

# On the possibility that rotation causes latitudinal abundance variations in stars

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**Abstract.** The effect of rotation of a star on the distribution of chemical species in radiative zones is discussed. Gravity darkening generates a large radiative force on heavy element ions which is directed toward the equatorial plane. Taking iron as an example, it is shown that this force may produce drift velocities similar to, and larger than, the typical velocities of bulk motion due to meridional circulation. This potentially allows large chemical abundance inhomogeneities to build up across a meridian over the lifetime of the star – particularly near the equatorial plane. This enhancement may be significantly reduced if the mass loss of the star is strongly metallicity dependent, in which case the mass-loss rate may be enhanced in the equatorial plane.

**Key words:** diffusion – stars: abundances – stars: evolution – stars: interiors – stars: rotation

## 1. Introduction

Abundance anomalies are observed in many different types of star – the photospheres of stars with effective temperatures hotter than  $\sim 6000\text{K}$  may have undergone chemical separation (Michaud 1970). It is generally accepted that this may be explained via the force on atoms by radiation: if this is larger than gravity, then the element will diffuse toward the surface, else it will settle. The force due to radiation pressure on elements in a star has been discussed at length by e.g. Michaud et al. (1976) Vauclair et al. (1978), Aleican & Artru (1990) and Gonzalez et al. (1995). Abundance anomalies have been discussed in a variety of chemically peculiar stars eg. Am and Fm stars (Charbonneau & Michaud 1991, Alecian 1996),  $\lambda$  Bootis stars (Michaud & Charland 1986) and magnetic Ap-Bp stars (Aleician & Vauclair, 1981, Michaud et al. 1981). When the star is also rotating, the surface abundance anomalies generated by radiation pressure may be destroyed by the action of turbulent motions within the star (Charbonneau 1992, Talon & Zahn 1997). These arise from shear motions created by the meridional motions which arise due to gravity darkening (see eg. von Zeipel 1924, Tassoul & Tassoul 1982, Zahn 1992)

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There is some observational evidence for *latitudinal* abundance variations of heavy elements in planetary nebulae (Balick et al. 1994). This is possibly explained by more highly processed material being ejected faster but at later times than the rest of the shell. However, the observations raise an interesting question: it is possible that there are stars which are chemically inhomogeneous in both radial and latitudinal directions?

Here a non-magnetic mechanism which generates latitudinal abundance variations is suggested and investigated. It is found that a significant latitudinal radiative acceleration on heavy elements exists in rotating stars due to gravity darkening (von Zeipel 1924), which causes ionic species to diffuse toward the equatorial plane. Competing with this equatorward drift are the meridional circulation currents, and turbulence generated via shear motions. The aim of this paper is solely to assess whether any latitudinal drift of heavy elements may occur in rotating stars.

The transport of chemicals is discussed in Sect. 2. In Sect. 3 the radiative force due to gravity darkening is calculated and in Sect. 4 the drift velocity of metals around an equipotential is derived. Numerical results of the diffusion velocity in stellar envelopes are presented in Sect. 5, and by considering the relevant timescales, Sect. 6 identifies the important regions within a star for diffusion to occur. A discussion is presented in Sect. 7, and conclusions given in Sect. 8.

## 2. Transport of chemicals

The conservation equation for a trace element of concentration  $c$  is

$$\rho \frac{\partial c}{\partial t} + \nabla \cdot \{-\rho \mathbf{D} \cdot \nabla c + \rho \mathbf{u} c + \rho \mathbf{v} c\} = 0 \quad (1)$$

where  $\mathbf{D}$  is the diffusivity tensor,  $\mathbf{u}$  is the global motion of the fluid (in this case the meridional motions generated via the rotation of the star), and  $\mathbf{v}$  describes any motion affecting only the contaminant. The concentration  $c$  is defined as the number density of ions compared to the number density of protons. Hereafter, the star is assumed to be axisymmetric, so that using a spherical coordinate system, for any  $X$ ,  $dX/d\phi = 0$ . Also, for rotating stars, the equipotential is not defined by  $r = \text{constant}$ . Therefore the “horizontal” direction is taken to be perpendicular

to the equipotential, and the “vertical” direction to be parallel to equipotential.

Following the work of Chaboyer & Zahn (1992) and Zahn (1992), Charbonneau (1992) examined the effect of anisotropic turbulent diffusion on the distribution of  $c(r, \theta)$  (Zahn 1975, Tassoul & Tassoul 1983, Zahn 1987), and found that if the horizontal Reynold number is  $R_H < 1$  then the equation above loses its bi-dimensional behaviour, and becomes one dimensional ie.  $c(r, \theta) \rightarrow c(r)$ . Essentially the horizontal turbulence becomes vigorous enough that it may redistribute the concentration of the trace element faster than meridional motions can generate an anisotropy. This also makes the angular momentum independent of  $\theta$  and gives rise to “shellular” rotation of the star (Zahn 1992).

