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Solar p-modes: possible source of excitation and frequency splitting

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Abstract. A new possible source of solar 5-minute oscillation is supposed. Weak periodic differential rotation (see Zaqarashvili 1997, hereafter paper I) generates exponentially growing slow magnetosonic waves with period of 22 yr in the radiative interior. The waves propagating with $\pm 45^0$ angles to the radial direction have maximal growth rates. The ideal MHD equations for slow and fast waves are linearly coupled due to the differential rotation. But linear coupling is inactive for the conditions in radiative interior. However preliminary non-linear analyses show energy resonant transformation from slow waves to the fast one. The period of resonant fast magnetosonic waves is shortest due to the large value of plasma β. The period is $\sim$ 5 minute for $\beta \approx 3 \times 10^{12}$. These fast magnetosonic waves transform into sound waves in unmagnetized convective envelope.

Frequencies of the sound waves are changed during propagation along the differentially rotating convective envelope. The frequency of the waves propagating in the direction of rotation is increased, but the frequency of the waves propagating in opposite direction of the rotation is decreased. It leads to a frequency splitting at the surface. The slight periodical variation of the differential rotation causes cyclic behaviour of the splitting. The rate and the phase of the splitting variation are in good agreement with the observations (Jimenez et al. 1994). Maximal splitting corresponds to the maximum of the solar activity.

Key words: MHD – waves – Sun: magnetic fields – Sun: oscillations

1. Introduction

Solar 5-minute oscillations were discovered by Leighton et al. (1962). Since then several theoretical models have been suggested for explanation of the physical mechanism of these oscillations (Schmidt & Zirker 1963; Moore 1967; Stix 1970; Ulrich 1970; Goldreich & Keeley 1977a,b; Brown 1984; Goldreich & Kumar 1988, 1990).

The discovery of the frequency splitting launched the new research field of helioseismology. The rotation of the solar envelope and the convective zone had been determined by imaging helioseismology (Brown & Morrow 1987; Christensen-Dalsgaard & Schow 1988; Libbrecht 1988). The helioseismological measurement of more deep rotation has been the subject of increasing interest during the last years (Christensen-Dalsgaard 1992; Toutain & Fröhlich 1992; Loudagh et al. 1993; Jimenez et al. 1994; Regulo et al. 1994; Toutain & Kosovichev 1994; Elsworth et al. 1995; Tomczyk et al. 1995; Harvey 1995; Lazrek et al. 1996). According to current thinking the splitting is caused by the reflection of p-modes on the rapidly rotating core.

Many observations (Woodard & Noyes 1985, Fossat et al. 1987; Gelly et al. 1988; Palle et al. 1989; Elsworth et al. 1990; Anguera Gubau et al. 1992; Bachmann & Brown 1993; Regulo et al. 1994) show that the frequencies of p-modes are correlated with the solar cycle. Recently Jimenez et al. (1994) found cyclic variation of the frequency splitting being higher at the maximum of the solar activity.

The mechanism explaining the frequency shift of p-modes was presented several years ago (Bogdan & Zweibel 1985; Zweibel & Bogdan 1986; Cambrell & Roberts 1989; Evans & Roberts 1990; Goldreich et al. 1991; Jain & Roberts 1994a,b). They suggest that the frequency variation is caused by the changing of the surface magnetic field during the solar cycle. However the problem of the frequency splitting variation (Jimenez et al. 1994) is still unsolved. The observed rate of the splitting variation requires significant changing of solar core rotation during the activity cycle. But no physical mechanism exists supporting such a significant changing of the rotation.

Recently it was supposed (see paper I) that the near-elliptical "path" of the Sun around the barycenter of the solar system causes weak periodic differential rotation in the interior. In other words the gravitation of the planets changes the solar MHD parameter (differential rotation) periodically. It leads to the parametric resonance in ideal MHD. The period of the "path" was taken as $\sim$11 yr (corresponding to the period of Jupiter's orbit). Then the solution of ideal, incompressible, linear MHD equations reveals that the periodic differential rotation reinforces Alfven oscillations of the fossil magnetic field in the radiative interior with period of 22 yr. It must be mentioned that the parametric resonance in Dynamo has been studied recently by Chiba & Tosa (1990), Hanasz et al. (1991), Schmitt & Rüdiger (1992).

In the present paper the compressibility of the medium is taken into account. Then linear MHD equations for slow and

Send offprint requests to: T. Zaqarashvili fast magnetosonic waves (hereafter SMW and FMW) are coupled due to the periodic differential rotation. The amplitude of SMW grows exponentially in time (with a period of 22 years). The waves propagating with $\pm 45^0$ angles ("main directions") to the vertical have maximal growth rates. The non-linear interaction of these resonant slow waves and nonresonant fast waves generates resonant harmonics of fast ones with great amplitude. The period of this FMW is shortest due to the large value of plasma β in the radiative interior. FMW propagating towards the surface run into unmagnetized convective envelope and transform into sound waves. Therefore p-modes may be generated in the solar radiative interior by large-amplitude oscillation of the fossil magnetic field.

Propagation of generated sound waves along differentially rotating convective envelope is investigated. The differential rotation influences the frequencies of the sound waves and changes their values (For example, Chagelishvili et al. 1994). This variation depends on the direction of the wave propagation with regard to the direction of the velocity inhomogeneity. Therefore the frequencies of the sound waves propagating along the above mentioned "main directions" (i.e. with ± 45 angles to the direction of inhomogeneity) have a different law of variation: one is increased and the other is decreased. It leads to the splitting of frequencies at the surface. The value of the splitting depends on the rate of differentiality. Therefore, the above mentioned periodical changing of the rate of the differential rotation causes the cyclic variation of the splitting.

