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On the flow topology in contact binaries

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Abstract. Using a simple model we investigate how the flow field in a contact binary can alter its topology under the influence of Coriolis forces. In the case studied, closed streamlines form over a relatively large area. This result causes us to review critically the argument that all surface streamlines can meet in a stagnation point, leading to a constant Jacobi energy over the whole surface. A general statement by Landau & Lifshitz concerning Bernoulli's constant is fully confirmed by the present investigation.

Key words: stars: binaries: close – hydrodynamics

1. Introduction

In recent discussions a strong difference of views has emerged concerning the correct fluid-dynamical treatment of the flow of gas over the surface of a contact binary. This difference of views concerns the question of whether there are any hydrodynamical arguments for assuming constancy of Jacobi energy (sum of potential and kinetic energy) over the system surface. The answer to this question could be of relevance for the theory of systems where very high velocities (in addition to the normal velocities of orbital and synchronised motions) seem to have been observed (Frasca et al., 1996).

My view (Hazlehurst 1997; hereafter H 97) that there are no hydrodynamical arguments favouring constant Jacobi energy derived largely from a consideration of the symmetry-breaking properties of the Coriolis forces. After referring to the rôle of these forces in this respect I went on to state that they might even lead to topological changes in the flow.

This view has however been sharply challenged (Kähler 1997; hereafter K 97) in a paper where the author sees no reason whatever for expecting a change in fluid topology from the above cause.

I shall therefore in this paper try to give a detailed picture of how the topological changes can actually occur. Since the differences of view described above do not involve the 'viscosity question' I shall retain the inviscid assumption in the interests of simplicity and, I hope, of transparency. I shall also assume that there is a steady circulation of gas through the system involving both supply and removal of material from the surface layers. This means that not all of the surface streamlines will close up in the surface itself. I shall use the simplest possible model, to be described in the next section, to represent this situation.

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2. A simple model

We shall in this paper assume the principle possibility of two stars being able to form a stable, stationary contact binary in which strong motions occur across the surface. We do not specify the evolutionary state of the system.

We shall regard the material involved in these motions as flowing from one star (say, the primary) to the other via the outer layers and then back again through the inside. Then, as seen from the surface, there would appear to be a source on the primary side of the system and a sink on the secondary side.

We start by agreeing with the statement in K 97 that the surface of a contact binary is topologically equivalent to a sphere. This being so, we can go a step further and attempt to represent the system by means of a rotating, roughly spherical object – at least as a topological analogue. A rather similar approach was used by Tassoul (1992); in our case we would require that the rotating object be supplied with a source on one side and a sink on the other.

In order to make the model 'realistic' (and not merely topologically similar) a rather large depth of contact for the system represented by the model would appear advantageous.

Finally, our simple model says nothing about the driving mechanism of the circulation and is merely intended to represent the situation as seen from the surface layers. Nevertheless we have in mind something like the thermally-driven circulation of Nariai (1976), Webbink (1977) and others.

3. Continuity of motions in the surface layer

In hydrodynamical considerations concerning the outer layers of a star it is practicable to take a mechanical rather than an optical definition of the surface. Consistent with both H 97 and K 97 we shall assume the surface to be an isobar:

$$P = constant \quad (1)$$

where the ‘constant’ in Eq. (1) has some small (normally very small) value.

Assuming the material to be an ideal gas we have:

$$P = \rho \frac{\Re T}{\mu} \tag{2}$$

so that the continuity equation:

$$div\,(\rho \boldsymbol{v}) = 0 \tag{3}$$

leads to:

$$\rho\, div \boldsymbol{v} + \frac{\mu}{\Re T}\boldsymbol{v}.grad\,P - \frac{\rho}{T}\boldsymbol{v}.grad\,T = 0 \tag{4}$$

where we have assumed constancy of μ.