In the current study, however, the velocity field  $\mathbf{v}$  may be non-zero due to the meridional forces on trace elements arising from the anisotropy of the radiation flux (see Sect. 2). To assess its likely effect in the presence of bulk meridional motions and diffusion, the ratio of the (turbulent) diffusive term to the radiation driven advective term across an equipotential  $s$  in Eq. 1 is examined:

$$\mathcal{R} = \frac{\nabla(\rho \mathbf{D} \cdot \nabla c)}{\nabla(\rho \mathbf{v} c)} \Big|_s \sim \left[ \frac{D_H}{v_\theta} \right] \frac{1}{r} h(c) \quad (2)$$

where  $D_H$  is the horizontal diffusion coefficient, the extra velocity component is  $\mathbf{v} = (0, v_s, 0)$  and  $h(c)$  is a function containing logarithmic derivatives of the concentration with respect to  $\theta$ . This is only an approximation as the unit vectors along an equipotential  $\hat{s}$  and  $\hat{\theta}$  are not parallel. However, this expression will suffice for the current study. Also it should be noted that there is no term in the angular momentum evolution equation equivalent to the radiation term  $\nabla(\rho \mathbf{v} c)$  in Eq. 1. Therefore, shellular rotation will still be achieved in the star, although the distribution of the trace element may be anisotropic.

The discussion of turbulent diffusion coefficients  $D_H$  has a long history, and the simple approximation due to Chaboyer & Zahn (1992) is adopted here:

$$D_H = \frac{|r u_r|}{C_H} \quad (3)$$

where  $C_H$  is a number of order unity and  $u_r$  is the radial component of the velocity of the bulk meridional circulation. With this replacement Eq. 2 becomes

$$\mathcal{R} \sim \left[ \frac{u_r}{v_s C_H} \right] h(c) \quad (4)$$

The main controlling parameter here is the ratio of the radial component  $u_r$  of the bulk meridional flow and the velocity  $v_s$  of the trace element due to the radiation as it slips through the plasma.

If  $\mathcal{R} > 1$  then the turbulent diffusion term will dominate the movement of trace elements, and so the star will become chemically homogeneous on equipotentials. However, if  $\mathcal{R} < 1$  then the distribution of the elements is determined by the extra drift velocity  $v_\theta$ . The criterion adopted in this paper for chemically inhomogeneous stars is that the extra drift velocity

$v_s$  is larger than the typical velocity of meridional motions  $u_r$ . In a recent numerical study of the a  $20M_\odot$  star, Urpin et al. (1996) found that for high rotational velocities, the circulation velocity is less than  $3 \times 10^{-5} \text{ cm s}^{-1}$ . This value is significantly lower than that from Eddington-Sweet theory, which fails close to the surface of the star. If velocities of  $v_s \gtrsim 10^{-5} \text{ cm s}^{-1}$  are produced due to the effects of rotation, then the star may develop metal-rich and metal-poor regions on the same equipotential.

### 3. Radiation forces

#### 3.1. Flux around the star

Assuming that the potential of the outer parts of a rotating star may be represented using the Roche approximation, the radius of equipotential surfaces  $r$  is dependent on the angle to the pole  $\theta$ , and is found by the solution of

$$\frac{r(\theta)}{r_{\text{pole}}} - \frac{\omega^2}{2GM} \sin^2 \theta \left( \frac{r(\theta)}{r_{\text{pole}}} \right)^3 = 1, \quad (5)$$

where  $\omega$  is the angular velocity,  $r_{\text{pole}}$  is the radius in the polar direction, and  $M$  is the mass contained within the equipotential surface. This has been solved for  $r(\theta)$  geometrically by Collins & Harrington (1966). Deep within the star the Roche approximation will be incorrect, although for the present discussion, it is adequate. Indeed the region in which it is possible to generate latitudinal metallic abundance anisotropies is found, *a posteriori*, to be the outer parts of the envelope and so  $M \approx M_*$ , and Eq. 5 will be a good representation.

The flux of the star  $f$  is proportional to the local gravity (von Zeipel 1924):

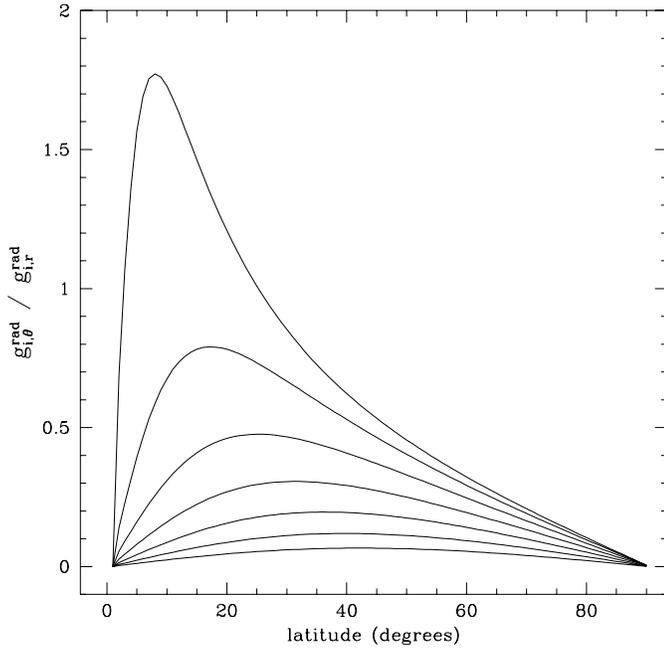
$$f \propto \nabla \left( -\frac{GM}{r(\theta)} - \frac{\omega^2 r(\theta)^2 \sin^2 \theta}{2} \right). \quad (6)$$

