and for $\Omega_{\rm S} = 1.8 \cdot 10^{-4}$ and $9 \cdot 10^{-5}\,{\rm s^{-1}}$ the calculations give $\tau_{\rm rel} \approx 3 \cdot 10^5$ and 10^6 years, respectively (compared to $\tau_{\rm KH} \approx 3.4 \cdot 10^4$ and $\tau_{\rm rel} \approx 5 \cdot 10^4$ years for the $30\,M_\odot$ model rotating with $\Omega_{\rm S} = 10^{-4}\,{\rm s^{-1}}$).

The meridional circulation velocity and the total diffusion coefficient are found to scale approximately as $\Omega_{\rm S}^2$, for example, for $\Omega_{\rm S} = 1.8 \cdot 10^{-4}\,{\rm s^{-1}}$ distributions of $\lg(D_{\rm eff} + D_{\rm v})$ versus M_r/M go about 0.6 ($\approx \lg 4$) above the curves plotted in Fig. 2c for $\Omega_{\rm S} = 9 \cdot 10^{-5}\,{\rm s^{-1}}$.

From Figs. 2c and 2d we infer that there is no a big difference (if one does not consider very deep layers adjacent to the convective core) between a state of rotation close to the uniform one and that approaching the asymptotic regime with respect to their ability (or, better to say, disability) to mix chemical elements. In particular, complete mixing of the radiative envelope is reached in neither case. Therefore, searching for an additional angular momentum transport mechanism (for instance, internal gravity waves; Zahn et al. 1997; see also the next section), which could support a state of nearly uniform rotation will hardly help to speed up mixing of chemical elements unless this new mechanism can effectively mix them itself. But it is important to note that even in the most unfavourable considered case of $M = 10\,M_\odot$, $\Omega_{\rm S} = 9 \cdot 10^{-5}\,{\rm s^{-1}}$ and $t = 8 \cdot 10^5$ years the turbulent diffusion (we have mentioned above that for rotation close to the steady-state one $D_{\rm eff}$ is much smaller than $D_{\rm v}$) turns out to be fast enough for *some* mixing, as diffusion-like abundance profiles in the envelope are built up and the surface abundance of ^{14}N is altered significantly by the end of the star's MS life (Fig. 3). To plot Fig. 3 we solved Eqs. (7) by the method and with the input physics described in Denissenkov et al. (1998, the nuclear kinetics network for 26 nuclides) for the fixed temperature and density distributions taken from our present ZAMS models. Note that the surface N abundance has begun to increase only after some delay time required for the diffusion wave to reach the surface (Fig. 3b, see also Talon et al. 1997).

Fig. 4 demonstrates that shortly after the rotation profile has begun to evolve, all the consistency criteria (14–17) are fulfilled, and only during the very early evolution we meet a situation when $K \lesssim D_{\rm h}$, but we ignored that.

5. Internal gravity waves

Another mechanism probably able to contribute to the angular momentum transport in stellar radiative zones is one associated with internal gravity waves (hereafter, IGWs) (Schatzman 1993; Zahn et al. 1997; Ringot 1998). In a single star the IGWs can be generated by turbulent motions of convective eddies (Press 1981; García López & Spruit 1991). They carry angular mo-