Now the concept of ‘surface streamlines’ used by both authors referred to above is clearly only meaningful if these streamlines actually lie in the surface; in view of Eq. (1) this means that the vectors $\boldsymbol{v}$ and grad P must be taken to be mutually perpendicular. Hence Eq. (4) simplifies to:

$$div\,\boldsymbol{v} - \frac{1}{T}\boldsymbol{v}.grad\,T = 0 \tag{5}$$

The assumption underlying Eq. (5) should perhaps be stated in more detail. It is that the ‘surface’ of the system can be taken to be one on which the pressure is constant and that, at points on that surface, the velocity of the flow is parallel to the surface and, hence, perpendicular to the pressure gradient. We comment on this assumption in the Appendix A.

In the approximation used in this paper we regard the motions as becoming essentially 2-dimensional “motions on a sphere” as the surface is approached. Here a new assumption is involved, namely that the radial component of the velocity does not contribute significantly to the divergence of the velocity on the surface. This approximation could be more serious than the one involving the constancy of surface pressure; it is also discussed in the Appendix A.

Introducing a polar coordinate system, r, θ, ϕ we then have on the surface $r = R$:

$$div\,\boldsymbol{v} = \frac{I}{Rsin\theta}\left(\frac{\partial\,v_\phi}{\partial\,\phi} + \frac{\partial}{\partial\,\theta}sin\theta v_\theta\right) \tag{6}$$

We next introduce the temperature distribution over the surface; we take for this the simple form:

$$\hat{\boldsymbol{\theta}}.\frac{gradT}{T} = -\frac{k}{R} \qquad \hat{\boldsymbol{\phi}}.gradT = 0 \tag{7}$$

(k = constant) where the unit vector $\hat{\boldsymbol{\theta}}$ is always assumed to point away from the source and towards the sink in the simple model of Sect. 2.

Substituting now from Eqs. (6) and (7) into Eq. (5) we find:

$$\frac{1}{sin\theta}\left(\frac{\partial v_\phi}{\partial\,\phi} + \frac{\partial}{\partial\,\theta}\,sin\theta v_\theta\right) + kv_\theta = 0 \tag{8}$$

Now the thermal capacity of the surface layers is extremely low so that k will be (mainly) determined by energy transport in the deeper layers. For good contact (as assumed in Sect. 2) we can expect k to be quite small. The question we therefore have to answer is whether we can simply drop the k-term in Eq. (8) or whether we must solve the full equation *first* and then go to the limit $k \to 0$ in the resulting full solutions.

Although we can not answer this question in general, we can answer it in particular cases. A particular solution of Eq. (8) of special interest here is:

$$v_\theta = \frac{Ae^{-k\theta}}{sin\theta} \qquad v_\phi = 0 \tag{9}$$

which corresponds to a source-sink pair with the source and sink located at $\theta = 0$ and $\theta = \pi$ respectively. Thus we see not only that the source and sink are diametrically opposite but that we can now define the polar axis of coordinates much more precisely as being the axis which joins source and sink. We also note that the k-term modulates the solutions but does not change their essential character.

Going now to the limit $k \to 0$ we find

$$v_\theta \to \frac{A}{sin\theta} \qquad v_\phi = 0 \tag{10}$$

so that, at least in this particular case, the ‘short cut’ of simply dropping the k-term in Eq. (8) seems to work. We shall assume without proof that this procedure is also viable for $v_\phi \neq 0$, or at least for those cases which we shall treat later in this paper.

Finally, we note that the use of the strictly two-dimensional approach of this section introduces the disadvantage that the velocity becomes arbitrarily high (see Eqs. (9) and (10)) in the immediate neighbourhood of the source or sink centre. This behaviour can however be avoided by using a more general approach. We shall discuss this aspect in Sect. 7.

4. Dynamical description of the flow

The motion of an inviscid fluid under the action of the total potential Φ is given by:

$$\frac{1}{2}\,gradv^2 - \boldsymbol{v}\times curl\boldsymbol{v} + 2\boldsymbol{\Omega}\times\boldsymbol{v} = -\,grad\Phi - \frac{1}{\rho}gradP \tag{11}$$

where $\boldsymbol{v}$ denotes the velocity in the rotating frame and stationarity is assumed.