Along the surface of any equipotential the local flux  $f$  is higher at the pole than the equator (gravity darkening). This difference between pole and equator is the root cause of the latitudinal variation of the metallic abundance. It is noted that this gravity darkening actually causes the meridional circulation, else energy would not be conserved in the star. However, the change in flux is not destroyed by the motions.

#### 3.2. Radiation force on a trace element

The force on ions due to radiation  $g_i^{\text{rad}}$  is dependent on the gradient of the flux passing through that point, i.e.  $g_i^{\text{rad}} \propto \nabla f$ . To facilitate the calculation of the force along an equipotential the ratio of the forces in the radial and latitudinal directions is now calculated. In the outer parts of the star, the luminosity is a constant, and hence the flux varies with distance  $r$  from the centre of the star as  $f \propto r^{-2}$ . As the radiative force is proportional to  $\nabla f$ , then the ratio of the latitudinal to radial components is

$$\frac{g_{i,s}^{\text{rad}}}{g_{i,r}^{\text{rad}}} \approx \left( \frac{df}{ds} \right) \left( \frac{df}{dr} \right)^{-1} = -\frac{r}{2} \frac{d \ln f}{ds} \quad (7)$$



**Fig. 1.** Ratio of latitudinal to radial radiative force defined from Eq. 4. The lines correspond to a rotation of 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 of break-up velocity, where the uppermost line corresponds to the highest rotation, decreasing downwards.

where  $s$  is the distance along an equipotential. Again, note this is only an approximation as the unit vectors  $\hat{s}$  and  $\hat{r}$  are not orthogonal. This ratio is shown in Fig. 1 for different rotation rates. As can be seen it is typically  $0.1 \rightarrow 1$ , and therefore, perhaps surprisingly, the radiative force around an equipotential is typically similar to the radial radiative acceleration.

Michaud et al. (1976) see also Alecian & Artru (1990) have discussed the radiative force due to lines on heavy elements. Their expressions, along with Eq. 7, lead to the latitudinal force on a trace element (with a negligible gradient in the concentration of the ions) of

$$g_{i,s}^{\text{rad}}(c \rightarrow 0) = -5.574 \times 10^7 \frac{T_{e4}^4}{A_i T_4} \left( \frac{R_*}{r} \right)^2 \Phi_i \left( \frac{r}{2} \frac{d \ln f}{ds} \right) \quad (8)$$

where  $T_4$  and  $T_{e4}$  are the temperature and effective temperature in units of  $10^4 \text{K}$  respectively,  $r$  is the radius and  $R_*$  is the stellar radius. The atomic parameters are enveloped in  $\Phi_i$ , and  $A_i$  is the atomic mass of the element  $i$ .

With the inclusion of a non-zero concentration of ions the driving lines become saturated, and the line force is reduced from Eq. 8:

$$g_{i,s}^{\text{rad}}(c \neq 0) = g_{i,s}^{\text{rad}}(c \rightarrow 0) \left( \frac{1}{1 + \frac{c}{c_0}} \right)^{1/2} \quad (9)$$

(Alecian 1985). The reference concentration  $c_0$  is

$$c_0 = 9.83 \times 10^{-23} \bar{\kappa} n_e T^{-1/2} \Psi_i^2 \quad (10)$$

where  $\bar{\kappa}$  is the Rosseland mean opacity,  $n_e$  is the number density of electrons and  $\Psi_i$  is an atomic parameter containing line broadening effects.

#### 4. Drift velocity

To calculate a typical order of magnitude of the drift velocity  $\mathbf{v}$  between the ions and rest of the plasma, the radiation force will be approximately balanced by the retarding frictional force. The diffusion velocity is

$$\mathbf{v} = D_{12} \left( -\nabla \ln c + A_i \frac{m_p \mathbf{g}_i^{\text{rad}}}{kT} + (1 - 2A_i + Z_i) \frac{m_p \mathbf{g}}{kT} - \alpha_T \nabla \ln T \right). \quad (11)$$

Here,  $m_p$  is the mass of a proton,  $k$  is Boltzmann's constant,  $Z_i$  is the charge of the element  $i$  in units of the fundamental charge  $e$ . The thermal diffusion coefficient is  $\alpha_T$  and the microscopic diffusion coefficient is  $D_{12}$ . The effective gravity is zero in the  $\hat{s}$  direction (from the definition of the equipotential), and so the third term in parentheses above is zero. Also, the thermal diffusion term is typically small compared to the radiative term and so the diffusion velocity is determined only by the radiative forces, and the concentration gradient.