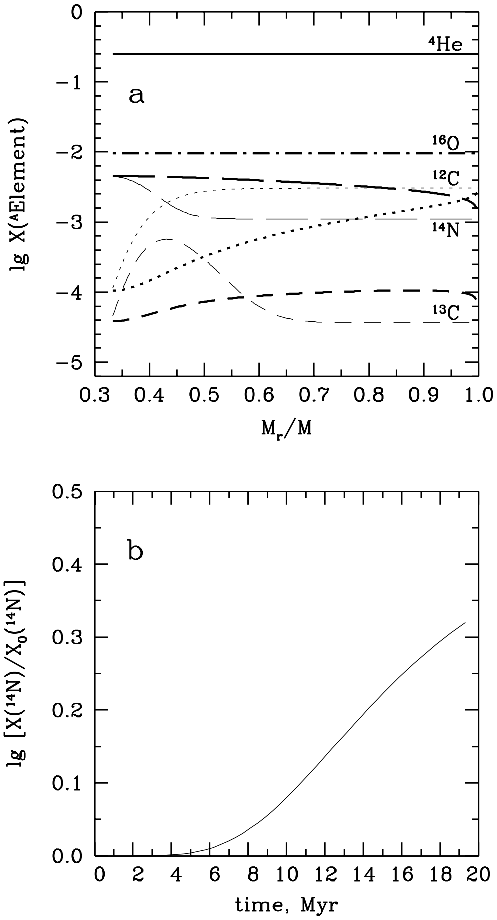

Fig. 3a and b. Panel **a**: Abundance distributions ($X_i = y_i A_i$, A_i is the atomic mass number) of the main CNO nuclides and of He in the radiative envelope of the $10\,M_\odot$ star by the end of its MS life are shown for two cases: without mixing (thin lines) and with mixing by turbulent diffusion (the meridional circulation plays an unimportant role here) induced by rotation with $V_{\rm S} = 230\,{\rm km\,s^{-1}}$ (thick lines). The diffusion coefficient profile for the calculations with mixing was taken from the nonstationary solution at $t = 8 \cdot 10^5$ years (Fig. 2a). Panel **b**: Increase of the surface N abundance with time in the calculations with mixing. The abundance of N begins to grow after some delay time which is required for the diffusion wave to reach the surface

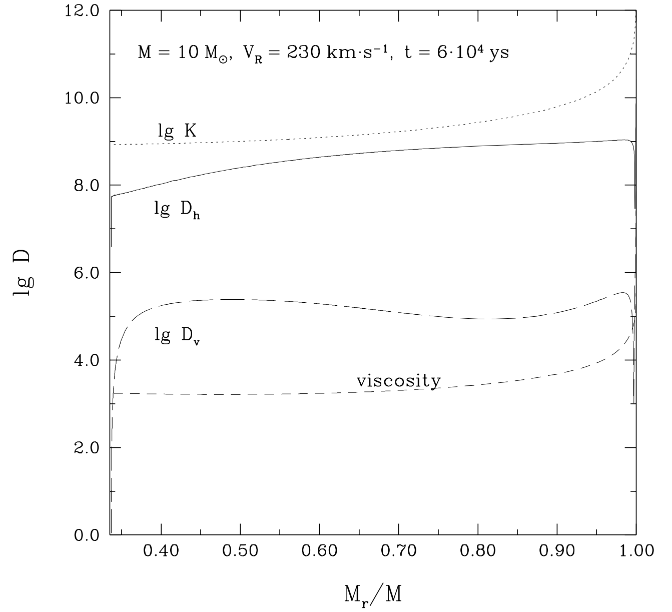

Fig. 4. An illustration of the fact that the consistency criteria (14–17) are fulfilled shortly after evolution of the rotation profile has begun

mentum through a radiative zone and deposit it locally at a place where some special conditions are met. Recently, Zahn et al. (1997) have proposed that the IGWs can strongly influence the evolution of the Ω-profile in the Sun (see, however, critical comments of Ringot (1998)). If a state of differential rotation is sustained in the Sun, by, for instance, meridional circulation, and Ω increases with depth, then a wave generated near the base of the solar convective envelope with a frequency ω will experience on its way inwards a Doppler frequency shifting with a resulting local frequency changing with depth as

$$\sigma(r) = \omega - m[\Omega(r) - \Omega_{\rm C}],$$

where $\Omega_{\rm C}$ is the angular velocity of the convective envelope ($\Omega_{\rm C} \le \Omega(r)$) and m the wave's azimuthal order. The special condition mentioned above is $\sigma(r) = 0$ which can be met at some radius $r^\star$ for a sufficiently large and positive mode number m. At $r = r^\star$ the IGW deposit some negative angular momentum to the local medium. In fact, the contribution of the IGWs to the angular momentum transport cannot be described either as pure diffusion or advection. It is a much more intricate process which requires the presence of asymmetry in propagation or dissipation of prograde (positive m) and retrograde (negative m) waves. Differential rotation and the Coriolis acceleration can produce such the asymmetries. For further physical and mathematical details the reader is refered to the paper of Zahn et al. cited above.

We have applied Zahn et al.'s results on the IGWs to the case of massive MS stars. The main qualitative differences as compared to the case of the Sun are the following: the IGWs are now generated near the convective core border, propagate outwards and carry positive angular momentum causing spinning-up of the radiative envelope.

In Fig. 5a approaching the steady-state rotation by our $10\,M_\odot$ model rotating with $\Omega_{\rm S} = 9 \cdot 10^{-5}\,{\rm s}^{-1}$ is shown in the diagram $\lg(\Delta\Omega/\omega_{\rm C})$ vs. $\lg(I/\omega_{\rm C}^4)$ where $\Delta\Omega = \Omega_{\rm C} - \Omega(r)$, $\omega_{\rm C}$ is the convective turnover frequency and

$$I = \int_{r_{\rm c}}^{r} K N_T^3 \frac{dr}{r^3}$$

a "damping" integral, the index "c" now refering to the convective core. Frequencies of the IGWs near the convective core border must lie between $\omega_{\rm C}$ and $N_{\rm C}$, the latter quantity being the Brunt-Väisälä frequency at the level where the IGWs are generated. On their way outwards the IGWs lose energy due to radiative leakage. The damping integral I appears in an approximate formula describing these losses. The diagram in Fig. 5a is similar to that plotted in Fig. 5 in Zahn et al. (1997). The solid and short-dashed straight lines divide it into seven domains where