In the second section the problem of excitation of resonant SMW and FMW in the radiative interior is investigated. In the third section the properties of sound waves propagating along differentially rotating convective envelopes are studied. In the last section remarks and conclusions are presented.

2. Resonant MHD waves in the radiative interior

It was obtained in paper I that the ellipticity of the Sun's motion around barycenter causes weak periodic (11 yr) differentiality in the rotation. This periodic shear of linear velocity generates resonant Alfven waves with a period of 22 yr. The influence of the periodic differential rotation

on the magnetosonic waves in the radiative interior is investigated in this section. A local Cartesian frame immovable with respect to the solar centre is used. The y-axis of the frame is directed along the tangent of the meridian toward the pole, the x-axis is toward the direction of the solar rotation, and the z-axis is normal to the solar surface.

2.1. Basic equations

The expression for linear velocity of the solar rotation at low latitudes was obtained as follows (see paper I):

$$V_x = V_{x0} + \alpha(t)z \tag{1}$$

where

$$\alpha = 2e cos\theta \Omega_0 sin(\Omega_0 t). \tag{2}$$

Here $\Omega_0 = 2\pi/t_0$ ($t_0 \approx 11$ yr), θ is the heliocentric latitude and e is the eccentricity of the Sun's "orbit" around barycenter. The origin of the frame is located at a fixed latitude, therefore θ is a constant parameter.

The fossil, unperturbed magnetic field is considered in the radiative interior. It has the toroidal component B_{x0} besides the poloidal one (Mestel & Weiss 1987; Charbonneau & MacGregor 1993).

Then linearized MHD equations have the following form:

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{V}_0 \cdot \boldsymbol{\nabla})\right] \boldsymbol{B}' = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{V}' + (\boldsymbol{B}' \cdot \boldsymbol{\nabla})\boldsymbol{V}_0 - \boldsymbol{B}_0 div \boldsymbol{V}', \tag{3a}$$

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{V}_0 \cdot \boldsymbol{\nabla})\right] \boldsymbol{V}' = -\frac{\boldsymbol{\nabla}}{\rho_0}\left[p' + \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{B}')}{4\pi}\right] + \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{B}'}{4\pi\rho_0} - (\boldsymbol{V}' \cdot \boldsymbol{\nabla})\boldsymbol{V}_0, \tag{3b}$$

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{V}_0 \cdot \boldsymbol{\nabla})\right] \rho' = -\rho_0 div \boldsymbol{V}', \tag{3c}$$

where ρ', p', $\boldsymbol{V}'$, $\boldsymbol{B}'$ are perturbations of the density, the pressure, the velocity and the magnetic field, respectively. $\boldsymbol{V}_0$ is the unperturbed velocity (Eq. (1)), $\boldsymbol{B}_0$ is the unperturbed magnetic field ($B_{x0},\ B_{y0}$) and ρ_0 is the density in the radiative interior.

Between pressure and density the simple adiabatic relation is taken:

$$p' = c_s^2 \rho', \tag{4}$$

where c_s is the sound speed.

The system of the differential equations (3) would be homogeneous without external periodic, inhomogeneous force, which is the reason for periodic shear of the velocity. In this case the dispersion relation can be easily obtained by normal modal theory. It has the following form (For example, Priest 1982):

$$\frac{\omega}{k} = \sqrt{\frac{1}{2}(c_s^2 + v_A^2) \pm \frac{1}{2}\sqrt{c_s^4 + v_A^4 - 2c_s^2 v_A^2 cos 2\theta_B}}, \tag{5}$$

where $v_A = B_0/\sqrt{4\pi\rho_0}$ is the Alfven speed and θ_B is the propagation angle. The sign + corresponds to FMW and the sign - corresponds to SMW. In the radiative interior $\beta = 8\pi p_0/B_0^2 \gg 1$ (i.e. $c_s^2/v_A^2 \gg 1$), therefore dispersion relations for fast and slow MHD waves have the forms:

$$\frac{\omega}{k} \approx c_s, \qquad \frac{\omega}{k} \approx v_A cos\theta_B \tag{6}$$

The difference in the frequencies of FMW and SMW depends on the value of β.

However, the system (3) is inhomogeneous in time and in the z coordinate. So the normal modal theory is not convenient for investigation of the stability of the system. Choosing the corresponding substitution of the new variables the space-dependent part of linearized MHD equations can be changed by temporal inhomogeneity. Then there is a possibility to study the temporal evolution of the spatial Fourier harmonics. This method was described in detail in paper I (also see Kelvin 1887; Chagelishvili et al. 1993 and other references in paper I).