If the mass of species 1 (the stellar plasma - mostly hydrogen and helium) is neglected compared to the mass of species 2 (the diffusing ions), and ion shielding is neglected in the computation of the Debye length, Eq. 40 of Paquette et al. (1986) becomes

$$D_{12} = \frac{3(2kT)^{5/2} m_p^{1/2}}{16\rho\pi^{1/2} Z_i^2 e^4 \Lambda_i} \quad (12)$$

where

$$\Lambda_i = \ln \left[ 1 + \frac{16(kT)^3 m_p}{4\pi Z_i^2 e^6 \rho} \right] \quad (13)$$

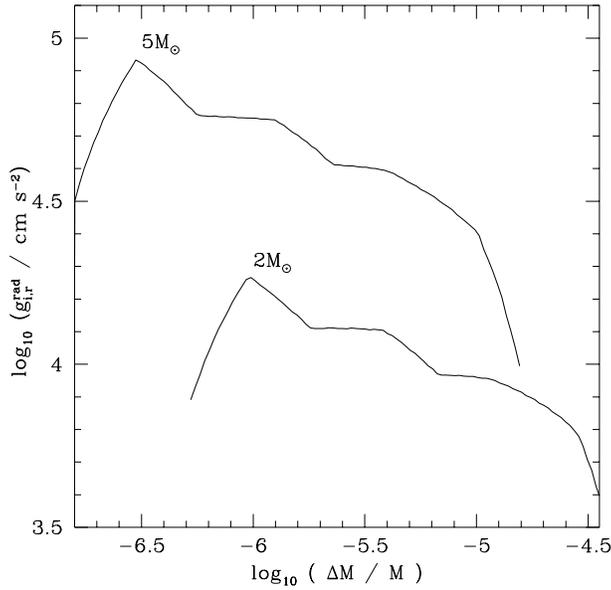
where  $\rho$  is the density. To obtain a simple expression for the diffusion velocity, an initially chemically homogeneous star is assumed, so that  $\nabla c = 0$  and Eq. 9 are good representations. The diffusion velocity is then

$$v_s = -2.2 \times 10^{-9} \frac{T_4^{1/2} T_{e4}^4}{\rho \left( 1 + \frac{c}{c_0} \right)^{1/2}} \left( \frac{R_*}{r} \right)^2 \frac{\Phi_i}{Z_i^2 \Lambda_i} \left( \frac{r}{2} \frac{d \ln f}{ds} \right) \quad (14)$$

This expression will overestimate the drift velocity as soon as a concentration gradient is established, as then  $\nabla c \neq 0$  in Eq. 11. However, with these points in mind, Eq. 14 is used as an estimate for the velocity in Eq. 4.

#### 5. Numerical estimates of the diffusion velocity

Here the typical drift velocity is calculated for two model stars. The stellar models have been provided by M. Salaris (private communication) and are computed according to the evolution code described in Salaris et al. (1997) and references therein. The two models correspond to  $(2.0M_\odot, 1.8R_\odot, T_{\text{eff}} = 8700\text{K})$  and  $(5.0M_\odot, 3.0R_\odot, T_{\text{eff}} = 16500\text{K})$  main sequence stars. These models are computed with no rotation and solar composition. Although below the models are used to represent non-solar composition rotating stars, it is unlikely that this slight inconsistency will introduce a large error.