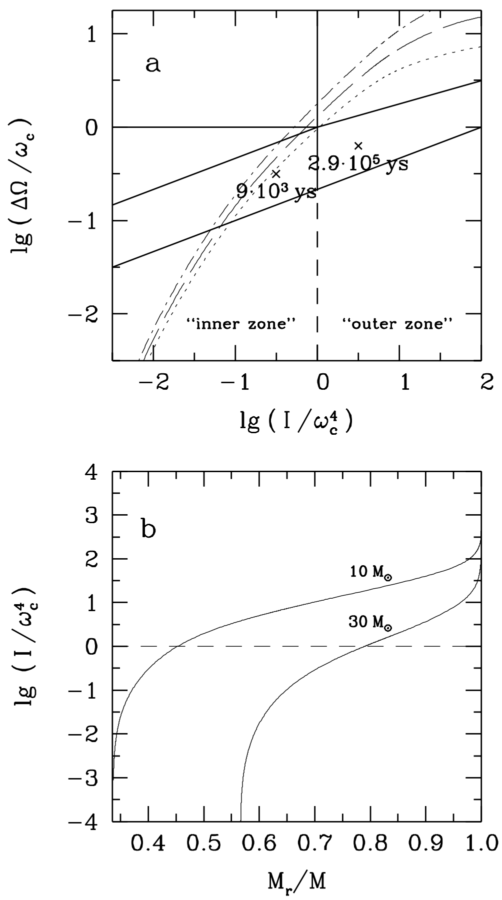

Fig. 5a and b. Panel **a**: Evolution of the rotation state of the 10 $M_\odot$ star with $V_S = 230\,\mathrm{km\,s^{-1}}$ in the diagram of Zahn et al. (1997, Fig. 5). Solid and dashed straight lines divide the diagram into seven domains where different approximate formulae for estimating the angular momentum flux carried by the internal gravity waves are applied in Zahn et al. (1997). Crosses are arbitrarily chosen points in the radiative envelope. Numbers near the crosses give the times required for the IGW flux to reach these points. Curves in panel **b** transform $\lg(I/\omega_c^4)$ into the usual relative mass coordinate

different approximate relations are used to estimate an angular momentum flux carried by the IGWs (formulae (A3), (A6), (A8–A11) in Zahn et al. (1997)). We used those relations to calculate characteristic times $\tau_{\rm IGWs}$ required for the angular momentum flux associated with the IGWs to reach two arbitrarily chosen points marked with crosses in Fig. 5a. The respective values of $\tau_{\rm IGWs}$ are given near the crosses. Fig. 5b transforms the coordinate $\lg(I/\omega_c^4)$ into the usual relative mass coordinate. We see that in the "inner zone" of the radiative envelope, defined here as a region where $I/\omega_c^4 < 1$, $\tau_{\rm IGWs} \ll \tau_{\rm rel} \sim 10^6$ years, whereas in the "outer zone", with $I/\omega_c^4 > 1$, $\tau_{\rm IGWs} \sim \tau_{\rm rel}$. It should be noted that $\tau_{\rm IGWs} \propto I/\omega_c^4$ (Zahn et al. 1997). Therefore, we infer that the "inner zone" is a part of the envelope where the angular momentum transport by the IGWs may dominate over that by the meridional circulation and turbulent diffusion.

In the considered scenario the IGWs can participate in redistributing the angular momentum until $\Delta\Omega \neq 0$ (following Zahn et al. (1997) we ignored the Coriolis acceleration) and, therefore, the IGWs tend towards establishing a state of uniform rotation. Thus, our analysis shows that the "inner zone" of the radiative envelope of a massive MS star may be a region where a state of nearly constant Ω is sustained by the IGWs, the size of this region increasing with stellar mass (Fig. 5b). However, as was explained in the preceding section, this cannot considerably accelerate mixing of chemical elements because of the efficient horizontal erosion.

A probably even more important contribution of the IGWs to the mixing of the massive MS stars could result from their non-linear behaviour and various hydrodynamical instabilities associated with it (Press 1981; García López & Spruit 1991). Unfortunately, this problem demanding extensive numerical 3D-simulations is still far from being solved.

6. Conclusion

We have calculated the evolution of the rotation profile accompanying the angular momentum transport by meridional circulation and turbulent diffusion in the radiative envelopes of MS model stars of 10 and $30\,M_\odot$. In these calculations we have closely followed the theoretical scheme of assumptions and simplifications proposed for the first time by Zahn (1992) and elaborated upon later by Talon & Zahn (1997).