By substitution of the new variables:

$$x_1 \;=\; x - V_{x0}t + 2ecos\theta cos(\Omega_0 t)z,\; y_1 = y,\; z_1 = z,$$
$$t_1 \;=\; t \tag{7}$$

and perform a Fourier analysis of Eqs. (3) with respect to x_1, y_1 and z_1:

$$\phi' \;=\; \int dk_{x_1}dk_{y_1}dk_{z_1}\hat{\phi}(k_{x_1},k_{y_1},k_{z_1},t_1) \times \exp\left[i(k_{x_1}x_1 + k_{y_1}y_1 + k_{z_1}z_1)\right] \tag{8}$$

the following system is obtained::

$$\frac{\partial \hat{b}_x}{\partial t_1} \;=\; iB_{y0}k_{y1}\hat{u}_x + \alpha\hat{b}_z - iB_{x0}[k_{y1}\hat{u}_y + K_z\hat{u}_z], \tag{9a}$$

$$\frac{\partial \hat{b}_z}{\partial t_1} \;=\; i(B_{x0}k_{x1} + B_{y0}k_{y1})\hat{u}_z, \tag{9b}$$

$$\frac{\partial \hat{u}_x}{\partial t_1} \;=\; -i\frac{k_{x1}c_s^2}{\rho_0}\hat{\rho} + i\frac{B_{y0}}{4\pi\rho_0 k_{y1}} \times \left[(k_{x1}^2 + k_{y1}^2)\hat{b}_x + k_{x1}K_z\hat{b}_z\right] - \alpha\hat{u}_z, \tag{9c}$$

$$\frac{\partial \hat{u}_y}{\partial t_1} \;=\; -i\frac{k_{y1}c_s^2}{\rho_0}\hat{\rho} - i\frac{B_{x0}}{4\pi\rho_0 k_{y1}} \times \left[(k_{x1}^2 + k_{y1}^2)\hat{b}_x + k_{x1}K_z\hat{b}_z\right], \tag{9d}$$

$$\frac{\partial \hat{u}_z}{\partial t_1} \;=\; -i\frac{K_z c_s^2}{\rho_0}\hat{\rho} + i\frac{1}{4\pi\rho_0 k_{y1}}\Big[K_z(B_{y0}k_{x1} - B_{x0}k_{y1})\hat{b}_x + (B_{x0}k_{x1}k_{y1} + B_{y0}k_{y1}^2 + B_{y0}K_z^2)\hat{b}_z\Big], \tag{9e}$$

$$\frac{\partial \hat{\rho}}{\partial t_1} \;=\; -i\rho_0\left[k_{x1}\hat{u}_x + k_{y1}\hat{u}_y + K_z\hat{u}_z\right], \tag{9f}$$

where $K_z = k_{z1} + 2k_{x1}ecos\theta cos(\Omega_0 t_1)$.

The system of the differential equations (9) describes the evolution of Fourier harmonics amplitudes represented by Eq. (8). From the expressions (7) and (8) the parameter determining the characteristic length of inhomogeneity of each Fourier harmonics along the axes x, y and z at any given moment of time can be written:

$$k_x = k_{x1},\; k_y = k_{y1},\; K_z = k_{z1} + 2k_{x1}ecos\theta cos(\Omega_0 t_1) \tag{10}$$

The wave number along the shear axis is a periodic function of time.

2.2. Analytical investigation

Magnetosonic waves are polarized in the plane, therefore for simplicity 2D case (x,z) is considered. It is assumed for a fossil magnetic field that $B_{y0} << B_{x0}$ (Mestel & Weiss 1987; Charbonneau & MacGregor 1993). Then from the system (9) one can get for $B_{y0} = 0$ and $k_x\hat{b}_x + K_z\hat{b}_z = 0$:

$$\frac{\partial \hat{b}_z}{\partial t} \;=\; ik_xB_{x0}\hat{u}_z, \tag{11a}$$

$$\frac{\partial \hat{u}_x}{\partial t} \;=\; -i\frac{k_xc_s^2}{\rho_0}\hat{\rho} - \alpha\hat{u}_z, \tag{11b}$$

$$\frac{\partial \hat{u}_z}{\partial t} \;=\; -i\frac{K_zc_s^2}{\rho_0}\hat{\rho} + i\frac{B_{x0}}{4\pi\rho_0 k_x}\left[k_x^2 + K_z^2\right]\hat{b}_z, \tag{11c}$$

$$\frac{\partial \hat{\rho}}{\partial t} \;=\; -i\rho_0[k_x\hat{u}_x + K_z\hat{u}_z]. \tag{11d}$$

If $\hat{u}_z$ from the Eq. (11a) substitute into Eq. (11c) and $\hat{u}_x$ from the Eq. (11d) substitute into Eq. (11b) using (11a) and (11c), one can obtain the following two differential equations (taking into account $\dot{K}_z = -k_x\alpha$ from the Eq. [10]):

$$\frac{\partial^2 \hat{b}_z}{\partial t^2} + V_A^2(k_x^2 + K_z^2)\hat{b}_z \;=\; k_xK_zB_{x0}c_s^2\frac{\hat{\rho}}{\rho_0}, \tag{12a}$$

$$\frac{\partial^2 \hat{\rho}}{\partial t^2} + c_s^2(k_x^2 + K_z^2)\hat{\rho} \;=\; \frac{B_{x0}}{4\pi}\frac{K_z(k_x^2 + K_z^2)}{k_x}\hat{b}_z + 2\frac{\rho_0\alpha}{B_{x0}}\frac{\partial \hat{b}_z}{\partial t}, \tag{12b}$$

These two equations are coupled. Equations of this type are well known in the general theory of oscillations (Magnus 1976; Kotkin & Serbo 1971). They describe the oscillation of two connected oscillator or mathematical pendulum. If the right hand side terms of these equations are independent in time then the initial conditions may be found in which the frequency of oscillations has one of the following values:

$$\omega = \sqrt{\frac{1}{2}(\omega_s^2 + \omega_A^2) \pm \frac{1}{2}\sqrt{(\omega_s^2 + \omega_A^2)^2 - 4\frac{k_x^2}{k_x^2 + k_z^2}\omega_s^2\omega_A^2}}, \tag{13}$$

where

$$\omega_s^2 = c_s^2(k_x^2 + k_z^2),\; \omega_A^2 = V_A^2(k_x^2 + k_z^2).$$

They are called as fundamental frequencies. In our case all terms are time-dependent, so that oscillation with one frequency is impossible. In any initial conditions the oscillation consists in two frequencies.