We now confine our considerations to motions in the surface layer. As in Sect. 3 we shall treat these as if they occurred over a spherical surface (see, however, below). We shall use spherical polars r, θ, ϕ (with $r = R$ on the surface) and the coordinate axis aligned in the source-sink direction i.e. perpendicular to the rotation axis. Taking the vector product of Eq. (11) with $\hat{r}$ leads to a further vector equation the components of which yield, after some reduction:

$$\frac{\partial}{\partial\theta}(\Phi + \frac{1}{2}v^2) = (\eta_r + 2\Omega sin\theta cos\phi)v_\phi \tag{12}$$

$$\frac{1}{sin\theta}\frac{\partial}{\partial\phi}(\Phi + \frac{1}{2}v^2) = -(\eta_r + 2\Omega sin\theta cos\phi)v_\theta \tag{13}$$

where we have again used Eq. (1). The quantity

$$\eta_r = \frac{1}{Rsin\theta}\left(\frac{\partial v_\theta}{\partial\phi} - \frac{\partial}{\partial\theta}sin\theta v_\phi\right) \tag{14}$$

represents the component of vorticity perpendicular to the surface.

As in Sect. 3 and as assumed earlier (H97 and K97) the pressure has been taken to be constant on the boundary. If this condition is not fulfilled, Eqs. (12) and (13) must be modified accordingly (for details see Appendix A).

It follows from Eqs. (12) and (13) that:

$$\frac{\partial}{\partial\phi}\left[(\eta_r + 2\Omega sin\theta cos\phi)\upsilon_\phi\right] = - \frac{\partial}{\partial\theta}sin\theta\left[(\eta_r + 2\Omega sin\theta cos\phi)\upsilon_\theta\right] \tag{15}$$

So far we have treated all motions as if they occurred over a spherical surface. The surface will however in practice adjust its form in order that the potential there can satisfy the above equations. As we have noted in H 97 this does not represent an inconsistency since the spherical approximation can still be used to tell us in which way the adjustment must occur. As in Tassoul 1992, the gravitational component of the potential can be taken to be spherically symmetrical in order to support the pseudospherical approach.

We see from the above equations that a necessary condition for constant Jacobi energy $\Phi + \frac{1}{2}\upsilon^2$ everywhere is:

$$\eta_r + 2\Omega sin\theta cos\phi = 0 \tag{16}$$

for non-zero velocities. This will also represent a sufficient condition, assuming that the continuity equation (see Sect. 3) can also be satisified.

We shall first consider the case where Eq. (16) is satisfied. This will serve as a useful basis for the discussion later of the more general case.

In order to see what Eq. (16) implies we substitute for η_r from Eq. (14) into Eq. (16) to give:

$$\frac{1}{Rsin\theta}\left(\frac{\partial\upsilon_\theta}{\partial\phi} - \frac{\partial}{\partial\theta}sin\theta\upsilon_\phi\right) + 2\Omega sin\theta cos\phi = 0 \tag{17}$$

This must now be combined with the continuity equation. Following the arguments of Sect. 3 we adopt this in the simplified form (k $\longrightarrow$ 0):

$$\frac{\partial\upsilon_\phi}{\partial\phi} + \frac{\partial}{\partial\theta}(sin\theta\upsilon_\theta) = 0 \tag{18}$$

We look for solutions of the form:

$$\upsilon_\theta = \frac{A}{sin\theta} + \upsilon_\theta^1 \tag{19}$$

where υ_θ^1 is restricted to be a *non-singular function*. Hence the solution includes, but does not entirely consist of, a source-sink pair. The additional function υ_θ^1 is included in order to allow us to satisfy the full set of equations. A solution of the form given in Eq. (19) should therefore permit us to study mathematically the simple model proposed in Sect. 2. Substituting from Eq. (19) into Eq. (18) we find:

$$\frac{\partial\upsilon_\phi}{\partial\phi} + \frac{\partial}{\partial\theta}sin\theta\upsilon_\theta^1 = 0 \tag{20}$$

We next attempt a separation of variables:

$$\upsilon_\theta^1 = sin\phi f(\theta) \tag{21}$$

leading to

$$\upsilon_\phi = cos\phi cos\theta f(\theta) + cos\phi sin\theta f'(\theta) \tag{22}$$

where the integration constant vanishes since υ_ϕ must be zero at the equator $(\phi = \pm\pi/2)$.