It should be noted that all our numerical results do not account for the gradient of the mean molecular weight. At the same time, as shown by expression (5), in a medium with a non-zero μ-gradient $D_{\rm V}$ can be reduced considerably. The difference in $D_{\rm V}$ with respect to the case of constant μ considered in the paper may amount to one or two orders of a magnitude. Thus the values of $\tau_{\rm mix}$ plotted in Figs. 1d and 2d may be greatly underestimated, especially near the convective core border. However, here we meet an apparent disagreement of the current theory with the observations of the He overabundances in O-B stars showing that in real stars additional mixing succeeds in overcoming the μ-gradient barrier. Unfortunately, this disagreement remains unexplained as yet.

One of our simplifications has been ignoring any mass loss by the stars. We consider this simplification as an unavoidable one at this stage of the analysis because it is still not clear which outer boundary conditions for Eq. (4) one should use in the presence of a stellar wind. Available semi-empirical formulae for the mass loss rates on the MS allow one to calculate only the angular momentum loss rate, i.e. actually give an outer boundary condition for the integral of Eq. (4). Bearing in mind that in the absence of a strong magnetic field an extended envelope of a mass losing massive MS star is most likely to possess a differential rotation it remains unclear which part of the angular momentum being lost is transfered to the stellar atmosphere by the meridional circulation and which one by the turbulent diffusion.

The situation becomes even more complicated if one wants to take into account the possibility that massive MS stars (presumably, those with $M \gtrsim 30\,M_\odot$) spend a considerable part of their MS life as objects embedded into a protostellar cocoon and thus acrete material instead of losing it (Beech & Mitalas 1994; Bernasconi & Maeder 1996).

On the basis of the results of calculations presented in the paper we have come to the following conclusions:

(*i*) the relaxation time $\tau_{\rm rel} \sim \tau_{\rm KH}\Omega_{\rm S}^{-2}$ required for a massive MS star to arrive at the state of stationary rotation is much shorter than the star's MS life-time $\tau_{\rm MS}$ for large enough values of $\Omega_{\rm S}$ which can be estimated *a priori* provided the quantities L, R and $\tau_{\rm MS}$ are known for an appropriate stellar model; if $\tau_{\rm rel} \ll \tau_{\rm MS}$ the assumption of the star being in the state of stationary rotation at every moment from the beginning of its MS evolution used by Talon et al. (1997) is quite reasonable and justified by our calculations; on the other hand, for sufficiently low values of $\Omega_{\rm S}$ the above assumption is no longer correct and we have to follow the evolution of the angular velocity profile by solving Eq. (4) simultaneously with the stellar evolution calculations;

(*ii*) qualitatively, the nonstationary solutions do not greatly differ from the stationary one in their ability to mix chemical elements; even for nearly uniform rotation the rate of mixing (in this case mainly sustained by the meridional circulation) is found to be much lower than the classical estimate $|rU|$, the reason for this being the effective horizontal erosion by the turbulent diffusion; no complete mixing of the envelope is possible;

(*iii*) despite of the reduced mixing rate, for a sufficiently large $\Omega_{\rm S}$ the turbulent diffusion (which becomes a dominant mixing mechanism for rotation close to the steady-state one) succeeds to build-up diffusion-like abundance profiles in the radiative envelope of a massive MS star before it leaves the MS; the surface abundances begin to decline from the initial ones after some delay time which is required for the diffusion wave to reach the stellar surface; recently, Lyubimkov (1996) has reported a similar delay in the appearance of He overabundances in the atmospheres of OB-stars; if additional mixing penetrates the convective core then the evolution of the surface He abundance may look like that of N shown in Fig. 3b;

(*iv*) internal gravity waves generated by convective eddies near the convective core border can successfully compete with the meridional circulation and turbulent diffusion in redistributing the angular momentum, especially in the inner part of the radiative envelope; the IGWs tend towards establishing a state of uniform rotation, however, this does not help to accelerate the mixing of chemical elements considerably (see point (*ii*) above); nonlinear effects associated with the IGWs may also initiate mixing of chemical elements in stellar radiative zones (Press 1981; García López & Spruit 1991); we are going to discuss some of these effects in a forthcoming paper.

We finish these conclusions without carrying out any detailed comparison of theory with observations because, from the one hand, the mixing mechanisms considered in the paper need further elaboration (cf. Maeder & Zahn 1998; Ringot 1998) before they can be incorporated as input physics into stellar evolution codes. On the other hand, observational data on the abundance anomalies in massive MS stars are still not definite enough to constrain a particular mixing mechanism (Lyubimkov 1996). Work in both directions is encouraged.

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