**Fig. 2.** Numerical estimates of the radial radiative acceleration  $g_{i,r}^{\text{rad}}$  for the two model stars.

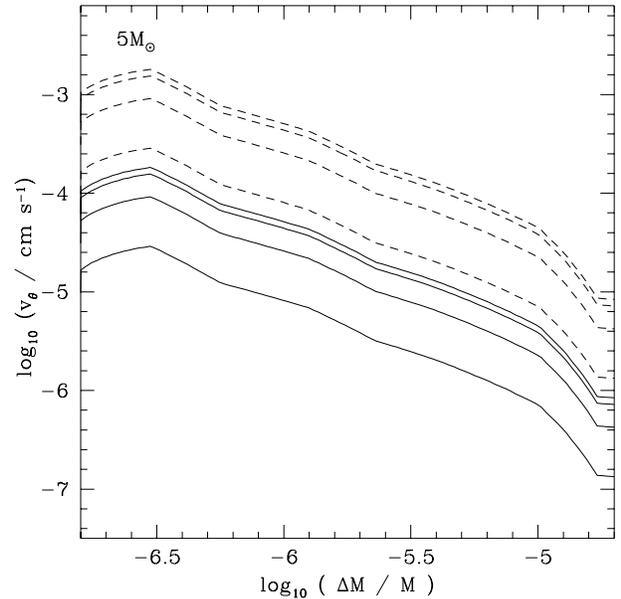
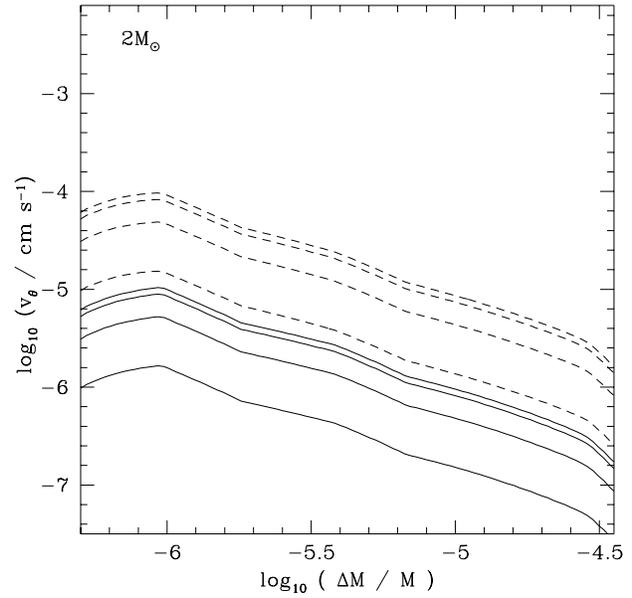
The calculation of the atomic parameters  $\Phi_i$  and  $\Psi_i$  are lengthy and cumbersome, and so the parameters given by Alecian et al. (1993) for iron are assumed (their Table 4), derived using Opacity Project data. These are for the states FeIX–FeXVII. For a given temperature in the model stars, the atomic parameters used are interpolated between those for the dominant, and next dominant ions (the temperatures at which ions are dominant is given in Eq. 3 of Alecian et al. 1993). These temperatures also give the applicable range used in the calculation:  $1.95 \times 10^4 K < T < 6.0 \times 10^5 K$  to cover all the ions used.

Although the models are calculated with solar metallicity, the drift velocity  $v_s$  is calculated for two values of iron abundance: solar abundance  $c = 3 \times 10^{-5}$  and  $10^{-2}$  solar. Although strictly this is inconsistent, the error associated in doing so will be very small.

In Fig. 2, the *radial* radiative acceleration is shown for the two stars with solar iron abundance, and should be compared with Fig. 4 of Alecian et al. (1993). It can be seen that the interpolation used here to calculate the atomic parameters is not introducing any large errors in the calculation.

Fig. 3 shows the value of  $v_s$  as a function of the fractional mass exterior to that point  $\Delta M/M$  for the two models. The stars is assumed to rotate at 0.7 of their break up velocities (0.89 of its break-up angular velocities), equal to  $v_{\text{rot}} = 260 \text{ km s}^{-1}$  and  $320 \text{ km s}^{-1}$  respectively for the  $2M_{\odot}$  and  $5M_{\odot}$  stars. The top panel of Fig. 3 corresponds to the  $5M_{\odot}$  star, and the lower to the  $2M_{\odot}$  star. In each panel, the lines refer to  $\theta = 10^\circ, 30^\circ, 50^\circ$  and  $70^\circ$  with the uppermost line corresponding to  $70^\circ$  decreasing downwards. The solid and dotted lines correspond to iron abundance of solar and  $10^{-2}$  solar respectively.

In three of the four models, the drift velocity exceeds  $10^{-5} \text{ cm s}^{-1}$  in the outer layers at some latitude. This indicates that there may be regions in the envelope where iron may start to diffuse round the star. For the  $5M_{\odot}$  star the outer  $10^{-5.5}$



**Fig. 3.** Numerical estimates of  $v_s$  from Eq. 14. The four lines in each panel refer to velocities at  $\theta = 10^\circ, 30^\circ, 50^\circ$  and  $70^\circ$  with the uppermost line corresponding to  $70^\circ$  decreasing downwards. The solid (dashed) lines correspond to solar ( $10^{-2}$  solar) abundance of iron.

( $10^{-4.7}$ ) of the solar ( $10^{-2}$  solar) composition model may suffer latitudinal drift of iron. For the  $2M_{\odot}$  star, at  $10^{-2}$  solar composition the situation is similar –  $v_s \gtrsim 10^{-5} \text{ cm s}^{-1}$  in the outer  $10^{-5}$  of the star. However for solar composition, there may be no effect as the radiative acceleration on iron is smaller. In this case the turbulence may homogenise the star faster than the metals may drift due to the radiation. Taken at face value, Fig. 3 indicates that stars rotating at significant fractions of their break-up velocity may be prone to latitudinal metallic drift in their outer envelopes.