Using Eqs. (17), (19), (21) and (22) we obtain:

$$f'' + 3cot\theta\, f' - 2f - 2\,\Omega\, R = 0 \tag{23}$$

for which the only non-singular solution is:

$$f(\theta) = constant = -\Omega R \tag{24}$$

Hence Eqs. (19), (21), (22) and (24) give:

$$\upsilon_\theta = \frac{A}{sin\theta} - \Omega R sin\phi \tag{25}$$

$$\upsilon_\phi = -\Omega R cos\theta cos\phi \tag{26}$$

We must next consider the source strength A. This should not be taken to be so large that the pseudospherical approximation breaks down. Let us suppose, somewhat arbitrarily, that the object introduced in Sect. 2 can be considered roughly spherical if for 99% of the surface the r.m.s. radius variation does not exceed 10% of the average radius. We find this criterion leads to:

$$A < 0.15\,\Omega R \tag{27}$$

where we have taken Ω^2 to be equal to one third of the quadratic angular velocity needed for rotational break-up (Whelan 1972). In order to lie reasonably safely within the limit given by Eq. (27) we shall assume that

$$A = 0.1\,\Omega R \tag{28}$$

in what follows.

5. The streamlines for constant Jacobi energy

In the chosen coordinate system the general equation of the streamlines is:

$$\frac{d\theta}{\upsilon_\theta} = \frac{sin\theta\, d\,\phi}{\upsilon_\phi} \tag{29}$$

Substituting for υ_θ and υ_ϕ from Eqs. (25) and (26) we find:

$$\frac{d\,\theta}{\frac{A}{sin\theta} - \Omega\, R\, sin\,\phi} = \frac{sin\,\theta\, d\,\phi}{-\,\Omega\, R\, cos\theta\, cos\phi} \tag{30}$$

Using the identity:

$$cos\theta\, cos\phi\, \frac{d\,\theta}{d\,\phi} = \frac{d}{d\,\phi}(sin\theta cos\phi) + sin\theta\, sin\phi \tag{31}$$

we can rewrite Eq. (30) as:

$$\frac{d}{d\,\phi}(sin\theta\, cos\phi) = \frac{-\,A}{\Omega\, R} \tag{32}$$

which integrates to give:

$$\Omega\, R\, sin\theta = A\frac{constant\,-\,\phi}{cos\phi} \tag{33}$$

or, for the choice of A given in Eq. (28):

$$sin\theta = 0.1\frac{constant\,-\,\phi}{cos\phi} \tag{34}$$

Depending on the choice of the "streamline constant" in Eq. (34) the streamlines are open or closed. The *limiting streamline* separating the two groups is given by:

$$sin\theta = 0.1\,\frac{\pi/2\,-\,\phi}{cos\phi} \tag{35}$$

The properties of this limiting streamline are as follows. The streamline begins at a stagnation point $(\theta = 5.74°; \phi = 90°)$ on the equator. It then moves inwards, at first at almost constant θ, and crosses the 'top' of the star at $\theta = 9.04°; \phi = 0°$. It then continues over to the other side, making this excursion as far as $\theta = 90°; \phi = -73.43°$. After passing through this extremity, it returns over the top of the star again, this time at $\theta = 170.96°; \phi = 0°$, finally returning to the equator again at a second stagnation point located at $\theta = 174.26°; \phi = 90°$.