Using the approximation $V_A \ll c_s$, from Eqs. (12a) and (12b) one can obtain fourth order differential equation:

$$\frac{\partial^4 \hat{b}_z}{\partial t^4} + \frac{2k_x\alpha}{K_z}\frac{\partial^3 \hat{b}_z}{\partial t^3} + c_s^2(k_x^2 + K_z^2)\frac{\partial^2 \hat{b}_z}{\partial t^2} - 2k_x\alpha K_zc_s^2\frac{\partial \hat{b}_z}{\partial t} + V_A^2c_s^2k_x^2(k_x^2 + K_z^2)\hat{b}_z = 0 \tag{14}$$

There are two time scales in Eqs. (12): one corresponds to SMW and the other - to FMW. For long time scale (SMW) one can get approximately from the Eq. (14):

$$\frac{\partial^2 \hat{b}_z}{\partial t^2} - \frac{2k_x\alpha K_z}{(k_x^2 + K_z^2)}\frac{\partial \hat{b}_z}{\partial t} + V_A^2k_x^2\hat{b}_z = 0 \tag{15}$$

This equation is similar to Eq. (37) in paper I. It can be easily compared with Mathieu's equation. Its solutions are well known. The modes with wave number:

$$k_x = \frac{\Omega_0}{2V_A} \tag{16}$$

have exponentially growing characters in time:

$$\hat{b}_z(t) = \sqrt{2}a_0\sqrt{\frac{k_x^2+K_z^2(0)}{k_x^2+K_z^2(t)}}e^{|\delta|t/4\pi}cos\left[\frac{\Omega_0 t}{2}+\frac{\pi}{4}\right], \quad (17)$$

where

$$\delta = 4\pi\Omega_0 ecos\theta\frac{k_x{\cdot}k_z}{k_x^2+k_z^2} \quad (18)$$

and a_0 is the value of $\hat{b}_z(t)$ at t=0. From the expression (18) it is seen that modes with:

$$|k_z| = |k_x| \quad (19)$$

have maximal growth rates. Corresponding frequency of density perturbation from Eq. (12b) is then:

$$w_+ \approx \frac{\Omega_0}{\sqrt{2}}\frac{c_s}{V_A} \quad (20)$$

The density perturbation has highest frequency with regards to the magnetic field perturbation. The difference in frequencies depends on the ratio c_s/V_A, i.e. on the plasma β in the solar interior.

The linear coupling of FMW and SMW in shear flows was found recently (Chagelishvili et al. 1996). They considered an inhomogeneous basic flow with constant linear shear. They found the effect of linear transformation of the magnetosonic waves one into another. Two necessary conditions must be satisfied for energy exchange between the waves: $\beta \approx 1$ and the rate of shear must be smaller. In our case the second condition is followed immediately, but β is greatest in the radiative interior. Therefore the effect of linear transformation is inactive for this region. However amplitudes of SMW have exponentially growing characters in time. Therefore the linear theory is not valid when some time is gone. The problem requires non-linear analyses. Non-linear interaction of MHD waves has been studied intensively during the recent years (Nakariakov & Oraevsky 1995; Nakariakov et al. 1996; Nakariakov & Roberts 1996; Joarder et al. 1997; Nakariakov et al. 1997). They found that most effective non-linear coupling takes place when the frequencies and wavenumbers of the interactive waves are connected by the resonant conditions:

$$w_1 + w_2 = w_3,\ k_1 + k_2 = k_3 \quad (21)$$

In our case, the index "1" corresponds to SMW with dispersion relation $w_1 = k_1 V_A$, where $w_1 = \Omega_0/2$. It has an exponentially growing amplitude. The index "2" corresponds to FMW with dispersion relation $w_2 = -k_2 c_s$ and the index "3" to FMW with dispersion relation $w_3 = k_3 c_s$. Using these dispersion relations, for $k_1 = \Omega_0/2V_A$, the wavenumbers of resonant triplet can be obtained:

$$k_2 = \frac{V_A - c_s}{2c_s}k_1,\ k_3 = \frac{V_A + c_s}{2c_s}k_1 \quad (22)$$

Third FMW will also have an exponentially growing amplitude. So resonant SMW interacting with FMW, which propagates in the opposite direction, generates resonant FMW with $k_3 = (V_A + c_s)k_1/2c_s$. Detailed non-linear investigation will be made in future.