## 6. Timescales

Is it likely that a star may come into equilibrium? This clearly depends on the timescales over which the relevant processes act. Here the important timescales are (i) the diffusion time (time taken for the ions to diffuse round that star), (ii) the main-sequence lifetime of the star, (iii) the lifetime of each layer (if mass-loss is present).

### 6.1. Diffusion, stellar and mass-loss timescales

For an asymmetry to occur in the elemental distribution along an equipotential, the metals must diffuse a significant distance from their initial latitude to the equator. This therefore defines a drift timescale  $\tau_D$ , dependent on latitude  $\pi/2 - \theta$ , which is that over which a latitudinal asymmetry in the metals will be generated:

$$\tau_D(\theta) \sim \frac{R_* (\pi/2 - \theta)}{v_s} \quad (15)$$

The star only has a finite time in which to allow metals diffuse around its equipotentials. The main sequence lifetimes for these models are  $\tau_{MS} = 9.5 \times 10^8 \text{ yr}$  and  $7.9 \times 10^7 \text{ yr}$  for the  $2M_\odot$  and  $5M_\odot$  star respectively.

The mass-loss timescale within the envelope depends on the mass exterior to that point ( $\Delta M$ ) and the mass loss rate  $\dot{M}$ . If the mass-loss rate  $\dot{M}$  is a function of azimuthal position  $\theta$  for rotating stars (for line-driven winds see Friend & Abbott 1982 and for radiation-driven dusty winds see Dorfi & Höfner 1996), then the mass-loss timescale also varies with  $\theta$ . However, as a first approach  $\tau_M(\theta)$  is taken to be independent of  $\theta$ :

$$\tau_M \sim \frac{\Delta M}{\dot{M}}. \quad (16)$$

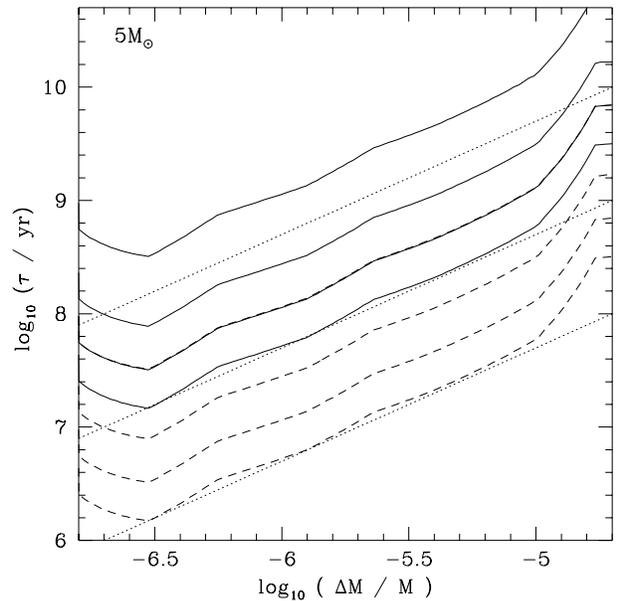
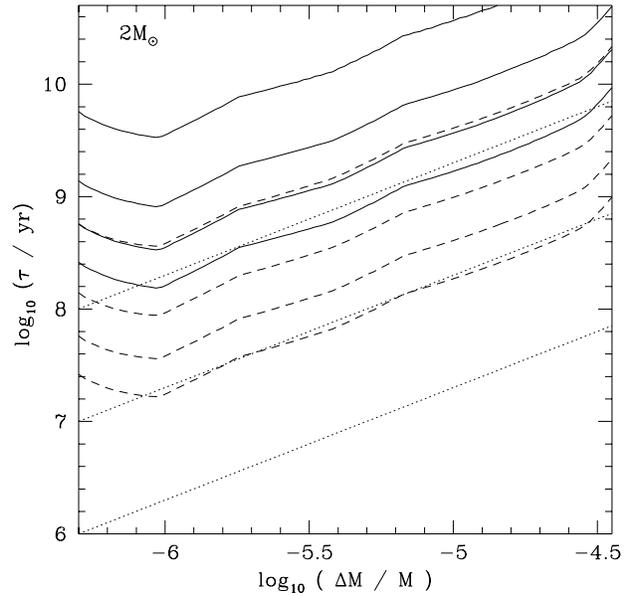
### 6.2. Where can the asymmetry exist?

In order for any significant metallic asymmetry to be generated, the diffusion timescale must be less than the mass-loss timescale. This ensures that the diffusion process has enough time to act before that layer of the star is lost in an outflow. The diffusion timescale must also be shorter than the typical main-sequence timescale else no significant asymmetry will build up over the lifetime of the star. Therefore, chemical abundance variations may be generated in regions for which the two inequalities

$$\tau_D \lesssim \tau_M, \quad \tau_D(\theta) \lesssim \tau_{MS} \quad (17)$$

are satisfied.