The very large loop which we have described above together with the remaining equatorial segment between $\theta = 174.26°$ and $\theta = 5.74°$, encircles all of the closed streamlines. Recalling that the sink and source are located at $\theta = 180°$ and $\theta = 0°$ respectively, we see that the system of closed streamlines covers almost the whole surface of the star. The 'open' streamlines connecting the source and sink are confined to a C-shaped narrow channel around the equator so that the material flowing from source to sink can only proceed along that side of the equator on which no stagnation points occur. The maximum width of the channel amounts to only $16.57°$.

This particular flow topology is a direct consequence of the action of the Coriolis forces. To see this, let us simply omit the Coriolis term in Eq. (11) (i.e. the third term on the L.H.S.) and repeat the analysis. We then find instead of Eqs. (25) and (26) the solution:

$$\upsilon_\theta = \frac{A}{sin\theta} \qquad\qquad \upsilon_\phi = 0 \tag{36}$$

corresponding to a *complete system* of open streamlines. One can hardly imagine a more dramatic change in the flow topology.

An immediate consequence of the change in flow topology caused by the Coriolis forces is that most points on the stellar surface are *not accessible* to a streamline coming from the source or going to the sink. Hence it is not valid to argue that the Jacobi energy *must* be constant over the surface of a contact binary on the grounds that all streamlines can be traced back to a point where they all come together. This argumentation was used for contact binaries of the 'reversing layer' type in K 97.

6. Velocity fields with non-constant Jacobi energy

Let us consider once more the situation described in Sect. 5. By far the greatest part of the star is covered with closed streamlines. Hence there is nothing to prevent us, in an ideal experiment, from introducing vorticity over this whole area. If viscosity is required for carrying out this experiment, it is of course removed immediately afterwards.

It is not difficult to show that, under the assumption that Eq. (18) is correct, the perpendicular component of the vorticity will, following the above experiment, remain constant on every streamline. We can imagine the vortices as moving around in an enclosed area ("vortex patch") thereafter.

The expression in brackets on the R.H.S. of Eq. (12) and Eq. (13) i.e. the total perpendicular vorticity, including the rotational contribution, is the relevant quantity in the above considerations. We note that, if this is non-zero, as it is following the above experiment, then the Jacobi energy on the L.H.S. of the above equations can not be constant. Outside the vortex patch, where the (total) vorticity is still zero, the Jacobi energy will remain constant as before.

In order to simplify matters as far as possible, let us assume that, in the enclosed region, the (perpendicular) vorticity is uniform. There is a vorticity jump (but not a velocity jump) along the edge of the vortex patch, including the stretch along the equator where the patches from the two hemispheres touch. The usual symmetry conditions across the equator are not affected by this.

In the enclosed region, therefore:

$$\eta_r \,+\, 2\Omega sin\theta cos\phi = F \tag{37}$$

where F is some constant. It follows that F is also constant on each streamline in this region. Now, before the extra vorticity was introduced we had:

$$\mathring{\eta}_r \,+ 2\Omega sin\theta cos\phi = 0 \tag{38}$$

from which it follows that:

$$\eta_r - \,\mathring{\eta}_r = F \tag{39}$$

Using **u** to denote the *change* in velocity we then have from Eq. (14):

$$\frac{1}{R\, sin\theta}\left(\frac{\partial u_\theta}{\partial\phi} - \frac{\partial}{\partial\theta}sin\theta u_\phi\right) = F \tag{40}$$

We look for solutions of the form:

$$u_\theta = sin\phi f(sin\theta cos\phi) \tag{41}$$

so that Eq. (18) then requires that:

$$u_\phi = cos\theta cos\phi f(sin\theta cos\phi) \tag{42}$$

where we have used the condition that u_ϕ vanishes at the equator.

Substituting from Eqs. (41) and (42) into Eq. (40) we then find, after some reduction:

$$f'(1 - sin^2\theta cos^2\phi) - 2f sin\theta cos\phi + RF = 0 \tag{43}$$

where f' denotes differentiation with respect to the complete argument $sin\theta cos\phi$.