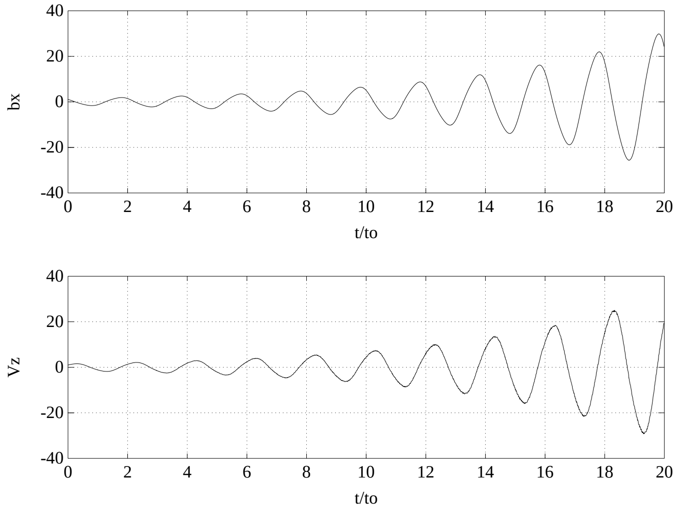

Fig. 1. Time evolution of the magnetic field and velocity perturbations in resonance case. $k_z = k_x = \frac{\Omega_0}{2V_A} - 0.1k_y,\ k_y \ll k_x$. Here $\beta = 2{\times}10^{10}$ and t_0=11 yr. The magnetic field has 22 yr period.

2.3. *Results of numerical simulation*

In searching a numerical solution to the ordinary differential equation system (9) it is convenient to work on the nondimensional form of the equations. The period of the shear t_0 = 11 yr is chosen for time to be unity. Variables and functions are nondimensionalized by writing:

$$v_x = \frac{i\hat{u}_x}{V_A},\ v_z = \frac{i\hat{u}_z}{V_A},\ b_x = \frac{\hat{b}_x}{B_0},\ v_y = \frac{i\hat{u}_y}{V_A},\ b_z = \frac{\hat{b}_z}{B_0},$$
$$\rho = \frac{\hat{\rho}}{\rho_0},\ \tau = t_1/t_0,\ k^1 = k_x l_0,\ k^2 = k_y l_0,\ k^3 = k_z l_0,$$
$$l_0 = V_A t_0,\ B_1 = \frac{B_{x0}}{B_0},\ B_2 = \frac{B_{y0}}{B_0}.$$

where $B_0 = \sqrt{B_{x0}^2 + B_{y0}^2}$. Analytical solution of such a 3D system for an incompressible medium (see paper I) revealed that resonant Alfven waves have a period two times larger than the period of shear i.e.

$$\omega_{AR} = \frac{(\boldsymbol{k}{\cdot}\boldsymbol{B}_0)}{B_0}V_A \approx \frac{\Omega_0}{2}. \quad (23)$$

Similar results were obtained analytically for SMW in 2D case in previous section. Numerical simulations of the linear differential equation system (9) reveals that it has exponentially growing solutions (see Fig. 1) for the wavenumbers satisfying the expression (23). This result was obtained in the following initial conditions, which correspond to initial SMW:

$$b_x = 1.0659,\ b_z = -1,\ v_x = -1.0659,$$
$$v_y = -1.0659,\ v_z = 1,\ \rho = 0.$$

Here $\gamma = 5/3$, $\beta = 2{\cdot}10^{10}$ and $B_{y0} = 0.1B_{x0}$.

The rate of the shear has a small value, therefore the instability region is small too. Solutions of the system have no growth rates for the wavenumbers slightly different from the expression

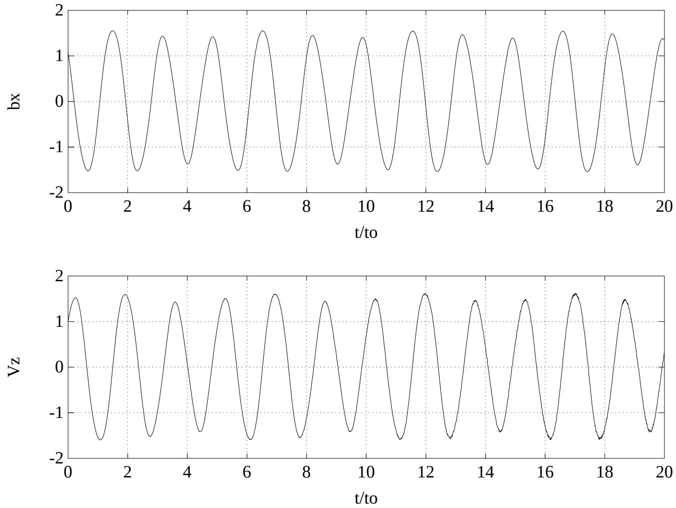

Fig. 2. Time evolution of the magnetic field and velocity perturbations in the case $k_z = k_x = 1.2\frac{\Omega_0}{2V_A} - 0.1k_y,\ k_y \ll k_x$. Here $\beta = 2{\times}10^{10}$ and t_0=11 yr.

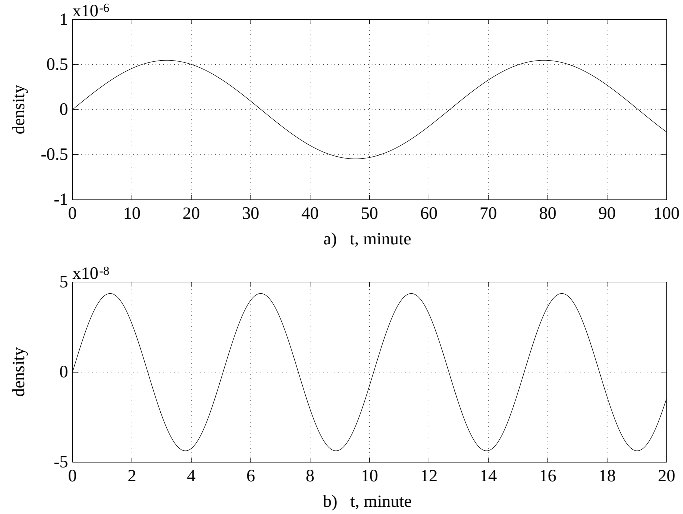

Fig. 3a and b. Short time interval for density perturbation in resonance case. **a** $\beta = 2{\times}10^{10}$ and time is normalized on 1 minute. The period of fast magnetosonic waves is ∼63 minutes. **b** $\beta = 3{\times}10^{12}$ and time is normalized on 1 minute. The period of fast magnetosonic waves is ∼5 min.