The timescales are displayed in Fig.4. The lines are the diffusion timescale around the star for  $\theta = 10^\circ, 30^\circ, 50^\circ$  and  $70^\circ$  with the lower line corresponding to  $70^\circ$  decreasing upwards. The solid lines are for solar iron abundance, and the dotted lines are for  $10^{-2}$  solar abundance. The oblique dashed lines are the mass loss timescale, corresponding to mass-loss rates of  $10^{-12} M_\odot \text{ yr}^{-1}$  (lower),



**Fig. 4.** The timescales in the star: the diffusion timescale around the star for  $\theta = 10^\circ, 30^\circ, 50^\circ$  and  $70^\circ$  with the lower line corresponding to  $70^\circ$  decreasing upwards. The solid (dashed) lines correspond to solar ( $10^{-2}$  solar) abundance of iron. The dotted lines are the mass-loss timescales corresponding to mass-loss rates of  $10^{-12} M_\odot \text{ yr}^{-1}$  (lower),  $10^{-13} M_\odot \text{ yr}^{-1}$ ,  $10^{-14} M_\odot \text{ yr}^{-1}$  (upper).

$10^{-13} M_\odot \text{ yr}^{-1}$ ,  $10^{-14} M_\odot \text{ yr}^{-1}$  (upper). Fig. 4 should be interpreted as follows: the region to the right and lower than the mass-loss timescale lines may, if the diffusion velocity is larger than the diffusive velocity ( $\sim$  the meridional velocity  $u_r$ , see Sect. 2) develop metallic abundance gradients around an equipotential. Therefore, the  $5M_\odot$  model may have enough time to develop a significant abundance gradient across most of the meridian in the outer regions is the mass-loss rate is  $\sim 10^{-13} M_\odot \text{ yr}^{-1}$ .

Fig. 4 suggests that during the main sequence lives of the  $10^{-2}$  solar abundance  $2M_{\odot}$  and both  $5M_{\odot}$  stars, the drift velocity due to the anisotropic flux in the star  $v_s$  is large enough that a latitudinal abundance gradient may be generated. However, this statement must be confined to stars which are rotating at significant fractions of break-up, and only applies to bands near to the equator.

Although this effect may produce metallic drift, it seems from Fig. 4 unlikely that the whole of an equipotential  $s$  will be involved. It appears that only regions close to the equator will be effected and consequently the metallic distribution will *not* come into equilibrium during the star's main-sequence lifetime. The distribution of metals is therefore difficult to calculate – indeed the dynamical modelling of the distribution is out of the scope of this paper and is flagged for further study.

## 7. Discussion

The results of the previous sections are startling. It has been found that in bands near the equator metals may drift toward the equatorial plane due to the anisotropic radiation field created when the star rotates. However, it is noted that if the mass-loss rate in the equatorial regions is strongly dependent on the metallicity, then the enhanced mass-loss rate will reduce the time that a given layer is bound to the star. Therefore the metallicity enhancement will not be able to build up to such a large level (as estimated below). In this case then a new equilibrium will be approached in which the magnitude of the metallicity enhancement is controlled by the enhanced mass-loss. This aspect of the problem is currently being investigated – the homogenising effect of the outer convection zone in the models has, so far, been neglected. This may homogenise the metallicity fast enough that no effect on the mass-loss rate may be observed.

The drift will slow and eventually stop when the logarithmic concentration gradient becomes comparable with the radiative force (see Eq. 11):

$$\frac{d \ln c}{d \theta} \approx A_i \frac{m_p g_{i,s}^{\text{rad}}}{kT} r \quad (18)$$

(again note this is an approximation as  $\hat{s}$  and  $\hat{\theta}$  are not parallel). As an example, the temperature is set to  $2.5 \times 10^5 \text{K}$  corresponding to  $\Delta M/M \sim 10^{-6}$  for both stars. The radii of the stars of  $r = 3.0R_{\odot}$  ( $5M_{\odot}$ ) and  $1.8R_{\odot}$  ( $2M_{\odot}$ ). From Fig. 2 the radial radiative force is  $\approx 10^{4.7}$  ( $10^{4.1}$ ) for the  $5M_{\odot}$  ( $2M_{\odot}$ ) star. If it is assumed that the radiative force around the star  $g_{i,s}^{\text{rad}}$  is a tenth of this (see Fig. 1) then the right-hand side of Eq. 18 is 1360 and 205 for the  $5M_{\odot}$  and  $2M_{\odot}$  stars respectively. These are clearly large numbers – indicative of a very large abundance build-up in the equatorial plane. However as shown in Sect. 6 it is unlikely that the whole of the meridian will come into equilibrium. It may be surmised, though, that significant abundance inhomogeneities can be generated (without being thwarted by the concentration gradient) through rotation.

In the course of this paper, radial motions due to radiative acceleration or gravitational settling have been neglected. It is likely that a radial component of drift velocity is present as well

as the meridional component focussed on here. Therefore, the motion of heavy element ions will not be solely be meridional – if the radial radiative acceleration exceeds gravity, then the drift will be toward the equator *and* toward the surface.