The particular integral of Eq. (43) of interest for us is:

$$f = \frac{RF}{1 + sin\theta cos\phi} \tag{44}$$

At very low latitudes this equation can only be regarded as an approximation since the assumption is made that for all higher latitudes than the one under consideration each parallel of latitude is filled with vortex elements. However since we shall only be interested in the qualitative consequence of introducing these vortex elements the approximation represented by Eq. (44) should be adequate.

Using Eqs. (25), (26), (41), (42), and (44) we now obtain for the total velocity:

$$v_\theta + u_\theta = \frac{A}{sin\theta} - Rsin\phi \left\{ \Omega - \frac{F}{1 + sin\theta cos\phi} \right\} \tag{45}$$

$$v_\phi + u_\phi = -Rcos\theta cos\phi \left\{ \Omega - \frac{F}{1 + sin\theta cos\phi} \right\} \tag{46}$$

The streamlines corresponding to this velocity field can be calculated by the same method as was used for the simple velocity field in Sect. 5. Once more using the identity given in Eq. (31) we find after integration that the limiting streamline is given by:

$$\phi = \frac{RF}{A} ln(1 + sin\theta cos\phi) - \frac{\Omega R}{A} sin\theta cos\phi + \pi/2 \tag{47}$$

provided that

$$F < \Omega - A/R \tag{48}$$

The analysis can be simplified if we consider only those cases for which, on the limiting streamline, the product $sin\theta cos\phi$ is everywhere sufficiently small to permit the expansion of the logarithm in Eq. (47) to first order. In these cases:

$$sin\theta \simeq A \frac{\pi/2 - \phi}{cos\phi} \frac{1}{R(\Omega - F)} \tag{49}$$

so that we see that the above approximation can be used when:

$$A << R(\Omega - F) \tag{50}$$

We further confirm, by comparing Eqs. (47) and (49), that Eq. (49) leads to exact results in the case $F = 0$.

As an illustrative example let us take $F = -0.5\Omega$ and $A = 0.1\Omega R$ (as before) and compare the results with those for $F = 0$ (see Sect. 5) which will be given in brackets below. Then we find for the locations of the stagnation points ($\phi = 90°$):

$$\theta = 3.82°(5.74°)\, and\, \theta = 176.18°(174.26°) \tag{51}$$

and for the extremity of the limiting streamline:

$$\phi = -78.68°(-73.43°)\, for\, \theta = 90° \tag{52}$$

We see that, relative to the case considered in Sect. 5 (in brackets) there has been a general increase in the area covered by the closed streamlines. Conversely a contraction of the region occupied by the 'open' streamlines has occurred. We shall consider conditions in this latter region in the following section.

7. Conditions where the surface streamlines meet

So far, we have used the idealization of a point source or sink within the context of a 2-dimensional flow field. The consequence of this is that the velocity increases to very high values as the source or sink centre is approached.

In order to connect with a proper 3-dimensional analysis let us write:

$$\boldsymbol{F} = \rho \boldsymbol{v} \tag{53}$$

and introduce the surface divergence:

$$S = \frac{1}{Rsin\theta} \left(\frac{\partial}{\partial\theta} sin\theta F_\theta + \frac{\partial}{\partial\phi} F_\phi \right) \tag{54}$$

still using the spherical approximation r = R for the surface itself.

We can now invert Eq. (54) and regard the quantity S as being the source-function for the 2-dimensional flow pattern on the surface. In particular we can, within a certain *small* distance measured along the surface from the source or sink centre, take S to be non-zero and finite (S>0 for source and S<0 for sink). Outside this 'near zone' we can assume S = 0 as previously and well outside it we can expect the solutions to be substantially the same as before.

Inside the 'near zones' however the situation has changed relative to that in the 2-dimensional case. Instead of containing a singularity the central portion of the source (or sink) region will now contain a stagnation point at which the 'open' surface streamlines can meet. At this meeting point the Jacobi energy will be the same for the various streamlines. Hence if it were possible to connect every point on the object surface by means of a streamline to the source or sink then it would be correct, as noted in K 97, to infer from Bernoulli's equation that the Jacobi energy must be constant over the whole surface.