(23) (Fig. 2). This result was obtained in the initial conditions:

$$b_x = 1.0716,\ b_z = -1,\ v_x = -1.0716,$$
$$v_y = -1.0716,\ v_z = 1,\ \rho = 0.$$

The expression (23) includes the wavenumbers along x and y axes. Therefore the radial wavenumber k_z is a free parameter. However it influences the value of the growth rate. Numerical simulations show that the maximal growth rate occurs for $|k_y| << |k_x| \approx |k_z|$. So it may be concluded that resonant SMW with maximal growth rates propagate in the plane x0z with $\pm 45^0$ angles to the radial direction.

The velocity perturbation has two frequencies (see Fig. 1): low frequency (with the period of 22-year) corresponding to SMW and highest frequency corresponding to FMW. The period of fast magnetosonic waves according to the expression (20) is then (for $\beta = 2{\times}10^{10}$):

$$t_s \sim 63\ minute \tag{24}$$

The short interval of the solutions for the density perturbation corresponding to different values of β is presented in Fig. 3.

The period is same as analytically obtained by expression (20) (for $\beta = 2{\times}10^{10}$). For more large values of β ($\sim 3{\times}10^{12}$) the period is ∼5 minute. There is surprising coincidence between the periods of FMW and solar global oscillations. Therefore one can suggest that the source of these oscillations is the large-scale magnetic field in the radiative interior. FMW generated in this region run into unmagnetized convective envelope and transform into sound waves. For $\beta = 3{\times}10^{12}$ and the temperature $T_0 = 2{\times}10^6$ K one can evaluate the wavelength of resonant SMW from the expression (16). It is equal to:

$$\lambda = \frac{2\pi}{\sqrt{k_x^2 + k_y^2 + k_z^2}} \approx 0.1\ R_0 \tag{25}$$

This is not far from the spatial scales over which p-modes remain coherent. Numerical simulation of the non-linear MHD equations with periodic shear flows will be made in the future.

3. Splitting of p-mode frequency and its cyclic variation

As was obtained in the previous section FMW with a 5-minute period may be excited for $\beta = 3{\times}10^{12}$ in the solar radiative interior. During propagation towards the solar surface they run into the unmagnetized convective envelope and transform into sound waves. The propagation of the sound waves along differentially rotating unmagnetized medium is investigated in this section. Previously used local nonmodal analysis for 2D case without magnetic field is performed. The frame is the same as in the previous section.

3.1. Mathematical formulation

It is suggested that the convective envelope rotates differentially (for example, Durney 1996). The linear velocity has the following form:

$$V_x = V_{x0} + (\alpha_0 + \alpha(t))z \tag{26}$$

where $\alpha_0 = const$ is the gradient of the linear velocity and $\alpha(t)$ has the expression (2). The medium is considered as unmagnetized. By the procedure described in Sect. 2 the following system of differential equations is obtained:

$$\frac{\partial \hat{u}_x}{\partial t} = -i\frac{k_x c_s^2}{\rho_0}\hat{\rho} - (\alpha_0 + \alpha(t))\hat{u}_z, \tag{27a}$$

$$\frac{\partial \hat{u}_z}{\partial t} = -i\frac{K_z c_s^2}{\rho_0}\hat{\rho}, \tag{27b}$$

$$\frac{\partial \hat{\rho}}{\partial t} = -i\rho_0[k_x\hat{u}_x + K_z\hat{u}_z], \tag{27c}$$

Here

$$K_z = k_z - k_x \alpha_0 t + 2k_x e cos\theta cos\Omega_0 t. \quad (28)$$

The system (27) may be changed by a third order differential equation for time-dependent shear flow (Eq. (26)). An analytical investigation of the equation is difficult. However a second order differential equation may be obtained for the constant shear. It takes a few hours for the sound waves to pass along the convective envelope. The time-dependent term in the expression (26) has the longest period (11 yr). So it will be a good approximation if one ignores this term in the system (27) and then evaluate the influence of slight changes in the differential rotation on the sound wave frequency. In this case the expression of vertical wave number is:

$$K_z = k_z - k_x \alpha_0 t$$

The propagation of the sound waves in a simple plane Couette flow was investigated recently by Chagelishvili et al. (1994). From the system (27) the following equation may be obtained:

$$\frac{\partial^2 \hat{u}_x}{\partial t^2} + \omega^2(t)\hat{u}_x = -c_s^2 k_x K_z const \quad (29)$$

where $\omega^2(t) = c_s^2(k_x^2 + K_z^2)$ and const is some constant of integration. The quantity $\omega(t)$ has the meaning of the angular frequency of the oscillations. So it is seen that the frequency of the sound waves propagating along shear flows varies with time.