It is difficult to accurately assess the effects of large latitudinal abundance gradients on the structure of a star. As soon as the abundances change at a given point, then the flux distribution will also change along with the local convective stability criterion. Convection smooths out the abundance overdensity, and provides some feedback to the ionic build up. It is possible that this feedback will regulate the ionic drift, modifying the structure of parts of the star.

Let us now consider the evolution of the outer envelope of the star during the main sequence. As the very outer layers are lost in a wind, then a given layer becomes slightly more diffuse and the layer moves to lower  $\Delta M/M$  (which will promote latitudinal ion drift). However, as the layer becomes convectively unstable then it is quickly made homogeneous. The presence of extra radiation-blocking ions in fact make the layer more unstable to convection, and so the equatorial regions of rotating stars can be expected to have a more extensive convective region. Due to this rapid convective homogenisation, it is very unlikely then that this latitudinal drift will be observed in the photosphere of the star during its main sequence lifetime (the possibility of a star having different chemical compositions on the same equipotential has already been considered using radial diffusion and magnetic fields by Michaud, Mègeessier & Charland 1981). The possibility of observing a direct manifestation of latitudinal abundance variation in post main-sequence phases of evolution is currently under study. Here the mass-loss rate may be large enough to prevent complete convective chemical homogenisation of regions which have generated abundance gradients during the main sequence.

It is clear that as the timescales over which a significant abundance gradient may be generated in parts of the star are similar to either mass-loss timescales or main-sequence lifetimes. Therefore the evolution of the star needs to be taken into account whilst considering the abundance time dependence – something which is not attempted here. Although difficult due to necessarily 2D nature of the calculation, it appears that time dependence of the abundance distribution needs to be included in models for rotating stars.

## 8. Conclusion

It has been shown that rotation may indirectly have a large effect on the latitudinal abundance distribution of metals via gravity darkening. Indeed, the effect is possibly active in all the model stars considered here. Stars of low metallicity are particularly prone to this effect. If the mass loss of the star is strongly metallicity dependent, then the effect in the outer layers may be curtailed somewhat. This may lead to an extra mass-loss rate enhancement in the equatorial plane.

Although several major points regarding the effects of meridional diffusion of ions toward the equator have simply

been touched on, the principle of metallic latitudinal distribution asymmetries has at least been put on quantitative basis.

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## References

- Alecian G., 1985, *A&A*145, 275  
 Alecian G., 1996, *A&A*310, 872  
 Alecian G., Artru M.-C., 1990, *A&A*234, 323  
 Alecian G., Michaud G., Tully J., 1993, *ApJ*411, 882  
 Alecian G., Vauclair, S., 1981, *A&A*101, 16  
 Chaboyer B., Zahn J.-P., 1992, *A&A*253, 173  
 Charbonneau P., 1992, *A&A*259, 134  
 Charbonneau P., Michaud G., 1991, *ApJ*370, 693  
 Collins G.W. II, Harrington J.P., 1966, *ApJ*146, 152  
 Dorfi E.A., Höfner S., 1996, *A&A*313, 605  
 Friend D.B., Abbott D.C., 1986, *ApJ*311, 701  
 Gonzalez J.-F., LeBlanc F., Artru M.-C., Michaud G., 1995, *A&A*297, 223  
 Michaud G., 1970, *ApJ*160, 641  
 Michaud G., Charland Y., 1986, *ApJ*311, 326  
 Michaud G., Charland Y., Vauclair S., Vauclair S., 1976, *ApJ*210, 447  
 Michaud G., Mégessier C., Charland Y., 1981, *A&A*103, 244  
 Michaud G., Tarasick D., Charland Y., Pelletier C., 1983, *ApJ*269, 239  
 Paquette C., Pelletier C., Fontaine G., Michaud G., 1986, *ApJS*61, 197  
 Salaris M., Dominguez I., Garcia-Berro E., et al., 1997, *ApJ*486, 413  
 Tassoul J.-L., Tassoul M., 1982, *ApJS*49, 317  
 Tassoul M., Tassoul J.-L., 1983, *ApJ*271, 315  
 Talon S., Zahn J.-P., 1997, *A&A*317, 749  
 Urpin V., Shalybkov D.A., Spruit H.C., 1996, *A&A*306, 455  
 Vauclair G., Vauclair S., Michaud G., 1978, *ApJ*223, 920  
 Zahn J.-P., 1975, *Mém. Soc. Roy. Sci. Liège*, 6e série 8, 31  
 Zahn J.-P., 1992, *A&A*265, 115  
 Zahn J.-P., 1987, In: Durney B.R., Sofia S. (eds.) *The Internal Solar Angular Velocity*. Reidel, p 201  
 von Zeipel H., 1924, *MNRAS*84, 665