However when a change of topology occurs closed streamlines cover a large part of the surface and the above argumentation can not be used. The Coriolis forces, being the cause of the topological change, are also the cause of the non-constancy (in general) of the Jacobi energy over the surface.

8. General discussion

In K 97, Sect. 2.6 various examples (Nariai (1976), Webbink (1977), Zhou & Leung (1990)) are quoted as fitting in well with the author's proposed scheme of constant Jacobi energy plus a complete system of 'open' surface streamlines to represent a thermally-driven circulation. Since the above series of references follows immediately after a critical comment directed at H 97 it is clear that we are obliged to take a position on these specific examples.

The situation regarding the first two examples listed above has already been discussed in H 97 – as long as Coriolis forces are neglected the condition of constant Jacobi energy is automatically satisfied; insofar the situation is not controversial. It remains to consider the third paper.

In the paper of Zhou & Leung Fig. 4 illustrates the equatorial flow field with the effect of the Coriolis forces clearly shown in the diagram. Now it is shown in H 97 and accepted in K 97 that constant Jacobi energy implies a very strong net retrograde motion in the surface layers. There is however no indication of such a net (i.e. phase-averaged) retrograde effect in the outer parts of the diagram, so that we must conclude that the Jacobi energy can not be a surface invariant. Furthermore the references to cyclones and anticyclones on opposite sides of the system would point to the presence of closed streamlines in the surface layers. Hence we can not argree that this model fits in well with the picture proposed in K 97.

We next consider the general argument brought in K 97 to support the view that, in a situation corresponding to thermally-driven circulation, a complete system of 'open' surface streamlines is to be expected. Since this same topology also characterizes the case of zero Coriolis forces, this is at the same time an argument against these forces causing any topological changes.

The basic consideration is that if we multiply the Coriolis forces everywhere by the factor Θ and allow Θ to increase gradually from zero then there is really no reason why an abrupt change in the 'solutions' should arise from this procedure. In principle, $\Theta = 1$ could be (and, for the purposes of Sect. 2.6, can be) reached in this way.

Let us consider this argument in relation to the simple model of the present paper. Then we find that, as the parameter Θ is gradually increased, the flow velocity does indeed change continuously all the way to $\Theta = 1$. However it is only necessary to consider the formation of the first closed streamline to see that topological changes can occur even when the velocity is changing continuously.

We therefore see that it is *not permissible* to assume that continuity of the velocity also implies preservation of the topology. This is the essential weakness in the 'Θ-argument' described above.

9. Conclusion

In this paper we investigated the special rôle of the Coriolis forces in influencing the flow topology over the surface of a contact binary. In order to follow the dynamical effects more easily, the surface geometry was drastically simplified and the mass flow was imagined to be generated by a source-sink pair.

We found that, whereas under neglect of Coriolis forces the surface is covered by streamlines connecting source and sink, the inclusion of these forces gave a quite different picture. Most of the surface was found to be covered by closed streamlines and the flow between source and sink was restricted to a narrow equatorial channel running around one side of the object only.

Freedom to change the vorticity distribution over the closed streamlines brought us also the freedom to influence the distribution of Jacobi energy over the part of the surface covered by closed streamlines. Hence there is no reason why in any given situation the Jacobi energy should be constant over the surface. This last result assumes that the Coriolis forces have been allowed for. It is not in conflict with the properties of the published models listed in the last section provided this important proviso is observed.

An argument, based upon analytical continuation, in favour of the view that Coriolis forces can not be expected to influence the flow topology, is subjected to criticism.

Returning now to the Jacobi energy (Bernoulli constant) situation our conclusion can best be summarized by noting that our results are exactly in line with the view expressed in Landau & Lifshitz (1959) that "In general the constant (Bernoulli's constant) takes different values for different streamlines". Indeed we regard our calculations as confirming that nothing should be added to, or taken from, this statement.