In the condition: $|\dot{\omega}(t)| \ll \omega^2(t)$ the solution of Eq. (29) is obtained (Chagelishvili et al. 1994). They found that the sound waves may extract energy from the shear flows. However in our case the rate of the shear and the propagation time are shorter, so the energy extraction is not significant.

3.2. *Splitting of p-mode frequency*

The frequency of the sound waves is expressed from Eq. (29) as follows:

$$\omega(t) = c_s\sqrt{k_x^2 + K_z^2} = c_s\sqrt{k_x^2 + (k_z - k_x\alpha_0 t)^2} \quad (30)$$

As was obtained in the second section the modes of resonant SMW with wave numbers $|k_x| = |k_z|$ have maximal growth rates. So excited sound waves propagate toward the surface with $\pm 45^0$ angles to the radial direction:i.e. they must have the horizontal wave numbers satisfying the expression

$$k_x \approx \pm k_z. \quad (31)$$

Then it is seen from the expression (30) that the frequencies of modes with wavenumbers (31) have different behaviours: one increases and the other decreases with time. Therefore it leads to the splitting of frequencies at the surface. If the rotation increases with depth then the coefficient of differentiality α_0 is negative. Therefore the frequency of p-modes with positive k_x increases and the frequency of p-modes with negative k_x decreases. The rate of the splitting depends on α_0 and on the propagation time. Let us roughly evaluate the time interval which needs for p-modes to pass along the convective envelope (width of H=2×10^{10} cm). If one takes the value of the temperature as 2×10^6 K at the upper part of the radiative interior and 6×10^3 K at the photosphere, then the temperature profile may be taken as $T(\xi) \approx T(0)e^{-5.8\xi}$, where $\xi = z/H$ (Priest 1982). Then after simple integration required time may be obtained as:

$$t_p \approx 90\ minute$$

Then it is possible to evaluate the differential rotation of the convective envelope by the observed splitting of p-mode frequency. From the expression (30) one can obtain:

$$\frac{\Delta\nu}{\nu} \approx \alpha_0 t_p \quad (32)$$

where ν is the frequency of p-modes, $\Delta\nu$ is splitting and $t_p \approx 90$ minute is the propagation time. Then the coefficient of the shear approximately is $\alpha_0 \sim 3.15\times 10^{-8}\ s^{-1}$ for observed splitting $\frac{\Delta\nu}{\nu} \sim 1.7\times 10^{-4}$ (Jimenez et al. 1994). It means that, the period of rotation is decreased by 0.5% at the depth of $2\cdot 10^{10}$ cm. This value slightly differs from the differential rotation theoretical evaluated by Durney (1996). Even such slight differential rotation leads to the observed splitting of p-mode frequencies.

The variation of the shear rate must influence the value of splitting. Time-dependent term in Eq. (26) has the following form at the equator:

$$\alpha(t) \approx 1.74\times 10^{-9} sin(\Omega_0 t)\ s^{-1} \quad (33)$$

Here $e \approx 0.048$ is taken for the eccentricity. Then the minimal and maximal values of the shear from (26) will be as follows:

$$\alpha_{0min} \approx 2.98\times 10^{-8}\ s^{-1}$$
$$\alpha_{0max} \approx 3.32\times 10^{-8}\ s^{-1}$$

Then one can easily evaluate the cyclic variation of the splitting from Eq. (32):

$$|\Delta\nu_{max} - \Delta\nu_{min}| \approx 0.11\Delta\nu \quad (34)$$

The result is in good agreement with observed cyclic variation of the splitting (Jimenez et al., 1994). Evolution of the magnetic field perturbation (i.e. cycle activity) and the variation of the differential rotation from Eq. (26) during several consequent cycles is presented in Fig. 4.

The splitting is maximal at the moment of maximal differentiality. Therefore it has its maximal value at the maximum of activity, as it is seen from Fig. 4. This result is also in good agreement with the observations (Jimenez et al. 1994).

4. Conclusion

Solar p-modes carry information about the solar internal structure. Therefore investigation of their properties has been a subject of scientific interest during recent years. The observation of the splitting of their frequencies was the origin of a new era in helioseismology. It was suggested that the splitting is caused by the reflection of p-modes on the rapidly rotating core. As was observed recently (Jimenez et al. 1994) the splitting has a

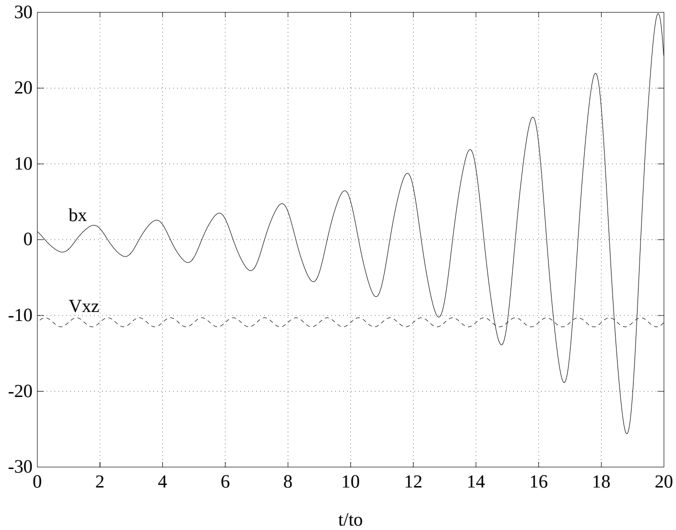

Fig. 4. Comparison of the magnetic field perturbation (from Fig. 1) and the rate of evaluated periodic differential rotation. The functions are nondimensionalized as $V_{xz} = t_0[\alpha_0 + \alpha(t)]$ and $b_x = \frac{\hat{b}_x}{B_0}$, where t_0=11 yr. It is clearly seen that the differentiality is maximal at the maximum of the magnetic field.

cyclic dependence: it is maximal during the maximum of solar activity. It requires significant variation of the solar core rotation during the activity cycle. However there is no physical mechanism supporting such variation.