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Appendix A: some comments on the assumptions

We first comment on the assumption of uniform surface pressure (see Sect. 3). This was made principally to preserve uniformity with previous papers (H97, K97 and references therein). Nevertheless it may reasonably be asked whether this simplified boundary condition is a) permissible and b) representative.

Let us see how this simplified boundary condition would fit in with, for example, the geostrophic approximation (see e.g. Tassoul 1992). Here the pressure gradient is regarded as being non-zero *on equipotentials*; this is however not in conflict with prescribing a constant pressure on the actual contact binary surface.

Nevertheless we must ask whether a departure from our simple boundary condition would be likely to change the flow pattern so much as to invalidate our general conclusions. To fix matters, let us assume that the specific entropy is uniform over the surface (barotropy); this will give us a situation which is thermally similar to that obtained by setting k = 0 in Sect. 3. If, as previously, the gas is assumed perfect, with constant molecular weight, then the thermal equation of state can be written:

$$P = KT^{2.5} \tag{A1}$$

Rather than returning with this equation to Sect. 3, with P now variable over the surface, it is more useful to go directly to the equations of motion. We then find that there is an extra term inside the brackets on the L.H.S. of Eqs. (12) and (13):

$$\Phi + \frac{1}{2}\upsilon^2 \longrightarrow \Phi + \frac{1}{2}\upsilon^2 + 2.5\frac{\Re}{\mu}K^{-0.4}P^{0.4} \tag{A2}$$

where the above triple sum is sometimes referred to as Bernoulli's integral. There is however no change on the R.H.S. of Eqs. (12) and (13).

For small surface pressures and supersonic motions *along* the surface the pressure-dependent correction term will be quite small so that its influence can be compensated by small changes either in Φ (slight change in surface shape) or in υ. Hence our conclusions based on the surface flow structure should not be seriously affected by possible variations in surface pressure. The question of the constancy of the Jacobi energy is now replaced by the question of whether Bernoulli's integral (triple sum) is constant over the whole surface, or whether it is just constant along the streamlines. Thus the controversy discussed in this paper still persists, but with slight changes in the details.

Somewhat more serious than the assumption of constant surface pressure is perhaps the neglect of the contribution of the radial velocity component to the velocity divergence; this is our 2-dimensional approximation of setting $\partial\upsilon_r/\partial r = 0$ everywhere on the surface except of course at the singular points where the source and sink are located.

We can improve on this highly idealized situation by allowing both source and sink to have a finite lateral extent *aR (a* small); we can then 'parametrize' this new picture (no longer 2-dimensional) by writing:

$$\frac{\partial\upsilon_r}{\partial r} = \frac{-\varepsilon cos\theta}{(sin^2\theta + a^2)^2} \quad \text{(A3)}$$

where ε is a small positive constant; the previous idealized case corresponds to $\varepsilon = 0$, $a = 0$. The minus sign in Eq. (A3) comes from the consideration that, for example, rising material (source) is decelerated on approaching the surface as far as the radial motion is concerned. Denoting as previously the source strength by A, and going to the limit $a \longrightarrow 0$ we confirm that the results of Sect. 3 are recovered if we identify:

$$\varepsilon = 2Aa^2 \quad \text{(A4)}$$

Keeping closely to the methods used in the text, but now assuming $a \neq 0$, we find that Eqs. (25) and (26) now become replaced by:

$$\upsilon_\theta = \frac{Asin\theta}{sin^2\theta + a^2} - \Omega Rsin\phi \quad \text{(A5)}$$

$$\upsilon_\phi = -\Omega Rcos\theta cos\phi \quad \text{(A6)}$$

It is worth noting that Eqs. (A5) and (A6) predict (for small *a*) the presence of stagnation points on the equator as already found in Sect. 5. There are however two further stagnation points very close to the geometrical poles $\theta = 0$ and $\theta = \pi$ which serve as a meeting point for those streamlines which do not close up on the surface. The structure of the flow in the region outside the source radius is however the same as for the simple case *a* = 0 calculated in Sect. 5.

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