A possible source of excitation, frequency splitting and cyclic variation of the splitting of solar p-modes is proposed in the present paper. Most of the phenomena occurring on the Sun have cyclic dependence. Therefore it can be suggested that they have a single origin. In paper I (Zaqarashvili 1997) it was supposed that weak 11 yr periodicity in the solar differential rotation (caused by the ellipticity of the Sun's motion around barycenter of the solar system) strengthens the Alfven oscillations with 22 yr period in the radiative interior. The linear coupling of FMW and SMW caused by periodic shear of unperturbed velocity is found. SMW have properties similar to the Alfven waves: the modes with 22 yr period grow exponentially in time. The corresponding FMW have shortest period. The period depends on the value of plasma β: it is equal to $\sim$5 minute for $\beta = 3{\times}10^{12}$. This phenomenon is similar to the problem of two connected mathematical pendulum, when one of them changes its length periodically. The amplitude of the pendulum with variable length grows exponentially (parametric resonance). The amplitude of the other pendulum has no growth rate in the linear approximation (in the case of significant deference in the periods of pendulums). But previous non-linear analyses show the energy transformation from resonant SMW to FMW. The resonant FMW with the wavenumber $k_3 = (V_A + c_s)k_1/2c_s$, where k_1 is wavenumber of resonant SMW, is generated by three-wave interaction. The modes propagating with $\pm 45^0$ angles to the radial direction have maximal growth rates. So there are two bands of the waves propagating towards and against the rotation. These FMW transform into sound waves in unmagnetized convective envelope.

The frequencies of these sound waves are changed during propagation along differentially rotating medium. The frequency of p-modes propagating in the direction of rotation is increased and the frequency of p-modes propagating in the direction opposite to that of rotation is decreased. So the splitting of frequencies appears at the surface. The rate of splitting depends on the value of differentiality and on the propagation time. The time required for p-modes to pass along convective envelope from the bottom ($2{\times}10^6$ K) to the surface ($6{\times}10^3$ K) is $\sim$90 minute. Then the shear of linear velocity was obtained from the observed splitting of frequencies:

$$\alpha_0 \approx 3.15{\times}10^{-8}\ s^{-1}$$

This is a small value. The period of rotation is decreased by 0.5% from the surface to the base of the convective zone at the depth of $2{\cdot}10^{10}$ cm. The rate of differential rotation differs slightly from the value obtained theoretically by Durney (1996) investigating its influence on the solar meridional motions.

As was supposed in paper I that solar differential rotation has a slight cyclic variation. The splitting depends on the rate of differentiality, therefore it must have a cyclic behaviour. Evaluated cyclic variation of the splitting is obtained as:

$$|\Delta\nu_{max} - \Delta\nu_{min}| \approx 0.11\Delta\nu$$

where $\Delta\nu_{max}$ is the maximal splitting and $\Delta\nu_{min}$ is the minimal one. This value is in good agreement with observed rate of splitting variation (Jimenez et al. 1994). It was found (Fig. 4) that the maximal splitting occurs at the maximum of the magnetic field as was shown by observation (Jimenez et al. 1994).

The first results of VIRGO (Fröhlich et al. 1997) show that solar activity enhances the excitation of oscillation. Also the amplitude depends on latitude and decreases towards the pole. It indicates that solar magnetic field may play a role in the excitation process of global oscillation.

If p-modes are generated by large-amplitude oscillation of the magnetic field in the radiative interior, then they must be coherent over the spatial scales corresponding to the wavelength of resonant Alfven and SMW. Solar p-modes remain coherent during 6-7 periods over the spatial scales $\sim 0.05\,R_0$. The wavelength of resonant SMW was evaluated as $\lambda \sim 0.1\,R_0$. It is seen that these two spatial scales are in good agreement.

The general opinion currently is that p-modes are reflected by the rapidly decreasing density as they travel toward the solar surface, and are turned around (refracted) by the increasing sound speed as they travel toward the solar centre. It was supposed that the observed splitting indicates the rapid rotation of the solar core. However recent observational analyses (For example, Lazrek et al. 1996) show rigid solar core rotation, contrary to the theory requiring faster rotation due to the angular momentum loss by the solar wind during the evolution of the Sun as a star of the main sequence. The mechanism of splitting presented in this paper does not exclude the existence of a rapidly rotating core. It also explains the cyclic variation of the splitting.

Another interesting point is the cyclic variation of p-mode frequencies. Many authors (Bogdan & Zweibel 1985; Zweibel

& Bogdan 1986; Cambrell & Roberts 1989; Evans & Roberts 1990; Goldreich et al. 1991; Jain & Roberts 1994a,b) suggest that the frequency variation is caused by the changing of the surface magnetic field during solar cycle. It is the most probable reason for this variation. However if p-modes are generated by the large-amplitude oscillation of the magnetic field in the radiative interior, then the cyclic variation of plasma β may influence their frequency. But evaluation of these changes by linear theory of perturbation is impossible. It must be studied by non-linear theory.

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